Section 12.3

1.) $\vec{v} = \overrightarrow{\langle 2, -4, \sqrt{5} \rangle}$, $\vec{u} = \overrightarrow{\langle -2, 4, -\sqrt{5} \rangle}$

a.) $\vec{v} \cdot \vec{u} = (-4) + (-16) + (-5) = -25$,

$|\vec{u}| = \sqrt{4+16+5} = \sqrt{25} = 5$,

$|\vec{v}| = \sqrt{4+16+5} = \sqrt{25} = 5$.

b.) $\cos\theta = \dfrac{\vec{v} \cdot \vec{u}}{|\vec{v}||\vec{u}|} = \dfrac{-25}{(5)(5)} = -1 \; (\to \theta = \pi)$

c.) $|\vec{u}| \cos\theta = (5)(-1) = -5$

d.) $\text{proj}_{\vec{v}} \vec{u} = \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} = \dfrac{-25}{(5)^2} \vec{v}$

$= -\vec{v} = \overrightarrow{\langle -2, 4, -\sqrt{5} \rangle}$

4.) $\vec{v} = \overrightarrow{\langle 2, 10, -11 \rangle}$, $\vec{u} = \overrightarrow{\langle 2, 2, 1 \rangle}$

a.) $\vec{v} \cdot \vec{u} = 4 + 20 + (-11) = 13$,

$|\vec{u}| = \sqrt{4+4+1} = \sqrt{9} = 3$,

$|\vec{v}| = \sqrt{4+100+121} = \sqrt{225} = 15$

b.) $\cos\theta = \dfrac{\vec{v} \cdot \vec{u}}{|\vec{v}||\vec{u}|} = \dfrac{13}{(15)(3)} = \dfrac{13}{45}$

c.) $|\vec{u}| \cos\theta = (3) \cdot \left(\dfrac{13}{45} \right) = \dfrac{13}{15}$

d.) $\text{proj}_{\vec{v}} \vec{u} = \left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2} \right) \vec{v} = \dfrac{13}{(15)^2} \vec{v}$

$= \dfrac{13}{225} \overrightarrow{\langle 2, 10, -11 \rangle} = \overrightarrow{\left(\dfrac{26}{225}, \dfrac{130}{225}, \dfrac{-143}{225} \right)}$

7.) $\vec{v} = \overrightarrow{(5,1)}$, $\vec{u} = \overrightarrow{(2,\sqrt{17})}$

a.) $\vec{v} \cdot \vec{u} = 10 + \sqrt{17}$,

$|\vec{u}| = \sqrt{4+17} = \sqrt{21}$,

$|\vec{v}| = \sqrt{25+1} = \sqrt{26}$

b.) $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{10+\sqrt{17}}{\sqrt{21}\sqrt{26}} = \dfrac{10+\sqrt{17}}{\sqrt{546}}$

c.) $|\vec{u}|\cos\theta = \cancel{\sqrt{21}} \cdot \dfrac{10+\sqrt{17}}{\cancel{\sqrt{21}}\sqrt{26}} = \dfrac{10+\sqrt{17}}{\sqrt{26}}$

d.) $\text{proj}_{\vec{v}}\vec{u} = \left(\dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v}$

$= \dfrac{10+\sqrt{17}}{26}\vec{v} = \overrightarrow{\left(\dfrac{5(10+\sqrt{17})}{26}, \dfrac{10+\sqrt{17}}{26}\right)}$

10.) $\vec{u} = \overrightarrow{(2,-2,1)}$, $\vec{v} = \overrightarrow{(3,0,4)}$, so

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{6+0+4}{\sqrt{9}\cdot\sqrt{25}} = \dfrac{10}{(3)(5)} = \dfrac{2}{3} \rightarrow$

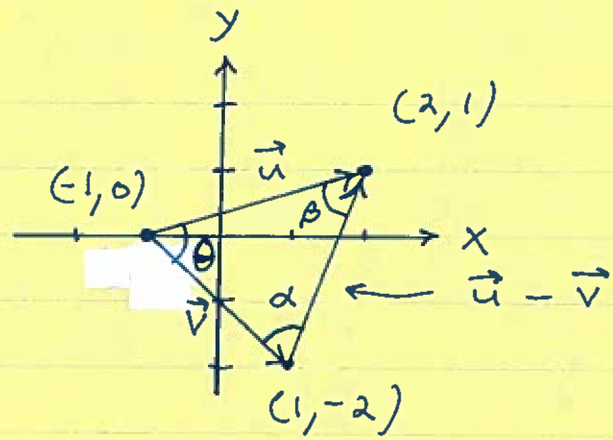
$\theta = \arccos\dfrac{2}{3} \approx 0.84$ radians

12.) $\vec{u} = \overrightarrow{(1,\sqrt{2},-\sqrt{2})}$, $\vec{v} = \overrightarrow{(-1,1,1)}$, so

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{-1+\sqrt{2}-\sqrt{2}}{\sqrt{5}\sqrt{3}} = \dfrac{-1}{\sqrt{15}} \rightarrow$

$\theta = \arccos\left(\dfrac{-1}{\sqrt{15}}\right) \approx 1.83$ radians

13.)



$\vec{u} = \overrightarrow{(3,1)}$, $\vec{v} = \overrightarrow{(2,-2)}$, $\vec{u}-\vec{v} = \overrightarrow{(1,3)}$;

angle between $\vec{u}$ and $\vec{v}$ satisfies

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{6-2}{\sqrt{10}\cdot\sqrt{8}} = \frac{4}{\sqrt{80}} = \frac{1}{\sqrt{5}} \longrightarrow$$

$$\theta = \arccos\frac{1}{\sqrt{5}} \approx 63.4^\circ \quad ;$$

angle between $-\vec{v}$ and $\vec{u}-\vec{v}$ satisfies

$$\cos\alpha = \frac{-\vec{v}\cdot(\vec{u}-\vec{v})}{|-\vec{v}||\vec{u}-\vec{v}|} = \frac{\overrightarrow{(-2,2)}\cdot\overrightarrow{(1,3)}}{|\overrightarrow{(-2,2)}||\overrightarrow{(1,3)}|}$$

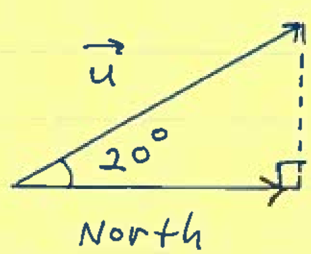
$$= \frac{-2+6}{\sqrt{8}\,\sqrt{10}} = \frac{4}{\sqrt{80}} = \frac{1}{\sqrt{5}} \longrightarrow$$

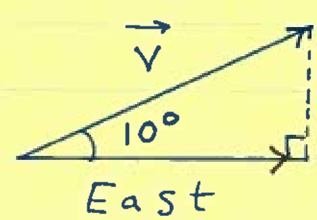
$$\alpha = \arccos\frac{1}{\sqrt{5}} \approx 63.4^\circ \quad ;$$

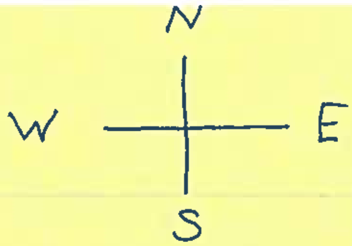
$$\theta + \alpha + \beta = 180^\circ \longrightarrow$$

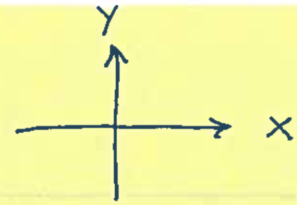
$$\beta = 180^\circ - \alpha - \theta \approx 53.2^\circ$$

16.)

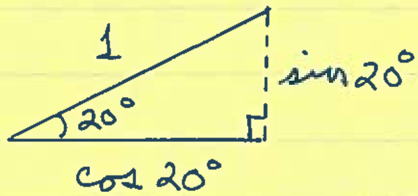


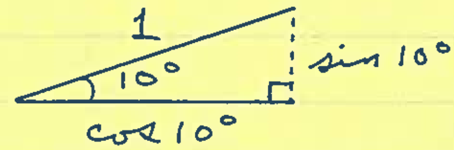






assume $\vec{u}$ and $\vec{v}$ are unit vectors in space. Then





so that

$$\vec{u} = 0\,\vec{i} + (\cos 20°)\,\vec{j} + (\sin 20°)\,\vec{k}$$

and

$$\vec{v} = (\cos 10°)\,\vec{i} + 0\cdot\vec{j} + (\sin 10°)\,\vec{k} \quad ;$$

the angle θ in the water main is the angle between

$-\vec{u}$ and $\vec{v}$

(since tails must be together !!)

so

$$\cos\theta = \frac{(-\vec{u})\cdot\vec{v}}{|-\vec{u}|\,|\vec{v}|} = \frac{0 + 0 - (\sin 20°)(\sin 10°)}{(1)\ (1)} \longrightarrow$$

$$\theta = \arccos(-\sin 20° \cdot \sin 10°) \approx 93.4°$$

17.) $\vec{v}_1 + \vec{v}_2 \perp \vec{v}_1 - \vec{v}_2$ iff

$(\vec{v}_1 + \vec{v}_2)\cdot(\vec{v}_1 - \vec{v}_2) = 0$ iff

$\vec{v}_1\cdot\vec{v}_1 - \vec{v}_1\cdot\vec{v}_2 + \vec{v}_2\cdot\vec{v}_1 - \vec{v}_2\cdot\vec{v}_2 = 0$ iff

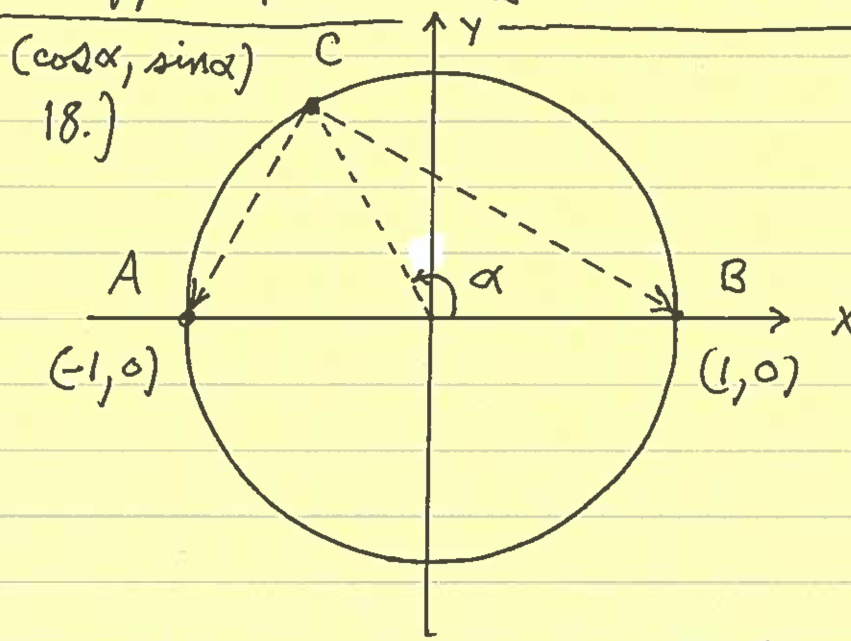
$|\vec{v}_1|^2 - \cancel{\vec{v}_1\cdot\vec{v}_2} + \cancel{\vec{v}_1\cdot\vec{v}_2} - |\vec{v}_2|^2 = 0$ iff

iff $|\vec{v}_1|^2 = |\vec{v}_2|^2$

iff $|\vec{v}_1| = |\vec{v}_2|$,

iff $\vec{v}_1$ and $\vec{v}_2$ have the same length

18.)



Assume radius $r = 1$.

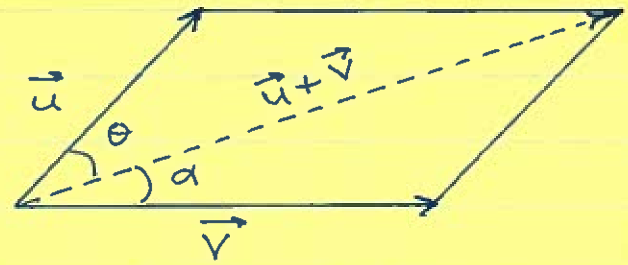
$\overrightarrow{CA} = \overrightarrow{(-1-\cos\alpha,\ 0-\sin\alpha)} = \overrightarrow{(-1-\cos\alpha,\ -\sin\alpha)}$,

$\overrightarrow{CB} = \overrightarrow{(1-\cos\alpha,\ 0-\sin\alpha)} = \overrightarrow{(1-\cos\alpha,\ -\sin\alpha)}$;

show $\overrightarrow{CA} \perp \overrightarrow{CB}$, i.e., show $\overrightarrow{CA} \cdot \overrightarrow{CB} = 0$:

$$\overrightarrow{CA} \cdot \overrightarrow{CB} = (-1-\cos\alpha)(1-\cos\alpha) + (-\sin\alpha)(-\sin\alpha)$$
$$= -1 + \cancel{\cos\alpha} - \cancel{\cos\alpha} + \cos^2\alpha + \sin^2\alpha$$
$$= -1 + 1 = 0$$

22.) Show that angles $\theta = \alpha$ if $|\vec{u}| = |\vec{v}|$:



$$\cos\theta = \frac{\vec{u}\cdot(\vec{u}+\vec{v})}{|\vec{u}|\,|\vec{u}+\vec{v}|} = \frac{\vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{u}+\vec{v}|}$$

$$= \frac{|\vec{u}|^2 + \vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{u}+\vec{v}|} \quad ;$$

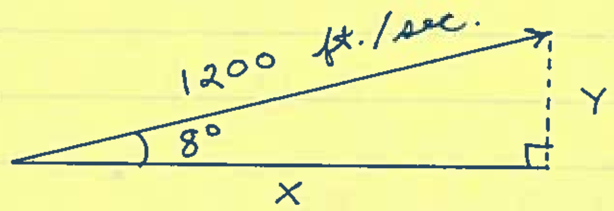
$$\cos\alpha = \frac{\vec{v}\cdot(\vec{u}+\vec{v})}{|\vec{v}|\,|\vec{u}+\vec{v}|} = \frac{\vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v}}{|\vec{v}|\,|\vec{u}+\vec{v}|}$$

$$= \frac{\vec{u}\cdot\vec{v} + |\vec{v}|^2}{|\vec{v}|\,|\vec{u}+\vec{v}|} = \frac{\vec{u}\cdot\vec{v} + |\vec{u}|^2}{|\vec{u}|\,|\vec{u}+\vec{v}|}$$

(since $|\vec{u}| = |\vec{v}|$) ;

then $\cos\theta = \cos\alpha \longrightarrow \theta = \alpha$.

23.)



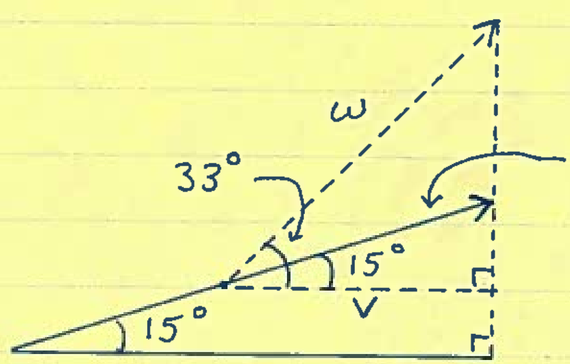
$\cos 8° = \dfrac{X}{1200} \longrightarrow X = 1200 \cos 8°$

≈ 1188.32 ft./sec. ;

$\sin 8° = \dfrac{Y}{1200} \longrightarrow Y = 1200 \sin 8°$

≈ 167.01 ft. /sec.

24.)



we have 2.5 lbs. of force here

$\cos 15° = \dfrac{V}{2.5} \longrightarrow V = 2.5 \cos 15°$;

$\cos 33° = \dfrac{V}{w} \longrightarrow$

$w = \dfrac{V}{\cos 33°} = \dfrac{2.5 \cos 15°}{\cos 33°} \approx 2.88$ lbs.

25.) a.) $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta \longrightarrow$

$|\vec{u} \cdot \vec{v}| = \big|\, |\vec{u}||\vec{v}| \cos\theta \,\big|$

$= |\vec{u}||\vec{v}| \cdot |\cos\theta|$

$\leq |\vec{u}||\vec{v}| \cdot (1) \longrightarrow$

$|\vec{u} \cdot \vec{v}| \leq |\vec{u}||\vec{v}|$.

6.) If $|\cos\theta| = 1$, then

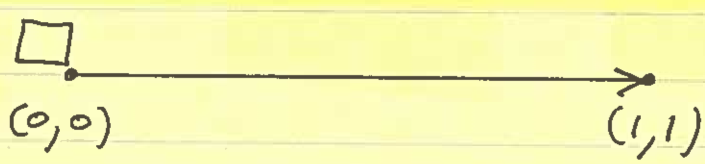
$$|\vec{u}\cdot\vec{v}| = |\vec{u}||\vec{v}|\cdot|\cos\theta|$$
$$= |\vec{u}||\vec{v}|\cdot(1)$$
$$= |\vec{u}||\vec{v}|.$$

This equality holds if $\theta = 0°$ (vectors have same direction) or if $\theta = 180°$ (vectors have opposite direction)

27.) Assume $\vec{u}_1$ is $\perp$ to $\vec{u}_2$ and $|\vec{u}_1| = 1 = |\vec{u}_2|$. If $\vec{v} = a\vec{u}_1 + b\vec{u}_2$, then

$$\vec{v}\cdot\vec{u}_1 = (a\vec{u}_1 + b\vec{u}_2)\cdot\vec{u}_1$$
$$= a\cdot(\vec{u}_1\cdot\vec{u}_1) + b\cdot(\vec{u}_2\cdot\vec{u}_1)$$
$$= a\,|\vec{u}_1|^2 + b\cdot(0) \quad (\text{since } \vec{u}_2 \perp \vec{u}_1)$$
$$= a(1)^2 = a$$

41.)

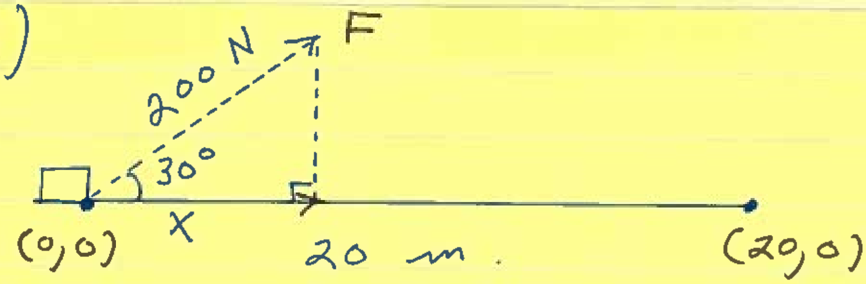


$$\vec{D} = \overrightarrow{(1-0,\, 1-0)} = \overrightarrow{(1,1)}$$

$$\vec{F} = \overrightarrow{(5,0)} \quad \text{so}$$

$$\text{Work} = \vec{F}\cdot\vec{D} = \overrightarrow{(5,0)}\cdot\overrightarrow{(1,1)} = 5+0 = 5 \text{ joules}$$

43.)



$$\cos 30^\circ = \frac{X}{200} \longrightarrow X = 200 \cos 30^\circ$$
$$= 200\left(\frac{\sqrt{3}}{2}\right) = 100\sqrt{3} \text{ N};$$

$$\text{Work} = (\text{force})(\text{distance})$$
$$= (100\sqrt{3})(20) \approx 3464 \text{ joules}$$

OR using vectors:

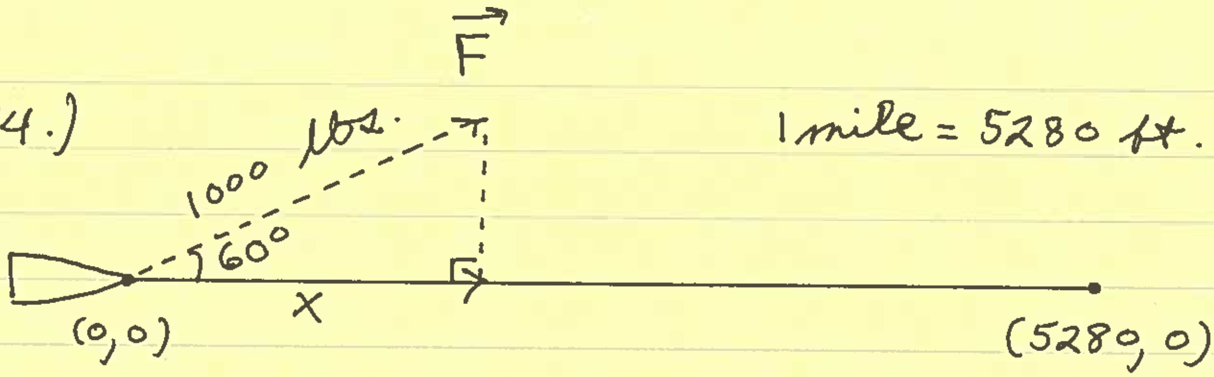
$$\vec{F} = \overrightarrow{(200 \cos 30^\circ, 200 \sin 30^\circ)}$$
$$= \overrightarrow{(200 \cdot \sqrt{3}/2,\ 200 \cdot 1/2)}$$
$$= \overrightarrow{(100\sqrt{3}, 100)} \text{ and}$$

$$\vec{D} = \overrightarrow{(20-0, 0-0)} = \overrightarrow{(20, 0)}, \text{ so}$$

$$\text{work} = \vec{F} \cdot \vec{D} = \overrightarrow{(100\sqrt{3}, 100)} \cdot \overrightarrow{(20, 0)}$$
$$= (100\sqrt{3})(20) + (100)(0) \approx 3464 \text{ joules}$$

44.)

1 mile = 5280 ft.



$$\cos 60° = \frac{x}{1000} \longrightarrow x = 1000 \cdot \cos 60°$$
$$= 1000\left(\tfrac{1}{2}\right) = 500 \text{ lbs.};$$

Work = (force)(distance)

$$= (500)(5280) = 2{,}640{,}000 \text{ ft.-lbs.}$$

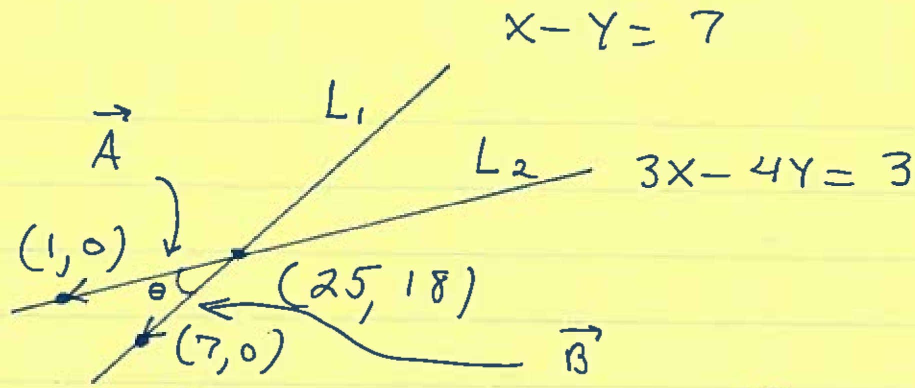
OR using vectors:

$$\vec{F} = \overrightarrow{(1000\cos 60°, 1000 \sin 60°)}$$
$$= \overrightarrow{\left(1000 \cdot \tfrac{1}{2}, 1000 \cdot \sqrt{3}/2\right)}$$
$$= \overrightarrow{(500, 500\sqrt{3})} \quad \text{and}$$

$$\vec{D} = \overrightarrow{(5280 - 0, 0 - 0)} = \overrightarrow{(5280, 0)}, \text{ so}$$

$$\text{Work} = \vec{F} \cdot \vec{D}$$
$$= \overrightarrow{(500, 500\sqrt{3})} \cdot \overrightarrow{(5280, 0)}$$
$$= (500)(5280) + (500\sqrt{3})(0)$$
$$= 2{,}640{,}000 \text{ ft.-lbs.}$$

49.)



$$\left.\begin{array}{r} 3x - 4y = 3 \\ x - y = 7 \end{array}\right\} \quad \left.\begin{array}{r} 3x - 4y = 3 \\ -4x + 4y = -28 \end{array}\right\}$$

$-x = -25 \rightarrow x = 25,\ y = 18$ is pt. of intersection ; pt. $(7,0)$ is on line L_1 and pt. $(1,0)$ is on line L_2 ; form vectors $\vec{A}$ and $\vec{B}$ and find angle θ between them :

$\vec{A} = \overrightarrow{(-24, -18)}$ and $\vec{B} = \overrightarrow{(-18, -18)}$, so

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|\,|\vec{B}|} = \frac{432 + 324}{\sqrt{900} \cdot \sqrt{648}} = \frac{756}{30\sqrt{648}} \rightarrow$$

$$\theta = \arccos\left(\frac{756}{30\sqrt{648}}\right) \approx 8.13^\circ$$