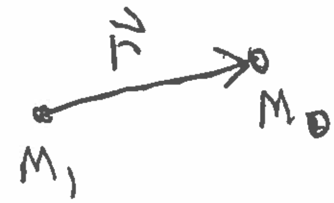
Incorporating Newton's Theory of gravity into the continuum theory of the compressible Euler Equations –

- Newtons Law for point masses:

$$\vec{F} = -G \frac{M_0 M_1}{r^2} \frac{\vec{r}}{r} \qquad (2 \text{ masses})$$

$\vec{r}$ pts betw the masses,



$$\vec{F} = M_0 \cdot \vec{a}$$

- Check: $\nabla \frac{1}{r} = -\frac{\vec{r}}{r^3}$

$$\phi = -GM_1 \frac{1}{r} \Rightarrow \vec{a} = -\nabla\phi \qquad \phi = \text{grav. potential}$$

In empty space, $\Delta\phi = -\operatorname{div}\nabla\phi = 0$

General Newton Law in empty space

$$\boxed{\Delta\phi = 0}$$

- When $\exists$ many masses pulling on M_0,

$$\vec{F} = M_0 \vec{a} = \sum_i - \frac{G M_i}{r_i^2} \frac{\vec{r}_i}{r_i}$$

In the continuum limit of a density of mass $\rho(\underset{\sim}{x})$, we get (Assume $\rho(x)$ compactly supported)

$$\phi(\underset{\sim}{x}) = \int_{\mathbb{R}^3} \frac{G}{|\underset{\sim}{x} - \underset{\sim}{y}|} \rho(\underset{\sim}{y})\, d^3y$$

$$\vec{a} = -\nabla \phi = \int_{\mathbb{R}^3} \frac{G}{|\underset{\sim}{x} - \underset{\sim}{y}|^3} (\underset{\sim}{x} - \underset{\sim}{y}) \rho(\underset{\sim}{y})\, d\underset{\sim}{y}$$

$\Rightarrow$ Take Laplacian of singular integral

$$\boxed{-\Delta \phi = 4\pi G \rho} \qquad (N)$$

continuum version of Newton's Law Grav.

- Classical Problem: (N) does not provide equations for how the masses ρ evolves – Assume $\rho, \vec{v}$ evolve by compressible Euler equations ... then

$$\rho_t + \mathrm{div}(\rho \vec{v}) = 0$$

$$(\rho v^i)_t + \mathrm{div}(\rho v^i \vec{v} + p e^i) = \underbrace{-\rho(\nabla \phi)^i}_{\substack{\text{gravitational}\\ \text{force that}\\ \text{changes the}\\ \text{momentum}}}$$

$$-\Delta \phi = 4\pi G \rho$$

- Einstein equations couple the gravitational field to the equations of motion thru

$$G = \frac{8\pi G}{c^4} T$$

$$\mathrm{div}\, T = 0 \qquad \text{(rel compressible Euler eqns)}$$