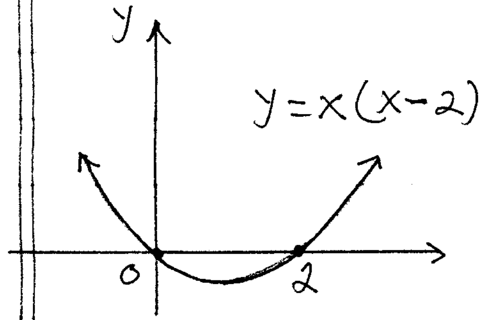
Section 13.1

1.) $\vec{r}(t) = (t+1)\vec{i} + (t^2-1)\vec{j} \rightarrow$

$\begin{cases} x = t+1 \rightarrow t = x-1 \\ y = t^2-1 \end{cases}$ (SUB) $\rightarrow y = (x-1)^2 - 1 \rightarrow$

$y = x^2 - 2x + \cancel{1} - \cancel{1} \rightarrow y = x(x-2)$ (parabola);



$\vec{v}(t) = \vec{r}\,'(t) = 1\cdot\vec{i} + 2t\cdot\vec{j} \xrightarrow{D}$

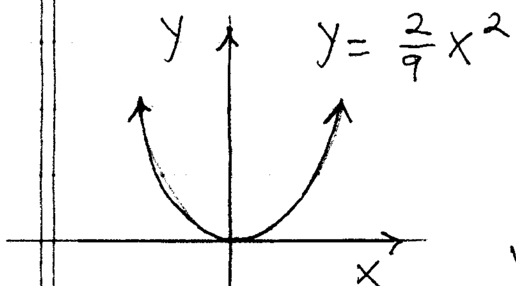
$\vec{a}(t) = \vec{v}\,'(t) = 0\cdot\vec{i} + 2\cdot\vec{j}$; then

$\vec{v}(1) = \vec{i} + 2\vec{j}$ and

$\vec{a}(1) = 2\vec{j}$

3.) $\vec{r}(t) = e^t\vec{i} + \frac{2}{9}e^{2t}\vec{j} \rightarrow$

$\begin{cases} x = e^t \\ y = \frac{2}{9}e^{2t} = \frac{2}{9}(e^t)^2 \end{cases}$ (SUB) $\rightarrow y = \frac{2}{9}x^2$ (parabola);



$\vec{v}(t) = \vec{r}\,'(t) = e^t\cdot\vec{i} + \frac{4}{9}e^{2t}\vec{j} \xrightarrow{D}$

$\vec{a}(t) = \vec{v}\,'(t) = e^t\cdot\vec{i} + \frac{8}{9}e^{2t}\cdot\vec{j}$; then

$v(\ln 3) = e^{\ln 3}\cdot\vec{i} + \frac{4}{9}\cdot e^{2\ln 3}\cdot\vec{j}$

$= 3\vec{i} + \frac{4}{9}e^{\ln 3^2}\cdot\vec{j} = 3\vec{i} + \frac{4}{9}\cdot 9\vec{j} \rightarrow \vec{v} = 3\vec{i} + 4\vec{j}$,

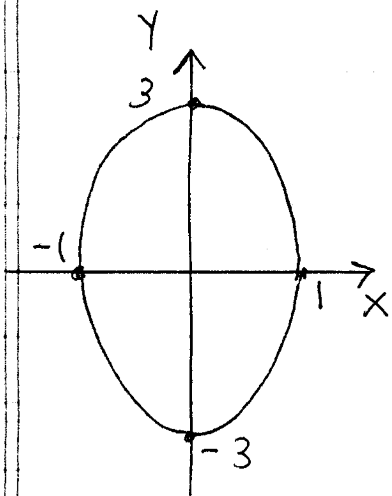
$\vec{a}(\ln 3) = e^{\ln 3}\vec{i} + \frac{8}{9}e^{2\ln 3}\cdot\vec{j} = 3\vec{i} + 8\vec{j}$

4.) $\vec{r}(t) = (\cos 2t)\vec{i} + (3\sin 2t)\cdot\vec{j} \rightarrow$

$\begin{cases} x = \cos 2t \\ y = 3\sin 2t \rightarrow \frac{y}{3} = \sin 2t \end{cases}$, then

$x^2 + \left(\frac{y}{3}\right)^2 = \cos^2 2t + \sin^2 2t = 1 \rightarrow x^2 + \left(\frac{y}{3}\right)^2 = 1$;

(ellipse) $\vec{v}(t) = (-2\sin 2t)\vec{i} + (6\cos 2t)\vec{j}$



$\xrightarrow{D} \vec{a}(t) = (-4\cos 2t)\vec{i} + (-12\sin 2t)\vec{j}$,

then

$\vec{v}(0) = (-2\sin 0)\vec{i} + (6\cos 0)\vec{j}$

$= 6\vec{j}$

$\vec{a}(0) = (-4\cos 0)\vec{i} + (-12\sin 0)\vec{j}$

$= -4\vec{i}$

6.) $\vec{r}(t) = \left(4\cos\frac{t}{2}\right)\vec{i} + \left(4\sin\frac{t}{2}\right)\vec{j} \xrightarrow{D}$

$\vec{v}(t) = \left(-2\sin\frac{t}{2}\right)\vec{i} + \left(2\cos\frac{t}{2}\right)\vec{j} \xrightarrow{D}$

$\vec{a}(t) = \left(-\cos\frac{t}{2}\right)\vec{i} + \left(-\sin\frac{t}{2}\right)\vec{j}$; if $t = \pi$

then $\vec{r}(\pi) = 0\cdot\vec{i} + 4\vec{j} = 4\vec{j}$,

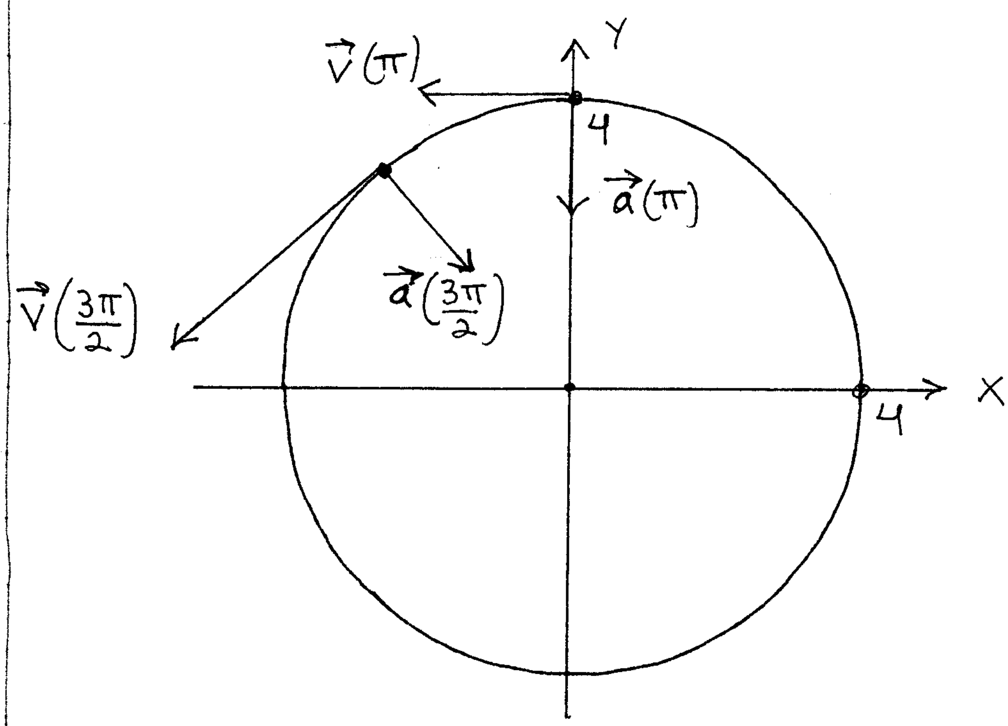
$\vec{v}(\pi) = -2\vec{i} + 0\cdot\vec{j} = -2\vec{i}$,

$\vec{a}(\pi) = 0\cdot\vec{i} + (-1)\vec{j} = -\vec{j}$; if $t = \frac{3\pi}{2}$

then $\vec{r}\left(\frac{3\pi}{2}\right) = \left(4\cdot\frac{-\sqrt{2}}{2}\right)\vec{i} + \left(4\cdot\frac{\sqrt{2}}{2}\right)\vec{j} = -2\sqrt{2}\,\vec{i} + 2\sqrt{2}\,\vec{j}$,

$\vec{v}\left(\frac{3\pi}{2}\right) = \left(-2\cdot\frac{\sqrt{2}}{2}\right)\vec{i} + \left(2\cdot\frac{-\sqrt{2}}{2}\right)\vec{j} = -\sqrt{2}\cdot\vec{i} - \sqrt{2}\cdot\vec{j}$,

$\vec{a}\left(\frac{3\pi}{2}\right) = \frac{\sqrt{2}}{2}\vec{i} + \frac{-\sqrt{2}}{2}\vec{j}$;



8.) $\vec{r}(t) = t\cdot\vec{i} + (t^2+1)\cdot\vec{j} \xrightarrow{D}$

$\vec{v}(t) = 1\cdot\vec{i} + 2t\cdot\vec{j} \xrightarrow{D}$

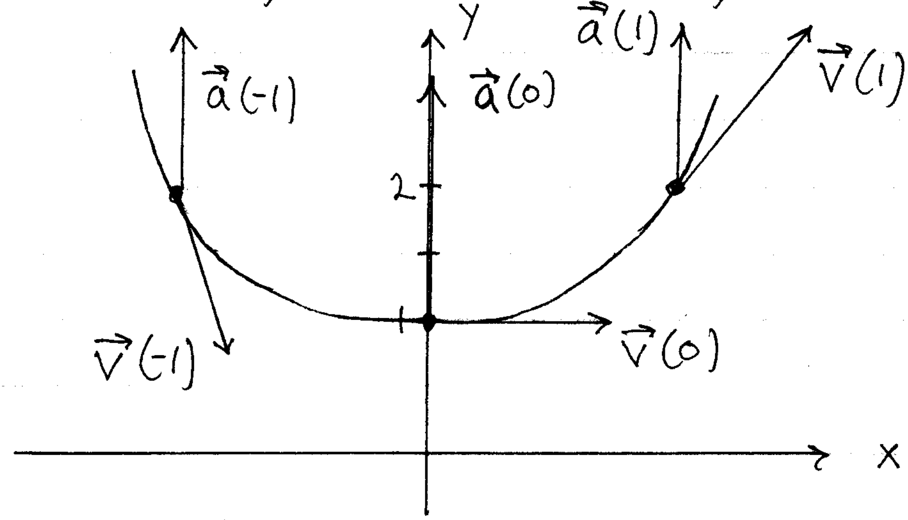
$\vec{a}(t) = 0\cdot\vec{i} + 2\cdot\vec{j}$; if $t=-1 \rightarrow$

$\vec{r}(-1) = -\vec{i} + 2\vec{j}$,

$\vec{v}(-1) = \vec{i} - 2\vec{j}$, $\vec{a}(-1) = 2\vec{j}$; if $t=0 \rightarrow$

$\vec{r}(0) = \vec{j}$, $\vec{v}(0) = \vec{i}$, $\vec{a}(0) = 2\cdot\vec{j}$; if $t=1 \rightarrow$

$\vec{r}(1) = \vec{i} + 2\vec{j}$, $\vec{v}(1) = \vec{i} + 2\vec{j}$, $\vec{a}(1) = 2\vec{j}$



10.) $\vec{r}(t) = (1+t)\vec{i} + \frac{1}{\sqrt{2}}t^2\cdot\vec{j} + \frac{1}{3}t^3\cdot\vec{k} \xrightarrow{D}$

$\vec{v}(t) = 1\cdot\vec{i} + \sqrt{2}t\cdot\vec{j} + t^2\vec{k} \xrightarrow{D}$

$\vec{a}(t) = 0\cdot\vec{i} + \sqrt{2}\cdot\vec{j} + 2t\cdot\vec{k}$; if $t=1$ then

$\vec{v}(1) = \vec{i} + \sqrt{2}\vec{j} + \vec{k}$ so speed is

$|\vec{v}(1)| = \sqrt{1^2 + (\sqrt{2})^2 + 1^2} = \sqrt{4} = 2$, then

direction of motion is

$\frac{\vec{v}(1)}{|\vec{v}(1)|} = \underline{\frac{1}{2}\vec{i} + \frac{1}{\sqrt{2}}\vec{j} + \frac{1}{2}\vec{k}}$ and

$\vec{v}(1) = 2\left(\frac{1}{2}\vec{i} + \frac{1}{\sqrt{2}}\vec{j} + \frac{1}{2}\vec{k}\right)$

12.) $\vec{r}(t) = (\sec t)\vec{i} + (\tan t)\vec{j} + \frac{4}{3}t\vec{k} \xrightarrow{D}$

$\vec{v}(t) = (\sec t \tan t)\vec{i} + (\sec^2 t)\vec{j} + \frac{4}{3}\vec{k} \xrightarrow{D}$

$\vec{a}(t) = (\sec t \cdot \sec^2 t + \sec t \tan t \cdot \tan t)\cdot\vec{i}$
$+ (2\sec t \cdot \sec t \tan t)\vec{j} + 0\cdot\vec{k}$

$= (\sec^3 t + \sec t \cdot \tan^2 t)\vec{i}$
$+ (2\sec^2 t \tan t)\vec{j}$; if $t = \pi/6$

$\vec{v}(\pi/6) = (\sec\frac{\pi}{6}\tan\frac{\pi}{6})\vec{i} + (\sec^2\frac{\pi}{6})\vec{j} + \frac{4}{3}\vec{k}$

$= \frac{2}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}\vec{i} + \left(\frac{2}{\sqrt{3}}\right)^2\vec{j} + \frac{4}{3}\vec{k}$

$= \frac{2}{3}\vec{i} + \frac{4}{3}\vec{j} + \frac{4}{3}\vec{k}$, so speed is

$|\vec{v}(\frac{\pi}{6})| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{4}{3}\right)^2} = \sqrt{\frac{4}{9} + \frac{16}{9} + \frac{16}{9}}$

$= \sqrt{\frac{36}{9}} = \sqrt{4} = 2$; and direction

of motion is $\frac{\vec{v}(\pi/6)}{|\vec{v}(\pi/6)|} = \frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$;

$\vec{v}\left(\frac{\pi}{6}\right) = 2\left(\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}\right)$.

13.) $\vec{r}(t) = (2\ln(t+1))\vec{i} + t^2\vec{j} + \frac{t^2}{2}\vec{k} \xrightarrow{D}$

$\vec{v}(t) = \frac{2}{t+1}\vec{i} + 2t\vec{j} + t\vec{k} \xrightarrow{D}$

$\vec{a}(t) = \frac{-2}{(t+1)^2}\vec{i} + 2\vec{j} + 1\vec{k}$; if $t = 1$

$\vec{v}(1) = 1\vec{i} + 2\vec{j} + 1\vec{k}$ so speed is

$|\vec{v}(1)| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$ and direction

of motion is

$$\frac{\vec{v}(1)}{|\vec{v}(1)|} = \frac{1}{\sqrt{6}}\vec{i} + \frac{2}{\sqrt{6}}\vec{j} + \frac{1}{\sqrt{6}}\vec{k} \quad ;$$

$$\vec{v}(1) = \sqrt{6}\left(\frac{1}{\sqrt{6}}\vec{i} + \frac{2}{\sqrt{6}}\vec{j} + \frac{1}{\sqrt{6}}\vec{k}\right) .$$

15.) $\vec{r}(t) = (3t+1)\vec{i} + \sqrt{3}\cdot t\,\vec{j} + t^2\vec{k} \xrightarrow{D}$

$\vec{v}(t) = 3\cdot\vec{i} + \sqrt{3}\,\vec{j} + 2t\,\vec{k} \xrightarrow{D}$

$\vec{a}(t) = 0\cdot\vec{i} + 0\cdot\vec{j} + 2\vec{k}$; if $t=0$ then

$\vec{v}(0) = 3\vec{i} + \sqrt{3}\,\vec{j} + 0\,\vec{k}$ and

$\vec{a}(0) = 0\cdot\vec{i} + 0\cdot\vec{j} + 2\,\vec{k} \rightarrow$

$|\vec{v}(0)| = \sqrt{3^2 + (\sqrt{3})^2} = \sqrt{12}$ and

$|\vec{a}(0)| = \sqrt{2^2} = 2$; then

$$\cos\theta = \frac{\vec{v}(0)\cdot\vec{a}(0)}{|\vec{v}(0)|\,|\vec{a}(0)|} = \frac{0+0+0}{\sqrt{12}\cdot 2} = 0 \quad \text{so}$$

$\theta = \frac{\pi}{2}$.

17.) $\vec{r}(t) = (\ln(t^2+1))\vec{i} + (\arctan t)\vec{j} + \sqrt{t^2+1}\cdot\vec{k} \xrightarrow{D}$

$$\vec{v}(t) = \frac{2t}{t^2+1}\vec{i} + \frac{1}{1+t^2}\vec{j} + \frac{t}{\sqrt{t^2+1}}\vec{k} \xrightarrow{D}$$

$$\vec{a}(t) = \frac{(t^2+1)(2) - 2t(2t)}{(t^2+1)^2}\vec{i} + \frac{-2t}{(1+t^2)^2}\vec{j}$$

$$+ \frac{\sqrt{t^2+1}\,(1) - t\cdot\frac{1}{\cancel{2}}(t^2+1)^{-1/2}(\cancel{2}t)}{t^2+1}\vec{k}$$

$$= \frac{2-2t^2}{(t^2+1)^2}\vec{i} + \frac{-2t}{(1+t^2)^2}\vec{j} + \frac{1}{(t^2+1)^{3/2}}\vec{k} \ ; \text{ if } t=0$$

$\vec{v}(0) = 0\vec{i} + 1\cdot\vec{j} + 0\cdot\vec{k}$ and
$\vec{a}(0) = 2\vec{i} + 0\cdot\vec{j} + 1\cdot\vec{k}$ so that
$|\vec{v}(0)| = \sqrt{1^2} = 1$ and $|\vec{a}(0)| = \sqrt{2^2+1^2} = \sqrt{5}$,
then
$$\cos\theta = \frac{\vec{v}(0)\cdot\vec{a}(0)}{|\vec{v}(0)|\,|\vec{a}(0)|} = \frac{0+0+0}{(1)(\sqrt{5})} = 0$$
so $\theta = \frac{\pi}{2}$.

19.) $\vec{r}(t) = (t-\sin t)\vec{i} + (1-\cos t)\vec{j} \xrightarrow{D}$
$\vec{v}(t) = (1-\cos t)\vec{i} + (\sin t)\vec{j} \xrightarrow{D}$
$\vec{a}(t) = (\sin t)\vec{i} + (\cos t)\vec{j}$, then
$\vec{v}(t)\perp\vec{a}(t)$ iff $\vec{v}(t)\cdot\vec{a}(t) = 0 \rightarrow$
$\sin t\,(1-\cos t) + \sin t\cdot(\cos t) = 0 \rightarrow$
$\sin t\cdot[1 - \cancel{\cos t} + \cancel{\cos t}] = 0 \rightarrow$
$\sin t = 0$ for $0 \le t \le 2\pi \rightarrow$
$t = 0, \pi, 2\pi$

20.) $\vec{r}(t) = (\sin t)\vec{i} + t\vec{j} + (\cos t)\vec{k} \xrightarrow{D}$
$\vec{v}(t) = (\cos t)\vec{i} + 1\cdot\vec{j} + (-\sin t)\vec{k} \xrightarrow{D}$
$\vec{a}(t) = (-\sin t)\vec{i} + 0\cdot\vec{j} + (-\cos t)\vec{k}$, then
$\vec{v}(t)\perp\vec{a}(t)$ iff $\vec{v}(t)\cdot\vec{a}(t) = 0 \rightarrow$
$-\sin t\cdot\cos t + 0 + \sin t\cdot\cos t = 0 \rightarrow 0=0$
so $\vec{v}(t)\perp\vec{a}(t)$ for all values of t

21.) $\int_0^1 [t^3\vec{i} + 7\vec{j} + (t+1)\vec{k}]\,dt$
$= \left(\frac{1}{4}t^4\Big|_0^1\right)\vec{i} + \left(7t\Big|_0^1\right)\vec{j} + \left(\frac{1}{2}t^2+t\right)\Big|_0^1\cdot\vec{k}$

$= \frac{1}{4}\vec{i} + 7\vec{j} + \frac{3}{2}\vec{k}$

24.) $\int_0^{\frac{\pi}{3}} \left[(\sec t \tan t)\vec{i} + (\tan t)\vec{j} + (2\cos t \sin t)\vec{k}\right] dt$

$= \left(\sec t \Big|_0^{\frac{\pi}{3}}\right)\vec{i} + \left(\ln|\sec t| \Big|_0^{\frac{\pi}{3}}\right)\vec{j} + \left(\sin^2 t \Big|_0^{\frac{\pi}{3}}\right)\vec{k}$

$= (\sec\frac{\pi}{3} - \sec 0)\vec{i} + (\ln|\sec\frac{\pi}{3}| - \ln|\sec 0|)\vec{j}$

$+ (\sin^2\frac{\pi}{3} - \sin^2 0)\vec{k}$

$= (2-1)\vec{i} + (\ln 2 - \cancel{\ln 1}^{0})\vec{j} + (\frac{3}{4} - 0)\vec{k}$

$= 1\cdot\vec{i} + \ln 2\cdot\vec{j} + \frac{3}{4}\vec{k}$

26.) $\int_0^1 \left[\frac{2}{\sqrt{1-t^2}}\vec{i} + \frac{\sqrt{3}}{1+t^2}\vec{k}\right] dt$

$= \left(2\arcsin t \Big|_0^1\right)\vec{i} + \left(\sqrt{3}\arctan t\Big|_0^1\right)\vec{k}$

$= (2\arcsin 1 - 2\arcsin 0)\vec{i}$

$+ (\sqrt{3}\arctan 1 - \sqrt{3}\arctan 0)\vec{k}$

$= (2\cdot\frac{\pi}{2} - 2\cdot 0)\vec{i} + (\sqrt{3}\cdot\frac{\pi}{4} - \sqrt{3}\cdot 0)\vec{k}$

$= (\pi)\vec{i} + \left(\frac{\sqrt{3}}{4}\pi\right)\vec{k}$

31.) $\vec{r}''(t) = -32\vec{k} \longrightarrow$

$\vec{r}'(t) = c_1\vec{i} + c_2\vec{j} + (-32t + c_3)\vec{k}$

and $\vec{r}'(0) = 8\vec{i} + 8\vec{j} + 0\cdot\vec{k}$

$c_1 = 8$, $c_2 = 8$, $-32(0) + c_3 = 0 \rightarrow c_3 = 0 \longrightarrow$

$\vec{r}'(t) = 8\vec{i} + 8\vec{j} + (-32t)\vec{k} \longrightarrow$

$\vec{r}(t) = (8t + c_1)\vec{i} + (8t + c_2)\vec{j} + (-16t^2 + c_3)\vec{k}$

and $\vec{r}(0) = 0\cdot\vec{i} + 0\cdot\vec{j} + 100\,\vec{k} \longrightarrow$

$8(0) + C_1 = 0,\ 8(0) + C_2 = 0,\ -16(0)^2 + C_3 = 100 \longrightarrow$

$C_1 = 0,\ C_2 = 0,$ and $C_3 = 100 \longrightarrow$

$$\vec{r}(t) = (8t)\,\vec{i} + (8t)\,\vec{j} + (100 - 16t^2)\,\vec{k}$$

33.) $\vec{r}(t) = (\sin t)\,\vec{i} + (t^2 - \cos t)\,\vec{j} + e^t\,\vec{k} \xrightarrow{D}$

$\vec{r}\,'(t) = (\cos t)\,\vec{i} + (2t + \sin t)\,\vec{j} + e^t\,\vec{k}$

and $t = 0 \longrightarrow$ point of tangency

is $(\sin 0,\ 0^2 - \cos 0,\ e^0) = (0, -1, 1)$ and

tangent vector is

$\vec{r}\,'(0) = (\cos 0)\,\vec{i} + (2(0) + \sin 0)\,\vec{j} + e^0\,\vec{k}$

$\quad = 1\cdot\vec{i} + 0\cdot\vec{j} + 1\cdot\vec{k}$ so

tangent line is given by

$$L: \begin{cases} x = 0 + (1)\,t \\ y = -1 + (0)\,t \\ z = 1 + (1)\,t \end{cases} \longrightarrow \begin{cases} x = t \\ y = -1 \\ z = 1 + t \end{cases} \quad \text{for } -\infty < t < \infty$$

34.) $\vec{r}(t) = (2\sin t)\,\vec{i} + (2\cos t)\,\vec{j} + 5t\,\vec{k} \xrightarrow{D}$

$\vec{r}\,'(t) = (2\cos t)\,\vec{i} + (-2\sin t)\,\vec{j} + 5\cdot\vec{k}$

and $t = 4\pi \longrightarrow$ point of tangency

is $(2\sin 4\pi,\ 2\cos 4\pi,\ 20\pi) = (0, 2, 20\pi)$

and tangent vector is

$\vec{r}\,'(4\pi) = (2\cos 4\pi)\,\vec{i} + (-2\sin 4\pi)\,\vec{j} + 5\,\vec{k}$

$\quad = 2\,\vec{i} + 0\cdot\vec{j} + 5\,\vec{k}$ so

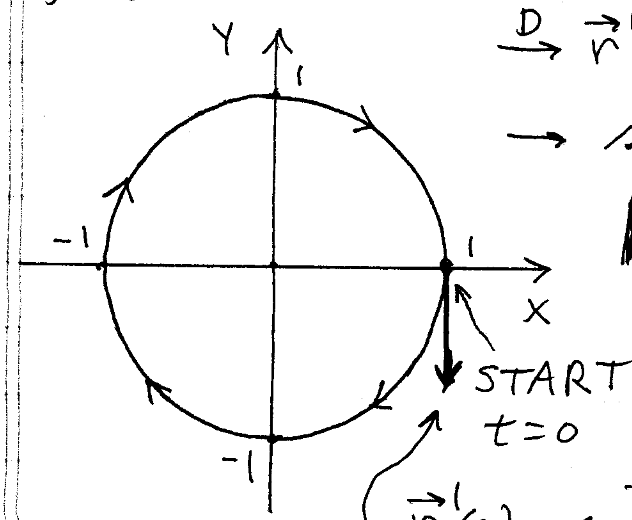
tangent line is given by

$$L: \begin{cases} x = 0 + (2)t \\ y = 2 + (0)t \\ z = 20\pi + (5)t \end{cases} \rightarrow \begin{cases} x = 2t \\ y = 2 \\ z = 20\pi + 5t \end{cases} \text{ for } -\infty < t < \infty$$

36.) $\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} + (\sin 2t)\vec{k} \xrightarrow{D}$
$\vec{r}\,'(t) = (-\sin t)\vec{i} + (\cos t)\vec{j} + (2\cos 2t)\vec{k}$
and $t = \frac{\pi}{2} \rightarrow$ point of tangency is
$(\cos\frac{\pi}{2}, \sin\frac{\pi}{2}, \sin\pi) = (0,1,0)$ and
tangent vector is
$\vec{r}\,'(\frac{\pi}{2}) = (-\sin\frac{\pi}{2})\vec{i} + (\cos\frac{\pi}{2})\vec{j} + (2\cos\pi)\vec{k}$
$= -1\cdot\vec{i} + 0\cdot\vec{j} + {}^-2\cdot\vec{k}$, so
tangent line is given by

$$L: \begin{cases} x = 0 + (-1)t \\ y = 1 + (0)t \\ z = 0 + (-2)t \end{cases} \rightarrow \begin{cases} x = -t \\ y = 1 \\ z = -2t \end{cases} \text{ for } -\infty < t < \infty$$

37.) d.) $\vec{r}(t) = (\cos t)\vec{i} + (-\sin t)\vec{j}$ for $t \geq 0$
$\xrightarrow{D} \vec{r}\,'(t) = (-\sin t)\vec{i} + (-\cos t)\vec{j}$



$\rightarrow$ speed is
$|\vec{r}\,'(t)|^2 = \sqrt{(-\sin t)^2 + (-\cos t)^2}$
$= \sqrt{\sin^2 t + \cos^2 t}$
$= \sqrt{1} = 1$, and

$\vec{r}\,'(0) = 0\cdot\vec{i} + (-1)\vec{j}$

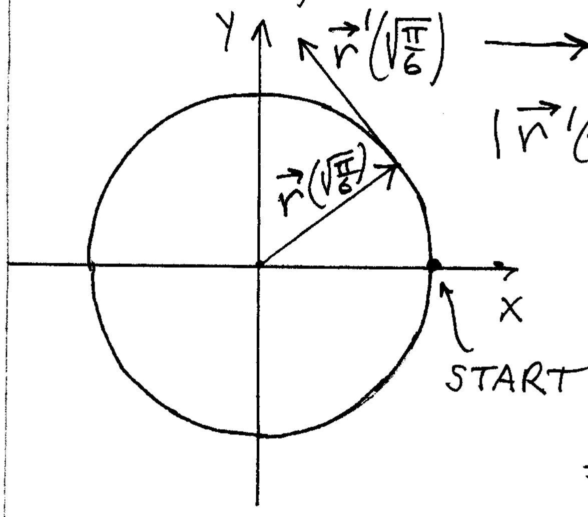
i.) YES, speed = 1

ii.) YES, since $\vec{r}\,'(t)$ has constant length we know $\frac{d}{dt}(\vec{r}\,'(t)) = \vec{r}\,''(t)$ is $\perp$ to $\vec{r}\,'(t)$ (Example from class)

iii.) The particle moves clockwise since $\vec{r}\,'(0) = -\vec{j}$.

37.) e.) $\vec{r}(t) = \cos(t^2)\cdot\vec{i} + \sin(t^2)\cdot\vec{j}$ for $t \geq 0$

$\rightarrow \vec{r}\,'(t) = -2t\sin(t^2)\cdot\vec{i} + 2t\cos(t^2)\cdot\vec{j}$



$\longrightarrow$ speed is

$$|\vec{r}\,'(t)| = \sqrt{(-2t\sin(t^2))^2 + (2t\cos(t^2))^2}$$
$$= \sqrt{4t^2\sin^2(t^2) + 4t^2\cos^2(t^2)}$$
$$= \sqrt{4t^2(\sin^2(t^2) + \cos^2(t^2))}$$
$$= \sqrt{4t^2(1)} = 2|t| = 2t\ ;$$

$\vec{r}\,'(0) = 0\cdot\vec{i} + 0\cdot\vec{j}$ (no good information)

$$\vec{r}\,'\left(\sqrt{\tfrac{\pi}{6}}\right) = \Big(-2\sqrt{\tfrac{\pi}{6}}\,\underbrace{\sin\tfrac{\pi}{6}}_{1/2}\Big)\vec{i} + \Big(2\sqrt{\tfrac{\pi}{6}}\,\underbrace{\cos\tfrac{\pi}{6}}_{\sqrt{3}/2}\Big)\vec{j}$$
$$= -\sqrt{\tfrac{\pi}{6}}\,\vec{i} + \sqrt{3}\sqrt{\tfrac{\pi}{6}}\,\vec{j}$$

i.) The speed of motion is $2t$, not a constant speed.

ii.) No, since velocity vector $\vec{r}\,'(t)$ does not have constant length

iii.) The particle moves counter-clockwise (SEE $\vec{r}\,'(\sqrt{\pi/6})$).

38.) $\vec{r}(t) = (2\vec{i} + 2\vec{j} + \vec{k})$
$+ \cos t \left(\frac{1}{\sqrt{2}}\vec{i} - \frac{1}{\sqrt{2}}\vec{j}\right)$
$+ \sin t \left(\frac{1}{\sqrt{3}}\vec{i} + \frac{1}{\sqrt{3}}\vec{j} + \frac{1}{\sqrt{3}}\vec{k}\right)$

$= \left(2 + \frac{1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{3}}\sin t\right)\vec{i}$
$+ \left(2 - \frac{1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{3}}\sin t\right)\vec{j}$
$+ \left(1 + \frac{1}{\sqrt{3}}\sin t\right)\vec{k} \longrightarrow$

$$\begin{cases} x = 2 + \frac{1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{3}}\sin t \\ y = 2 - \frac{1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{3}}\sin t \\ z = 1 + \frac{1}{\sqrt{3}}\sin t \end{cases} \longrightarrow z - 1 = \frac{1}{\sqrt{3}}\sin t$$

$$\rightarrow \begin{cases} x = 2 + \frac{1}{\sqrt{2}}\cos t + (z-1) \\ y = 2 - \frac{1}{\sqrt{2}}\cos t + (z-1) \end{cases} \quad \text{(ADD)}$$

$\rightarrow x + y = 4 + 2z - 2$

$\rightarrow \boxed{x + y - 2z = 2}$ (a plane);

now find distance between (x, y, z) and $(2, 2, 1)$:

$$L = \sqrt{(x-2)^2+(y-2)^2+(z-1)^2}$$

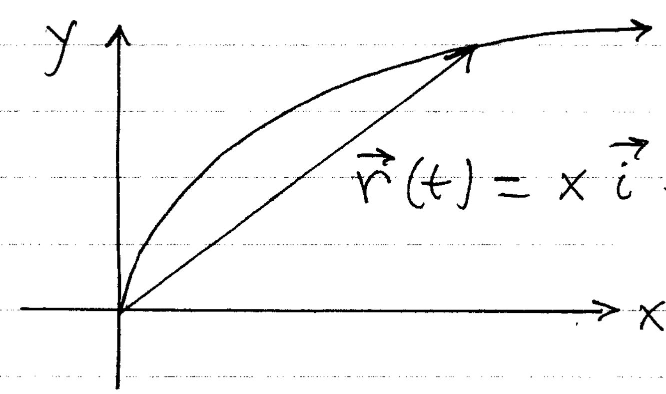
$$= \sqrt{\left(\tfrac{1}{\sqrt2}\cos t+\tfrac{1}{\sqrt3}\sin t\right)^2 + \left(-\tfrac{1}{\sqrt2}\cos t+\tfrac{1}{\sqrt3}\sin t\right)^2 + \left(\tfrac{1}{\sqrt3}\sin t\right)^2}$$

$$= \sqrt{\tfrac12\cos^2 t+\cancel{\tfrac{2}{\sqrt6}\cos t\sin t}+\tfrac13\sin^2 t + \tfrac12\cos^2 t-\cancel{\tfrac{2}{\sqrt6}\cos t\sin t}+\tfrac13\sin^2 t + \tfrac13\sin^2 t}$$

$= \sqrt{\cos^2 t+\sin^2 t} = \sqrt{1} = 1$;
thus points (x, y, z) lie in the plane $x+y-2z=2$, and are 1 unit away from point $(2,2,1)$, a circle of radius 1 centered at $(2,2,1)$.

40.) (NEXT after 41.)

41.)



$y^2 = 2x$ or $x = \frac12 y^2$

$\vec{r}(t) = x\,\vec{i} + y\,\vec{j} = x(t)\,\vec{i} + y(t)\,\vec{j}$

then

velocity vector is

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$\vec{r}\,'(t) = x'(t)\,\vec{i} + y'(t)\,\vec{j}$ so

$\text{speed} = |\vec{r}\,'(t)| = \sqrt{(x'(t))^2 + (y'(t))^2} = 5$

$\rightarrow \boxed{(x'(t))^2 + (y'(t))^2 = 25}$; and

$\boxed{(y(t))^2 = 2x(t)} \xrightarrow{D} 2 \cdot y(t)\,y'(t) = 2x'(t)$

$\rightarrow \boxed{(y(t))^2 (y'(t))^2 = (x'(t))^2}$; then

(SUB) $2x(t)\left(25 - (x'(t))^2\right) = (x'(t))^2$

$\rightarrow$ (Solve for $x'(t)$.) $\longrightarrow$

$50\,x(t) - 2x(t)\cdot(x'(t))^2 = (x'(t))^2 \rightarrow$

$50\,x(t) = 2x(t)\cdot(x'(t))^2 + (x'(t))^2 \rightarrow$

$50\,x(t) = (2x(t)+1)(x'(t))^2 \rightarrow$

$x'(t)^2 = \dfrac{50x(t)}{2x(t)+1} \rightarrow \boxed{x'(t) = \sqrt{\dfrac{50x(t)}{2x(t)+1}}}$;

if $x=2$, then

$x'(t) = \sqrt{20} = \boxed{2\sqrt{5}}$ and

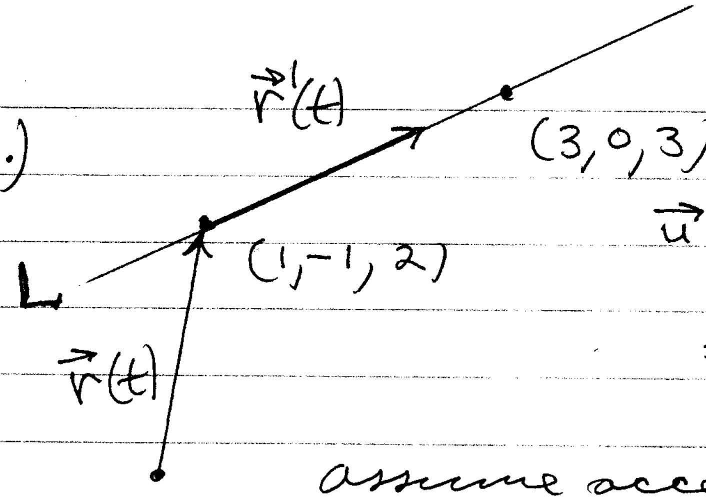
$y'(t) = \sqrt{25 - (x'(t))^2} = \sqrt{25-20} = \boxed{\sqrt{5}}$;

thus at $(2,2)$ velocity is

$\vec{r}(t) = x'(t)\cdot\vec{i} + y'(t)\,\vec{j}$

$= 2\sqrt{5}\,\vec{i} + \sqrt{5}\,\vec{j}$.

40.)



direction vector

$\vec{u} = (3-1)\vec{i} + (0-(-1))\vec{j} + (3-2)\vec{k}$
$= 2\vec{i} + 1\cdot\vec{j} + 1\cdot\vec{k}$

assume acceleration vector is

$\vec{r}''(t) = 2\vec{i} + 1\cdot\vec{j} + 1\cdot\vec{k} \rightarrow$

$\vec{r}'(t) = (2t + c_1)\vec{i} + (t + c_2)\vec{j} + (t + c_3)\vec{k}$; then

$\vec{r}'(0) = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$ (the initial velocity vector) is a multiple of $\vec{u}$, say s

$s\vec{u} = s(2\vec{i} + 1\cdot\vec{j} + 1\cdot\vec{k}) = 2s\vec{i} + s\vec{j} + s\vec{k}$,

i.e. $\vec{r}'(0) = s\vec{u}$ so that

$c_1 = 2s$, $c_2 = s$, and $c_3 = s \rightarrow$

$\underline{\vec{r}'(0) = 2s\vec{i} + s\vec{j} + s\vec{k}}$; and

initial speed is 2 so

$|\vec{r}'(0)| = \sqrt{(2s)^2 + s^2 + s^2} = 2 \rightarrow$

$6s^2 = 4 \rightarrow s = 2/\sqrt{3}$; thus

$\vec{r}'(t) = \left(2t + \frac{4}{\sqrt{3}}\right)\vec{i} + \left(t + \frac{2}{\sqrt{3}}\right)\vec{j} + \left(t + \frac{2}{\sqrt{3}}\right)\vec{k}$ then

$\vec{r}(t) = \left(t^2 + \frac{4}{\sqrt{3}}t + c_1\right)\vec{i} + \left(\frac{t^2}{2} + \frac{2}{\sqrt{3}}t + c_2\right)\vec{j}$
$+ \left(\frac{1}{2}t^2 + \frac{2}{\sqrt{3}}t + c_3\right)\vec{k}$

and $\vec{r}(0) = 1\cdot\vec{i} + (-1)\vec{j} + 2\cdot\vec{k}$ (Given)

so $\vec{r}(0) = c_1\vec{i} + c_2\vec{j} + c_3\vec{k}$, then

$c_1 = 1$, $c_2 = -1$, $c_3 = 2$ and

$$\vec{r}(t) = \left(t^2 + \tfrac{4}{\sqrt{3}}t + 1\right)\vec{i} + \left(\tfrac{1}{2}t^2 + \tfrac{2}{\sqrt{3}}t - 1\right)\vec{j} + \left(\tfrac{1}{2}t^2 + \tfrac{2}{\sqrt{3}}t + 2\right)\vec{k}$$

43.) $\vec{r}(t) = (3\cos t)\vec{j} + (2\sin t)\vec{k} \xrightarrow{D}$

$\vec{r}\,'(t) = (-3\sin t)\vec{j} + (2\cos t)\vec{k} \rightarrow$

$\vec{r}\,''(t) = (-3\cos t)\vec{j} + (-2\sin t)\vec{k}$.

Let $F(t) = |\vec{v}(t)| = |\vec{r}\,'(t)| = \sqrt{(-3\sin t)^2 + (2\cos t)^2}$

$\rightarrow F(t) = \sqrt{9\sin^2 t + 4\cos^2 t}$

$= \sqrt{5\sin^2 t + 4\underbrace{(\sin^2 t + \cos^2 t)}_{1}}$

$= \sqrt{5\sin^2 t + 4}$;

the maximum value of

$F(t) = |\vec{v}(t)| = \sqrt{5(1)^2 + 4} = 3$;

the minimum value of

$F(t) = |\vec{v}(t)| = \sqrt{5(0)^2 + 4} = 2$; let

$G(t) = |\vec{a}(t)| = |\vec{r}\,''(t)|$

$= \sqrt{(-3\cos t)^2 + (-2\sin t)^2}$

$= \sqrt{9\cos^2 t + 4\sin^2 t}$

$= \sqrt{5\cos^2 t + 4(\cos^2 t + \sin^2 t)}$

$= \sqrt{5\cos^2 t + 4}$;

the maximum value of

$G(t) = |\vec{a}\,'(t)| = \sqrt{5(1)^2 + 4} = \sqrt{9} = 3$;

the minimum value of

$G(t) = |\vec{a}\,'(t)| = \sqrt{5(0)^2 + 4} = \sqrt{4} = 2$.

45.) Theorem: If $\vec{v}(t) \cdot \vec{v}'(t) = 0$, then $|\vec{v}| = C$, a constant.

Proof: If $\vec{v}(t) \cdot \vec{v}'(t) = 0 \rightarrow$
$\vec{v}'(t) \cdot \vec{v}(t) = 0$ then
$$\vec{v}(t) \cdot \vec{v}'(t) + \vec{v}'(t) \cdot \vec{v}(t) = 0 \rightarrow$$
$$D(\vec{v}(t) \cdot \vec{v}(t)) = 0 \rightarrow$$
$$\vec{v}(t) \cdot \vec{v}(t) = c_1, \text{ a constant} \rightarrow$$
$$|\vec{v}(t)|^2 = c_1 \rightarrow$$
$$|\vec{v}(t)| = \sqrt{c_1} = C, \text{ a constant.}$$
QED

48.) Theorem: If $\vec{u}(t) = a\vec{i} + b\vec{j} + c\vec{k}$, a constant vector, then $\vec{u}'(t) = 0 \cdot \vec{i} + 0 \cdot \vec{j} + 0 \cdot \vec{k}$.

Proof: $\vec{u}(t) = a\vec{i} + b\vec{j} + c\vec{k} \xrightarrow{D}$
$$\frac{d}{dt}\vec{u}(t) = \frac{d}{dt}(a)\vec{i} + \frac{d}{dt}(b)\vec{j} + \frac{d}{dt}(c)\vec{k}$$
$$= 0 \cdot \vec{i} + 0 \cdot \vec{j} + 0 \cdot \vec{k}.$$

QED.