

Math 16A

Kouba

The Derivative of Sin x

FACTS: 1.) $\lim_{w \rightarrow 0} \frac{\sin w}{w} = 1$

2.) $\lim_{w \rightarrow 0} \frac{\cos w - 1}{w} = 0$

3.) $\sin(A + B) = \sin A \cos B + \cos A \sin B$

Let $f(x) = \sin x$. Then $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

$$= \lim_{\Delta x \rightarrow 0} \frac{\sin(x + \Delta x) - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\sin x \cdot \cos(\Delta x) + \cos x \cdot \sin(\Delta x) - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\sin x \cdot (\cos(\Delta x) - 1) + \cos x \cdot \sin(\Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \left\{ \sin x \cdot \left(\frac{\cos(\Delta x) - 1}{\Delta x} \right) + \cos x \cdot \left(\frac{\sin(\Delta x)}{\Delta x} \right) \right\}$$

$$= \sin x \cdot (0) + \cos x \cdot (1)$$

$$= \cos x$$