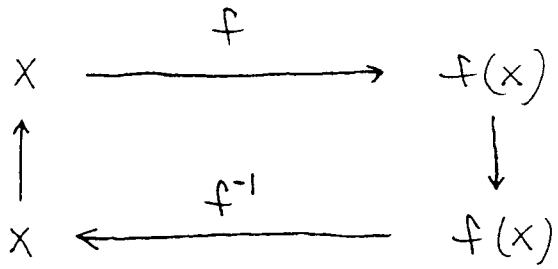
Inverse Function Summary

I. If $f(x)$ is a one-to-one function, then $f(x)$ has an inverse function, $f^{-1}(x)$, which satisfies:

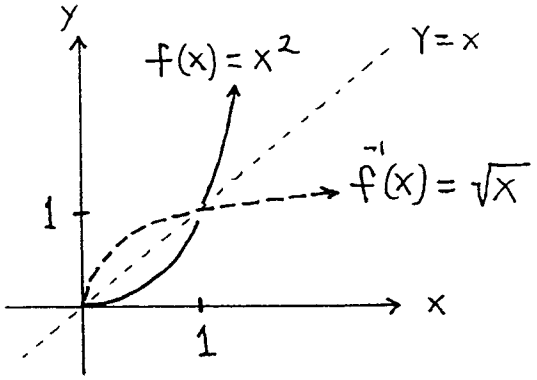
$$f\left(f^{-1}(x)\right) = x \quad \text{and} \quad f^{-1}\left(f(x)\right) = x .$$



a. The domain of $f^{-1}(x)$ is the range of $f(x)$.
b. The domain of $f(x)$ is the range of $f^{-1}(x)$.

<u>Ex</u>: Let $f(x) = x^2$ for $x \geq 0$, then

$$f^{-1}(x) = \sqrt{x} ;$$

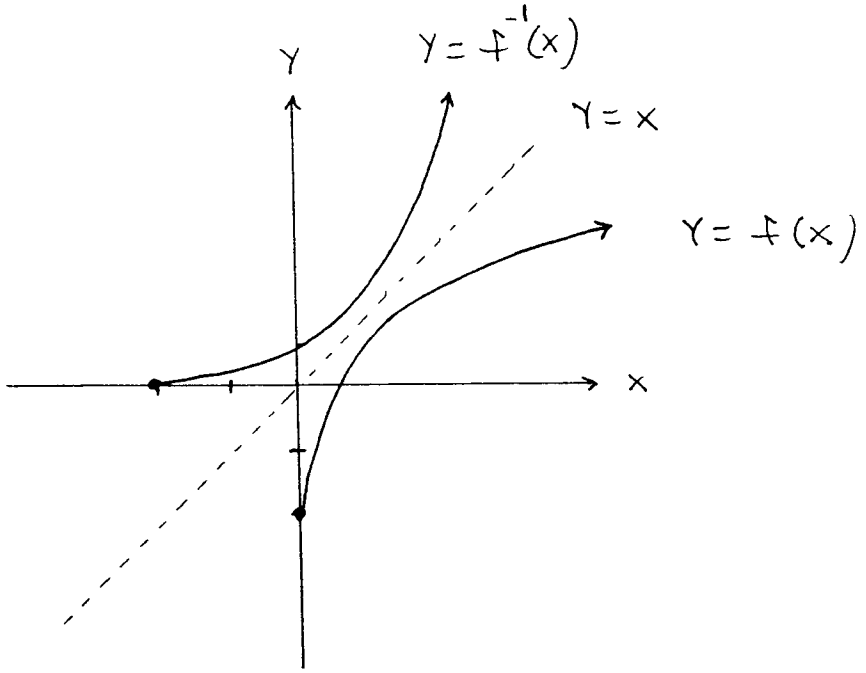


$f(3) = 9$ and $f^{-1}(9) = 3$

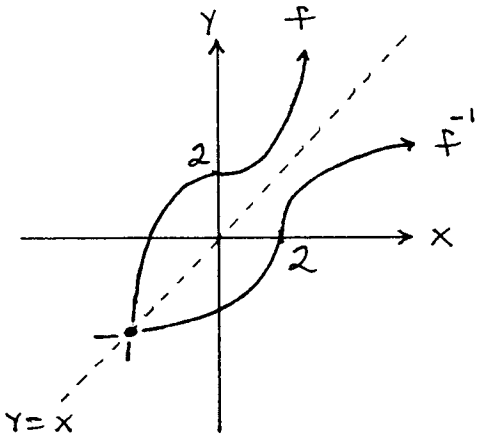
so that

$$f\left(f^{-1}(9)\right) = f(3) = 9 \quad \text{and} \quad f^{-1}\left(f(3)\right) = f^{-1}(9) = 3 .$$

II. If $f(x)$ and $f^{-1}(x)$ are graphed on the same axes, they are symmetric with respect to the line $Y = X$



Ex: Graph $f(x) = x^5 + 2$ to verify that it is one-to-one, and find its inverse:



$f(f^{-1}(x)) = x$ so that

$(f^{-1}(x))^5 + 2 = x \rightarrow$

$(f^{-1}(x))^5 = x - 2 \rightarrow$

$f^{-1}(x) = (x-2)^{1/5}$.

Extra Problems: Find $f^{-1}(x)$ for

1.) $f(x) = 7 - \frac{2}{x}$, where $x \neq 0$

2.) $f(x) = \frac{x+3}{x-1}$, where $x \neq 1$

3.) $f(x) = x^2 - 6x$, where $x \leq 3$