Inflection Point — a Biological Interpretation

Ex: Assume that the number of elk in an isolated herd in Yellowstone National Park at time t (years) is given by

$$N(t) = 400 - \frac{4500}{t^2 + 30} \quad \text{for } t \geq 0 .$$

Note that $N(0) = 250$ so initially there are 250 elk in the herd. Since

$$\lim_{t \to +\infty} N(t) = 400 ,$$

we would expect that after a "long period of time" the number of elk in the herd would stabilize at 400. Differentiation gives

$$N'(t) = 4500\,(t^2 + 30)^{-2}(2t) = \frac{900t}{(t^2+30)^2} ,$$

$$\begin{array}{l} 0 \quad + \quad + \\ \hline t=0 \\ N = 250 \text{ elk} \end{array} \quad N'$$

; since the units of $N'(t)$ are elk per year, the following examples show the rate at which the herd is increasing at specific times:

$N'(1) = 9.4$ elk per year,
$N'(5) = 14.9$ elk per year,
$N'(10) = 5.3$ elk per year.

Differentiating again,

$$N''(t) = \frac{9000\,[30 - 3t^2]}{(t^2+30)^2}.$$

$+ \quad 0 \quad -$ $\;N''$

$t=0$ $\quad t = \sqrt{10} \approx 3.16$ years

$N \approx 287$ elk

$N' = 17.8$ elk/yr.

What is the biological significance of this inflection point? It represents the time at which the size of the elk herd is increasing most rapidly. The graph of $N(t)$ follows.