

Math 16A

Kouba

Derive the Product Rule Using the Limit definition of the Derivative

Let  $F(x) = f(x)g(x)$  . It's derivative is

$$\begin{aligned} F'(x) &= \lim_{\Delta x \rightarrow 0} \frac{F(x + \Delta x) - F(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x)g(x + \Delta x) - f(x)g(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x)g(x + \Delta x) - f(x + \Delta x)g(x) + f(x + \Delta x)g(x) - f(x)g(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \left[ \frac{f(x + \Delta x)g(x + \Delta x) - f(x + \Delta x)g(x)}{\Delta x} + \frac{f(x + \Delta x)g(x) - f(x)g(x)}{\Delta x} \right] \\ &= \lim_{\Delta x \rightarrow 0} \left[ f(x + \Delta x) \cdot \frac{g(x + \Delta x) - g(x)}{\Delta x} + \frac{f(x + \Delta x) - f(x)}{\Delta x} \cdot g(x) \right] \\ &= f(x) \cdot g'(x) + f'(x) \cdot g(x) \\ &= f(x) \cdot g'(x) + f'(x) \cdot g(x) \end{aligned}$$