You have a small business which sells boxes of greeting cards. Assume that the demand x for boxes is inversely proportional to the square of the price p of a box of cards. If you charge \$20 per box, 125 boxes are sold. Your initial investment is \$750 and the cost to you for each box is \$5. Find the price p and the number of boxes x which will result in the maximum profit to you.

x: demand p: price

$$X = \frac{C}{p^2} \quad \text{and} \quad p = \$20, \ X = 125 \text{ boxes} \quad \text{so}$$

$$125 = \frac{C}{400} \rightarrow C = 50,000 \quad \text{so}$$

$$X = \frac{50,000}{p^2} \quad \text{or} \quad \text{price} \ p = \sqrt{\frac{50,000}{X}} \ ;$$

$$\text{cost} \ C = 750 + 5X \quad \text{so} \quad \text{profit}$$

$$P_r = (\text{revenue}) - (\text{cost})$$

$$= pX - (750 + 5X)$$

$$= \sqrt{\frac{50,000}{X}} \cdot X - 750 - 5X = \sqrt{50,000} \cdot \sqrt{X} - 750 - 5X \ ;$$

$$P_r' = \sqrt{50,000} \cdot \frac{1}{2\sqrt{X}} - 5 = 0 \rightarrow \cdots \rightarrow X = 500 \text{ boxes}$$

$$\frac{+ \quad 0 \quad -}{X = 500 \text{ boxes}} \ p'$$

$$p = \$10$$

and max. profit is

$$P_r = \$1750$$