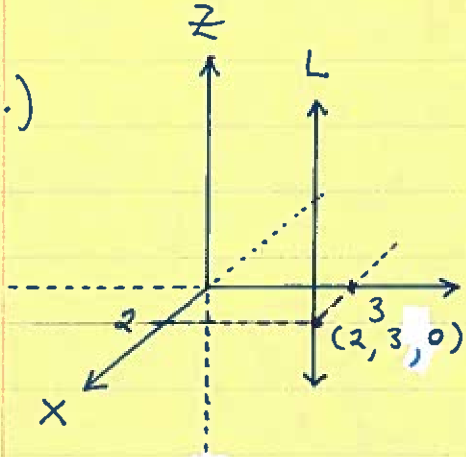
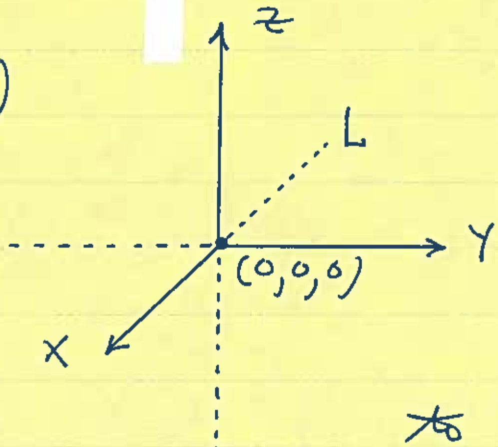
Section 12.1

1.)



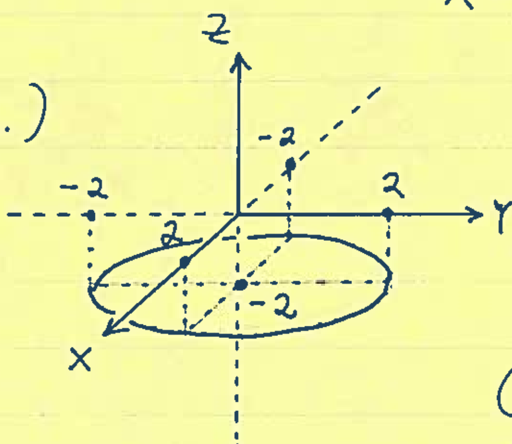
The set of points with $X=2$ and $Y=3$ is the line L passing through the point $(2,3,0)$ and parallel to the z-axis

3.)



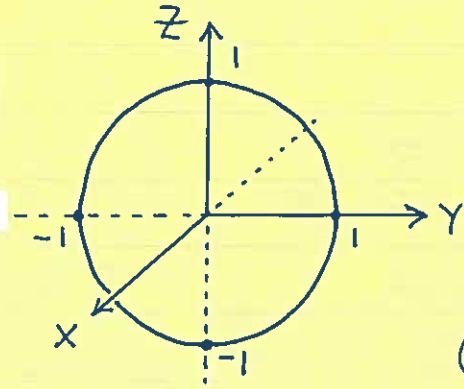
The set of points with $Y=0$ and $Z=0$ is the line L passing through the point $(0,0,0)$ and parallel to the X-axis (L is the X-axis)

6.)



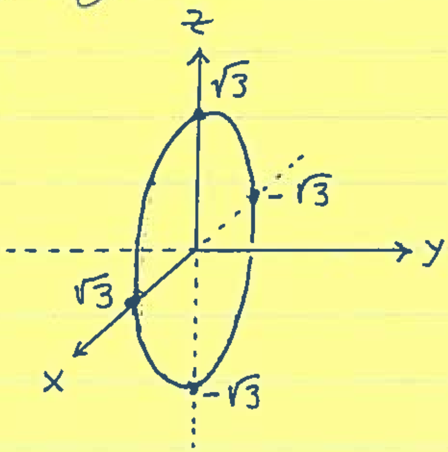
The set of points with $X^2+Y^2=4$ and $Z=-2$ is the set of points on the circle $X^2+Y^2=4$ (center $(0,0)$, radius 2) lying in ~~the~~ plane $Z=-2$ (parallel to the XY-plane)

8.)



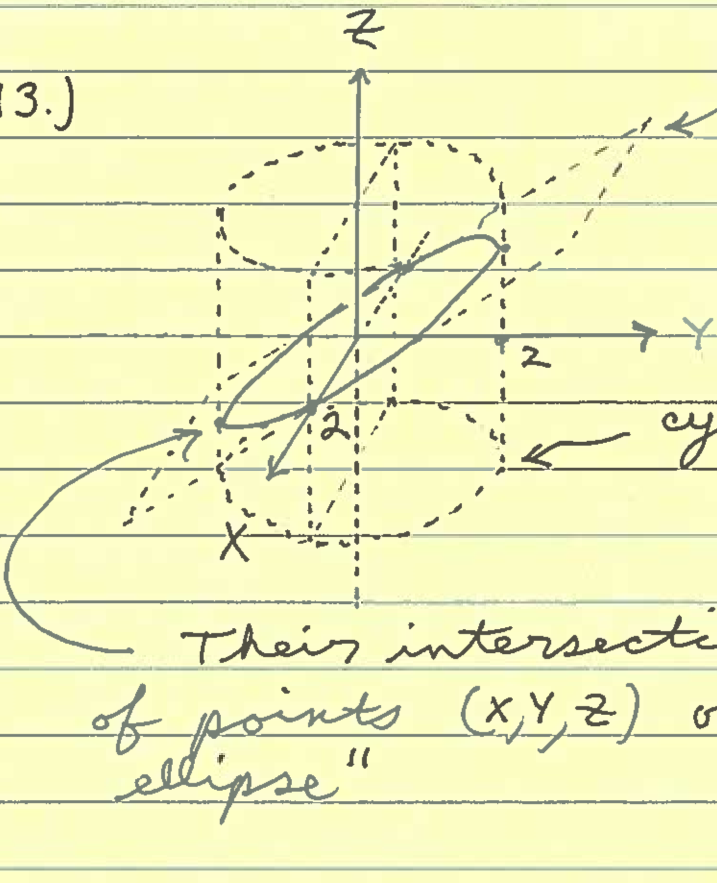
The set of points with $Y^2+Z^2=1$ and $X=0$ is the set of points on the circle $Y^2+Z^2=1$ (center $(0,0)$, radius 1) lying in the plane $x=0$ (yz-plane)

12.)



The set of points with $x^2+(y-1)^2+z^2=4$ and $y=0 \rightarrow x^2+(-1)^2+z^2=4$ $\rightarrow x^2+z^2=3=(\sqrt{3})^2$ is the set of points lying on the circle $x^2+z^2=3$ (center $(0,0)$, radius $\sqrt{3}$) and in the plane $y=0$ (xz-plane)

13.)

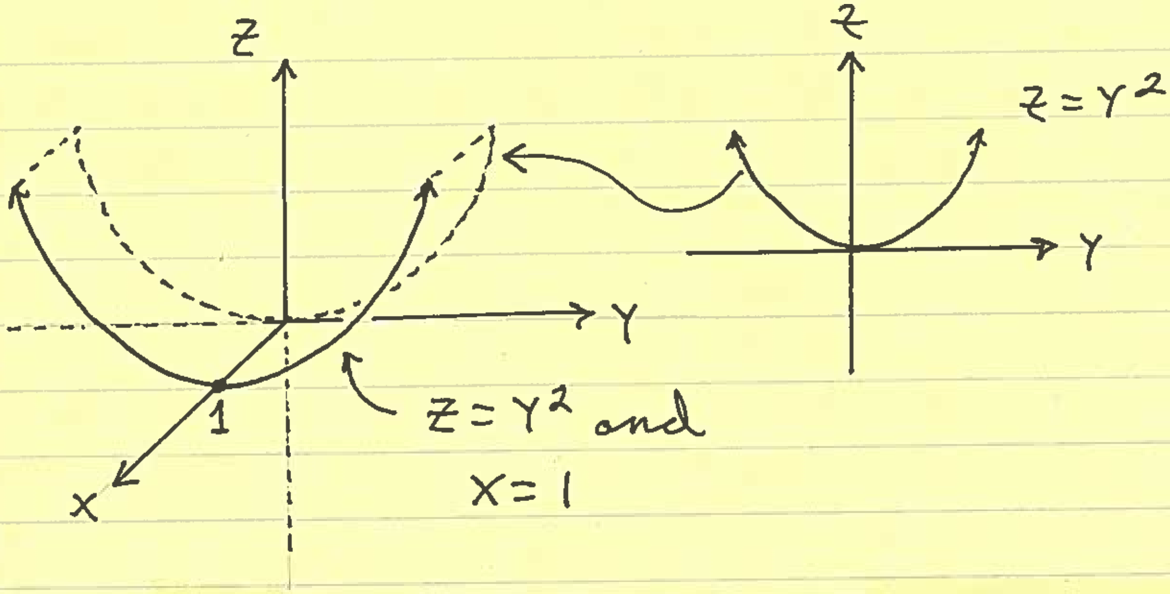


tilted plane $z=y$

cylinder of radius 2 : $x^2+y^2=2^2$

Their intersection is the set of points (x,y,z) on the "tilted ellipse"

16.)

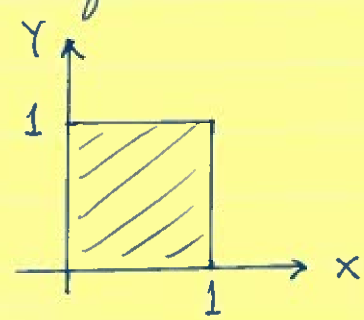


17.) a.) $X \geq 0$, $Y \geq 0$, $Z = 0$: The set of points in the 1st quadrant of the XY-plane

b.) $X \geq 0$, $Y \leq 0$, $Z = 0$: The set of points in the 4th quadrant of the XY-plane

18.) a.) $0 \leq X \leq 1$: The set of points lying on and between the parallel planes $X = 0$ (YZ-plane) and $X = 1$.

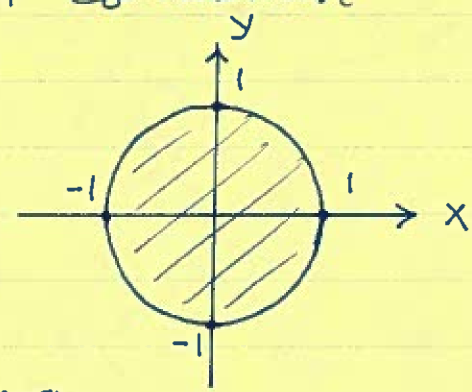
b.) $0 \leq X \leq 1$, $0 \leq Y \leq 1$: The set of points on and inside the vertical (parallel to Z-axis) square column passing through the given 1 by 1 square in the XY-plane.



c.) $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$: The set of points on and inside the 1 by 1 by 1 cube in the 1st octant.

20.) a.) $x^2 + y^2 \le 1$, $z = 0$: The set of points lying on and inside the circle $x^2 + y^2 = 1$ (center $(0,0)$, radius 1) in the plane $z = 0$ (xy-plane)

c.) $x^2 + y^2 \le 1$: The set of points on and inside the vertical (parallel to z-axis) circular column passing through the given circle of radius 1



21.) b.) $x^2 + y^2 + z^2 = 1$, $z \ge 0$: The set of points lying on or inside the top half of the sphere $x^2 + y^2 + z^2 = 1$ (center $(0,0,0)$, radius 1)

22.) a.) $x = y$, $z = 0$: The set of points lying on the line $x = y$ in the plane $z = 0$ (xy-axis)

b.) $x = y$: The set of points on the plane passing through the line $x = y$ (in the xy-plane) and parallel to the z-axis.

26.) a.) $X=3$ b.) $Y=-1$ c.) $Z=2$

27.) a.) $Z=1$ b.) $X=3$ c.) $Y=-1$

28.) a.) $X^2+Y^2=2^2$, $Z=0$
b.) $Y^2+Z^2=2^2$, $X=0$
c.) $X^2+Z^2=2^2$, $Y=0$

30.) a.) $(X+3)^2+(Y-4)^2=1^2$, $Z=1$
b.) $(Y-4)^2+(Z-1)^2=1^2$, $X=-3$
c.) $(X+3)^2+(Z-1)^2=1^2$, $Y=4$

31.) a.) $Y=3$, $Z=-1$
b.) $X=1$, $Z=-1$
c.) $X=1$, $Y=3$

32.) all points (X,Y,Z) equidistant from $(0,0,0)$ and $(0,2,0)$:

$$\sqrt{(X-0)^2+(Y-0)^2+(Z-0)^2}=\sqrt{(X-0)^2+(Y-2)^2+(Z-0)^2}$$

$$\rightarrow \cancel{X^2}+Y^2+\cancel{Z^2}=\cancel{X^2}+(Y-2)^2+\cancel{Z^2}$$

$$\rightarrow \cancel{Y^2}=\cancel{Y^2}-4Y+4 \rightarrow 4Y=4 \rightarrow$$

$\boxed{Y=1}$ (a plane parallel to the XZ-plane)

34.) all points (X,Y,Z) 2 units from $(0,0,1)$ and 2 units $(0,0,-1)$:

$$\sqrt{(x-0)^2+(y-0)^2+(z-1)^2} = 2 \quad \text{and}$$

$$\sqrt{(x-0)^2+(y-0)^2+(z+1)^2} = 2 \longrightarrow$$

$$x^2+y^2+(z-1)^2 = 4 \quad \text{and}$$

$$x^2+y^2+(z+1)^2 = 4 \longrightarrow$$

$$\cancel{x^2}+\cancel{y^2}+(z-1)^2 = \cancel{x^2}+\cancel{y^2}+(z+1)^2 \longrightarrow$$

$$\cancel{z^2}-2z+\cancel{1} = \cancel{z^2}+2z+\cancel{1} \longrightarrow 4z=0$$

$$\longrightarrow \quad z=0 \ ; \ \cancel{\text{then}}$$

$$x^2+y^2+(0-1)^2 = 4 \longrightarrow$$

$$\boxed{x^2+y^2 = 3 \quad \text{and} \quad z=0}$$

36.) $0 \le x \le 2,\ 0 \le y \le 2,\ 0 \le z \le 2$

37.) $z \le 0$

38) $x^2+y^2+z^2=1$ and $z \ge 0$

39.) a.) $(x-1)^2+(y-1)^2+(z-1)^2 < 1^2$

b.) $(x-1)^2+(y-1)^2+(z-1)^2 > 1^2$

42.) $D = \sqrt{(2-\text{-}1)^2+(5-1)^2+(0-5)^2}$

$= \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$

43.) $D = \sqrt{(4-1)^2+(-2-4)^2+(7-5)^2}$

$= \sqrt{9+36+4} = \sqrt{49} = 7$

47.) $(x-(-2))^2+(y-0)^2+(z-2)^2=(2\sqrt{2})^2$
$\rightarrow$ center $(-2,0,2)$, radius $2\sqrt{2}$

52.) $(x-0)^2+(y-(-1))^2+(z-5)^2=2^2 \rightarrow$
$x^2+(y+1)^2+(z-5)^2=4$

55.) $x^2+y^2+z^2+4x-4z=0 \rightarrow$
$(x^2+4x+\underline{\underline{4}})+y^2+(z^2-4z+\underline{\underline{4}})=4+4 \rightarrow$
$(x+2)^2+y^2+(z-2)^2=8=(2\sqrt{2})^2 \rightarrow$
center $(-2,0,2)$, radius $2\sqrt{2}$

58.) $3x^2+3y^2+3z^2+2y-2z=9 \rightarrow$
$3x^2+(3y^2+2y\quad)+(3z^2-2z\quad)=9 \rightarrow$
$3x^2+3(y^2+\frac{2}{3}y\quad)+3(z^2-\frac{2}{3}z\quad)=9 \rightarrow$
$x^2+(y^2+\frac{2}{3}y\quad)+(z^2-\frac{2}{3}z\quad)=3 \rightarrow$
$x^2+(y^2+\frac{2}{3}y+\frac{1}{9})+(z^2-\frac{2}{3}z+\frac{1}{9})=3+\frac{1}{9}+\frac{1}{9} \rightarrow$
$(x-0)^2+(y+\frac{1}{3})^2+(z-\frac{1}{3})^2=\frac{29}{9}=\left(\frac{\sqrt{29}}{3}\right)^2 \rightarrow$
center $(0,-1/3,1/3)$, radius $\frac{\sqrt{29}}{3}$

59.) a.) Distance from (x,y,z) and point $(x,0,0)$ (on the x-axis):

$$D=\sqrt{(x-x)^2+(y-0)^2+(z-0)^2}$$

$$=\sqrt{y^2+z^2}.$$

b.) Distance from (X,Y,Z) and point $(0,Y,0)$ (on the Y-axis):

$$D=\sqrt{(X-0)^2+(Y-Y)^2+(Z-0)^2}$$

$$=\sqrt{X^2+Z^2}$$

c.) Distance from (X,Y,Z) and point $(0,0,Z)$ (on the Z-axis):

$$D=\sqrt{(X-0)^2+(Y-0)^2+(Z-Z)^2}$$

$$=\sqrt{X^2+Y^2}$$

60.) a.) Distance from (X,Y,Z) and point $(X,Y,0)$ (on XY-plane):

$$D=\sqrt{(X-X)^2+(Y-Y)^2+(Z-0)^2}$$

$$=\sqrt{Z^2}=|Z|$$

b.) Distance from (X,Y,Z) and point $(0,Y,Z)$ (on YZ-plane):

$$D=\sqrt{(X-0)^2+(Y-Y)^2+(Z-Z)^2}=\sqrt{X^2}=|X|$$

c.) Distance from (X,Y,Z) and point $(X,0,Z)$ (on XZ-plane):

$$D=\sqrt{(X-X)^2+(Y-0)^2+(Z-Z)^2}=\sqrt{Y^2}=|Y|$$

62.) Distance P to A $= \sqrt{(3-2)^2+(1-(-1))^2+(2-3)^2}$

$= \sqrt{1+4+1} = \sqrt{6}$,

Distance P to B $= \sqrt{(3-4)^2+(1-3)^2+(2-1)^2}$

$= \sqrt{1+4+1} = \sqrt{6}$

63.) The set of points on the plane $Y=1$ is equidistant from the planes $Y=-1$ and $Y=3$.

64.) Let (x,y,z) be equidistant from point $(0,0,2)$ and the xy-plane:

Distance (x,y,z) to $(0,0,2) = \sqrt{(x-0)^2+(y-0)^2+(z-2)^2}$

$= \sqrt{x^2+y^2+(z-2)^2}$;

Distance (x,y,z) to xy-plane $= z \rightarrow$

$z = \sqrt{x^2+y^2+z^2-4z+4} \rightarrow$

$\cancel{z^2} = x^2+y^2+\cancel{z^2}-4z+4 \rightarrow$

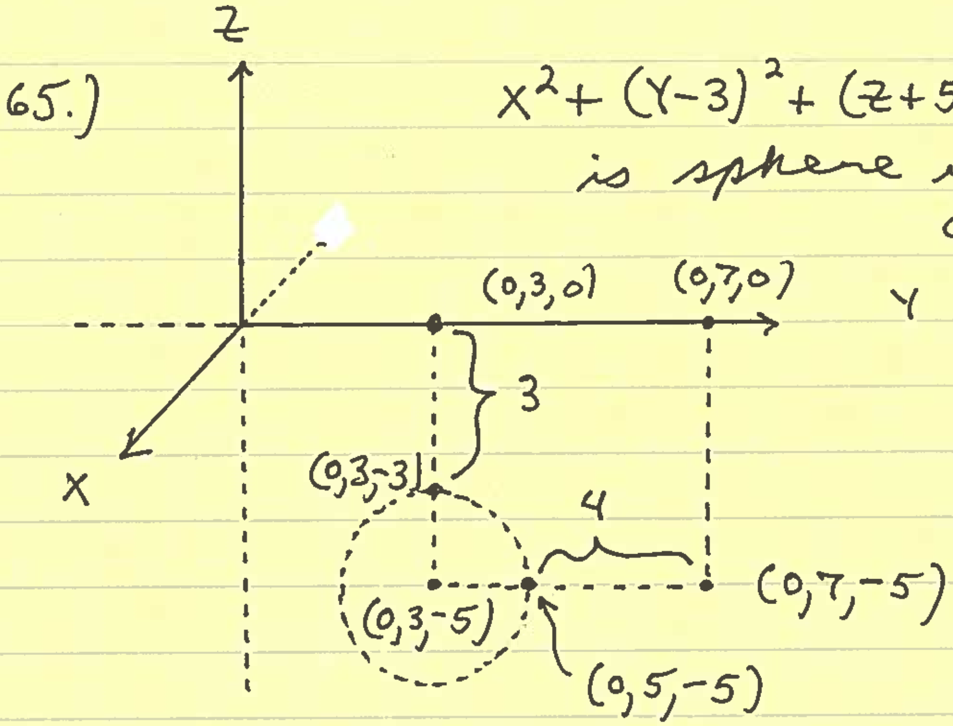
$4z = x^2+y^2+4 \rightarrow$

$z = \frac{1}{4}(x^2+y^2+4)$.

65.)

$$x^2 + (y-3)^2 + (z+5)^2 = 2^2$$

is sphere with center $(0,3,-5)$, radius $r = 2$



a.) The distance from sphere to xy-plane is 3

b.) The distance from sphere to point $(0,7,-5)$ is 4

66.) Distance (x,y,z) to $(0,0,0)$ is

$$L_1 = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2} = \sqrt{x^2 + y^2 + z^2};$$

Distance (x,y,z) to $(0,4,0)$ is

$$L_2 = \sqrt{(x-0)^2 + (y-4)^2 + (z-0)^2} = \sqrt{x^2 + (y-4)^2 + z^2};$$

Distance (x,y,z) to $(3,0,0)$ is

$$L_3 = \sqrt{(x-3)^2 + (y-0)^2 + (z-0)^2} = \sqrt{(x-3)^2 + y^2 + z^2};$$

Distance (x,y,z) to $(2,2,-3)$ is

$$L_4=\sqrt{(x-2)^2+(y-2)^2+(z+3)^2} \quad ;$$

Set $L_1 = L_2 \longrightarrow$

$$\sqrt{x^2+y^2+z^2}=\sqrt{x^2+(y-4)^2+z^2} \longrightarrow$$

$$\cancel{x^2}+\cancel{y^2}+\cancel{z^2}=\cancel{x^2}+\cancel{y^2}-8y+16+\cancel{z^2} \longrightarrow$$

$$8y=16 \rightarrow \boxed{y=2} \quad ;$$

Set $L_1 = L_3 \longrightarrow$

$$\sqrt{x^2+y^2+z^2}=\sqrt{(x-3)^2+y^2+z^2} \longrightarrow$$

$$\cancel{x^2}+\cancel{y^2}+\cancel{z^2}=\cancel{x^2}-6x+9+\cancel{y^2}+\cancel{z^2} \longrightarrow$$

$$6x=9 \rightarrow x=\frac{9}{6} \rightarrow \boxed{x=\frac{3}{2}} \quad ;$$

Set $L_1 = L_4 \longrightarrow$

$$\sqrt{x^2+y^2+z^2}=\sqrt{(x-2)^2+(y-2)^2+(z+3)^2} \longrightarrow$$

$$\cancel{x^2}+\cancel{y^2}+\cancel{z^2}=\cancel{x^2}-4x+4+\cancel{y^2}-4y+4+\cancel{z^2}+6z+9 \longrightarrow$$

$$0=-4\left(\frac{3}{2}\right)+\cancel{4}-\cancel{4}(2)+\cancel{4}+6z+9 \longrightarrow$$

$$0=-6+9+6z \longrightarrow 6z=-3 \longrightarrow$$

$$z=\frac{-3}{6} \longrightarrow \boxed{z=\frac{-1}{2}} .$$