

1.) (15 pts.) Use  $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$  to determine the derivative of  $f(x) = x^2 - 3x + 4$ .

$$\begin{aligned}
 f'(x) &= \lim_{\Delta x \rightarrow 0} \frac{((x + \Delta x)^2 - 3(x + \Delta x) + 4) - (x^2 - 3x + 4)}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x \cdot \Delta x + (\Delta x)^2 - 3x - 3 \cdot \Delta x + 4 - x^2 + 3x - 4}{\Delta x} \\
 &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x \cdot (2x + \Delta x - 3)}{\Delta x} \\
 &= 2x + 0 - 3 \quad \text{so} \quad f'(x) = 2x - 3
 \end{aligned}$$

2.) Consider the function  $f(x) = \frac{x}{x-2}$  and assume that its derivative is  $f'(x) = \frac{-2}{(x-2)^2}$ .

a.) (2 pts.) What is  $f(4)$ ?  $f(4) = \frac{4}{4-2} = 2$

b.) (2 pts.) What is  $f'(4)$ ?  $f'(4) = \frac{-2}{(4-2)^2} = -\frac{1}{2}$

c.) (5 pts.) Find an equation of the line which is tangent to the graph of  $f$  at  $x = 4$ .

$x = 4, y = 2$  and tangent slope is  $m = f'(4) = -\frac{1}{2}$   
 so  $y - 2 = -\frac{1}{2}(x - 4)$  OR  $y = -\frac{1}{2}x + 4$

d.) (6 pts.) Find an equation of the line which is perpendicular to the graph of  $f$  at  $x = 4$ .

$x = 4, y = 2$  and  $\perp$  slope is  $m = 2$   
 (negative reciprocal of  $f'(4) = -\frac{1}{2}$ ) so  
 $y - 2 = 2(x - 4)$  OR  $y = 2x - 6$