☑ Math 180
Introduction to General Relativity
Winter 2018
Blake Temple MSB 3148
Bainer 1060 10-10:50
Office Hrs: MWF 2:30-3:30 Appt

☑ Einstein 1915 (Introduction) $G = KT$

Einstein Curvature Tensor

Const of Nature $K = \frac{8\pi G}{c^4}$

stress Energy Tensor

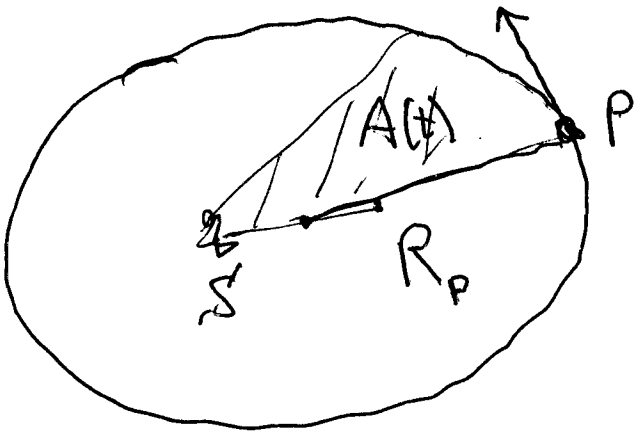
"Energy & the flow of energy creates spacetime curvature" ($E = mc^2$ so everything converts to energy)

T measures: "Energy density momentum density & their fluxes $\equiv$ $\frac{\text{energy}}{\text{vol}}$, $\frac{\text{energy}}{\text{area time}}$"

G measures spacetime curvature.

☒ Picture: Einstein's Theory replaced Newton's Laws as the fundamental explanation for gravity —

Q: Why do the planets orbit the sun?



Kepler's 3 Laws: (1609)

(1) Planets move in elliptical orbits around the Sun with sun at focus of ellipse

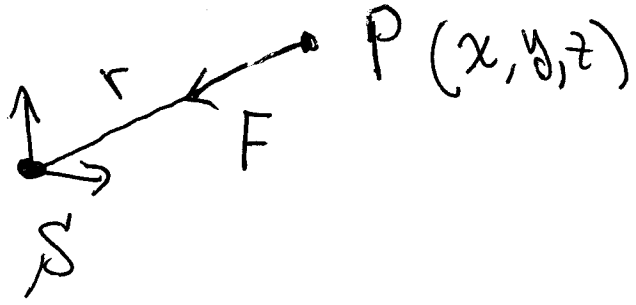
(2) Planets sweep out equal area in equal time

$$\frac{dA}{dt} = \text{const}$$

(3) $\frac{T_p^2}{R_p^3} = \text{Const indept of planet.}$

• Newton 1687 (Principia): Derived Kepler's Laws from principle that the sun was pulling on the planets with an inverse square force ≈ spring (Hooke's idea)

Newton Force Law $\vec{r}(t) = \overrightarrow{(x(t), y(t), z(t))}$ position of planet



$$\vec{F} = -G \frac{M_S M_P}{r^2} \frac{\vec{r}}{r}$$

"Inverse square force pointing from planet back toward sun"

$$\vec{F} = M_P \cdot \vec{a}, \quad \vec{a} = \ddot{\vec{r}} = \frac{d^2}{dt^2}\left[\vec{r}(t)\right]$$

$$M_P \ddot{\vec{r}} = -G \frac{M_S M_P}{r^3} \vec{r}$$

Newton: $\boxed{\ddot{\vec{r}} = -GM_s \frac{\vec{r}}{r^3}}$

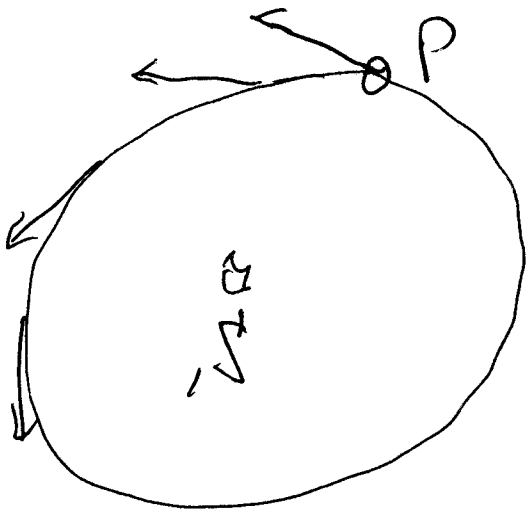
"The equation for motion of planet is independent of the planet!"

I.e., a feather & the earth follow the same path if given same initial condt's!"

I.e. "The sun pulls on every object with a different force (depending on mass), but it all works out so every object follows the same path!"

Einsteins Idea - Makes more sense that the sun is creating the paths, not producing forces like springs.

- Einstein: The sun is curving the space around it and the planet is moving in a straight line in a curved space



free fall path is an ellipse!

Q: How do you make sense of this?

"The space itself is the unknown since the space creates the curves"

Q: What are the equations that determine the space?

Answer: Its not space, but spacetime that is curved.

the equations (1915 after 9 years of struggle)

$$G = \frac{8\pi t}{c^4} T$$

Q: Whats the unknown?

Ans: The gravitational metric tensor

$g_{ij}(\vec{x}, t)$ ← 4×4 symmetric matrix
⇒ 10 unknown fuctions depending on $(\vec{x}, t)$

$ij = 0, 1, 2, 3$

$\underset{\sim}{x} = (t, \vec{x})$

Q: How many pages of 12 pt font text to write out equations w/o using any summation sign's etc? ≈ 250 pages!

- Q: How do you understand something so complicated?

Ans: Tensors for notation

Geometry for understanding their meaning

Nutshell: $\frac{\partial^2}{\partial x^i \partial x^j} g_{ij} = T(S, P, U)$

$\frac{\partial^2}{\partial x^i \partial x^j} g_{ij}$: 2nd derivatn of metric tensor

measure Curvature

$T(S, P, U)$: T measure energy density & $\frac{\text{mom}}{\text{vol}} = \frac{\text{energy}}{\text{area time}}$

Topics:

1. Metric / Manifolds / Tensors
2. Special Relativity
3. Connection
4. Curvature (Riemann)
5. Einsteins Equation $G = \kappa T$
6. Schwarzschild metric (Black Holes)
7. Cosmology

Idea: Present the Correct framework (tensors) to understan Einsteins theory, But skip technical details.