Section 14.1

1.) d.) $f(x,y) = x^2 + xy^3 \rightarrow$

$f(3,-2) = (3)^2 + 3(-2)^3 = 9 - 24 = -15$

4.) c.) $f(x,y,z) = \sqrt{49 - x^2 - y^2 - z^2} \rightarrow$

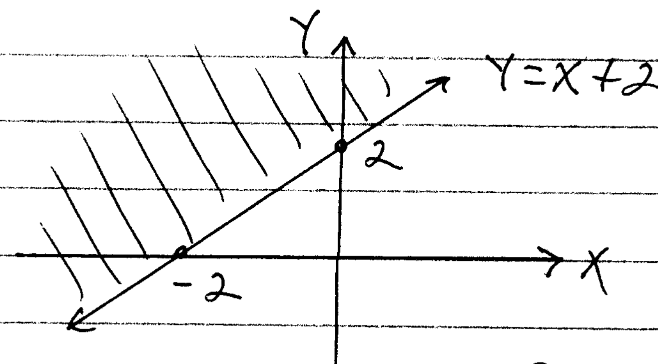
$f(-1,2,3) = \sqrt{49 - (-1)^2 - (2)^2 - (3)^2}$

$= \sqrt{49 - 1 - 4 - 9} = \sqrt{35}$

5.) $f(x,y) = \sqrt{y - x - 2}$; need $y - x - 2 \geq 0$

$\rightarrow y \geq x + 2$ so

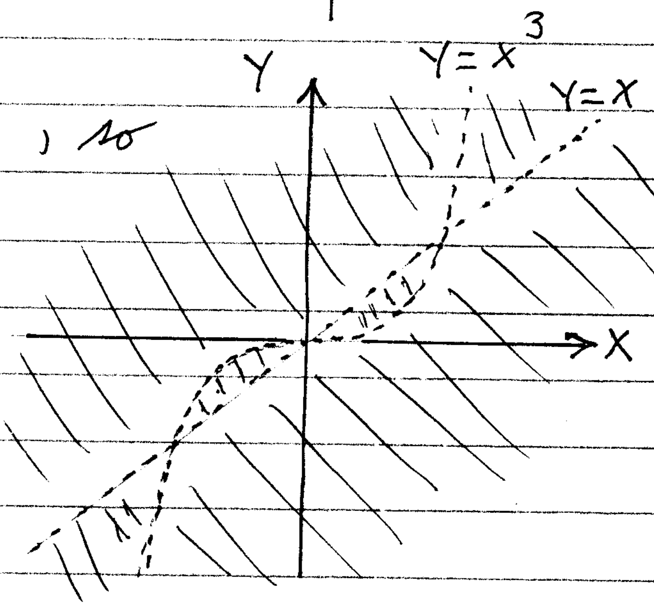
<u>Domain</u> : all pts. (x,y) on or above the line $y = x + 2$



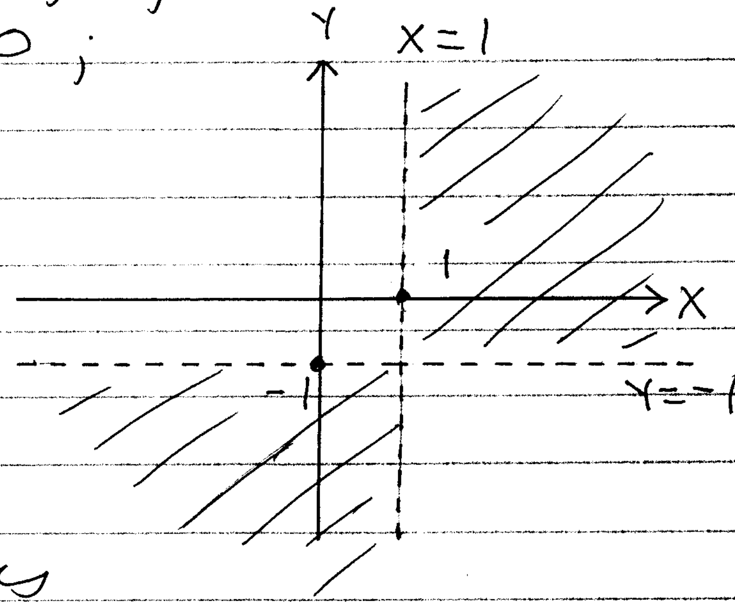
7.) $f(x,y) = \dfrac{(x-1)(y+2)}{(y-x)(y-x^3)}$, so

$y \neq x$, $y \neq x^3$;

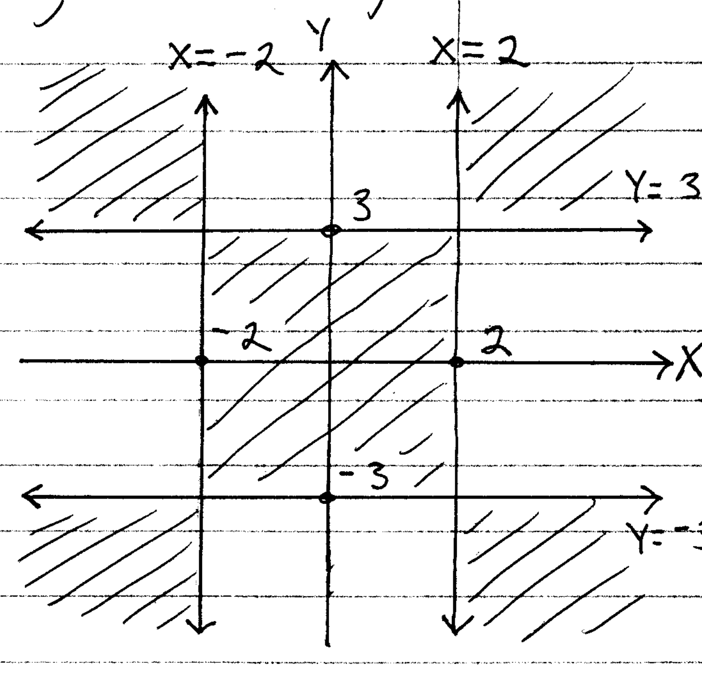
<u>Domain</u> : all pts. (x,y) NOT on graphs $y = x$, $y = x^3$



10.) $f(x,y) = \ln(xy + x - y - 1)$
$= \ln((x-1)(y+1))$;
need $(x-1)(y+1) > 0$;
$(x-1)(y+1) = 0$
when $x = 1$ or $y = -1$;
now choose test point in each of the 4 regions ;
<u>Domain</u>: all pts. (x,y) in the shaded regions



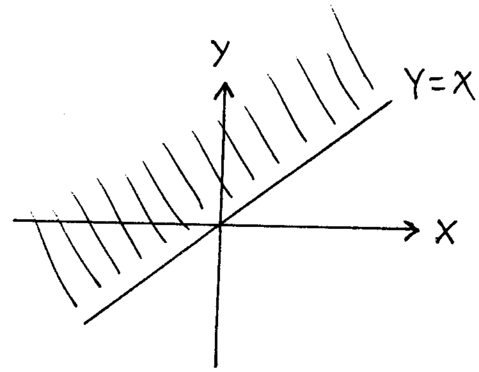
11.) $f(x,y) = \sqrt{(x^2-4)(y^2-9)} = \sqrt{(x-2)(x+2)(y-3)(y+3)}$,
need $(x-2)(x+2)(y-3)(y+3) \geq 0$;
$(x-2)(x+2)(y-3)(y+3) = 0$
when $x = 2$, $x = -2$, $y = 3$, or $y = -3$;
now choose test point in each of the 9 regions ;
<u>Domain</u>: all pts. (x,y) on the lines, $x = 2$, $x = -2$, $y = 3$, $y = -3$ or in shaded regions



Section 14.1

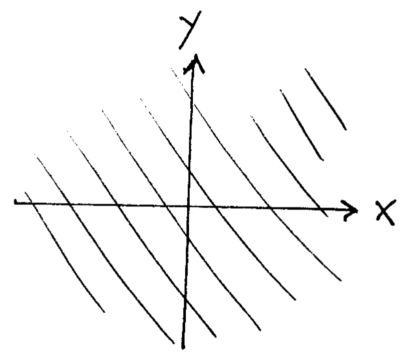
18.) $f(x,y) = \sqrt{y-x}$

a.) <u>Domain</u>: $y - x \geq 0$.
so all pts. (x,y) with
$y \geq x$



b.) Consider all pts. $(0,y)$ for $0 \leq y < \infty$;
for these pts. the z-value is
$z = \sqrt{y}$ and $0 \leq z < \infty$. It follows
(since $\sqrt{x-y} \geq 0$) that the <u>Range</u> of f
is $0 \leq z < \infty$.

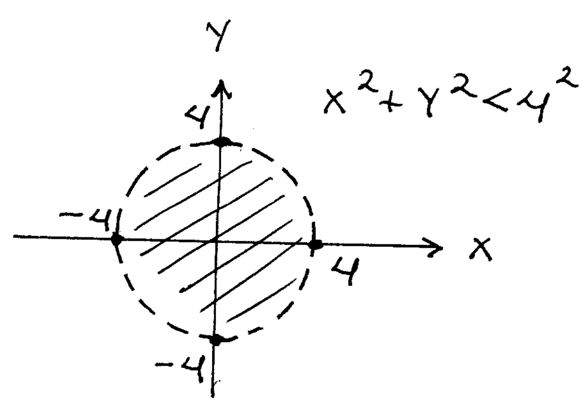
19.) $f(x,y) = 4x^2 + 9y^2$



a.) <u>Domain</u>:
all pts. (x,y)

b.) Consider all pts.
$(x,0)$ for $-\infty < x < \infty$; for these pts. the
z value is $z = 4x^2$ and $0 \leq z < \infty$.
It follows (since $4x^2 + 9y^2 \geq 0$) that
the <u>Range</u> of f is $0 \leq z < \infty$.

23.) $f(x,y) = \dfrac{1}{\sqrt{16 - x^2 - y^2}}$



a.) <u>Domain</u>: $16 - x^2 - y^2 > 0$
$\rightarrow$ $x^2 + y^2 < 16$ so
domain is set of pts.
(x,y) <u>inside</u> the circle

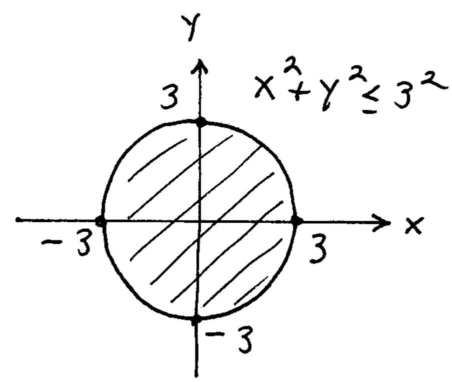
$$x^2 + y^2 = 4^2$$

b.) Consider all pts. $(x,0)$, where $-4 < x < 4$; for these pts. the z-value is $z = \frac{1}{\sqrt{16-x^2}}$; note that $z = \frac{1}{4}$ if $x = 0$ and

$$\lim_{x \to 4^-} z = \lim_{x \to 4^-} \frac{1}{\sqrt{16-x^2}} = \frac{1}{\sqrt{0}} = +\infty \; ;$$

the z-values range for $z = \frac{1}{4}$ to ∞; since $\frac{1}{4} \le \frac{1}{\sqrt{16-x^2-y^2}}$, it follows that the <u>Range</u> of f is $\frac{1}{4} \le z < \infty$.

24.) $f(x,y) = \sqrt{9-x^2-y^2}$

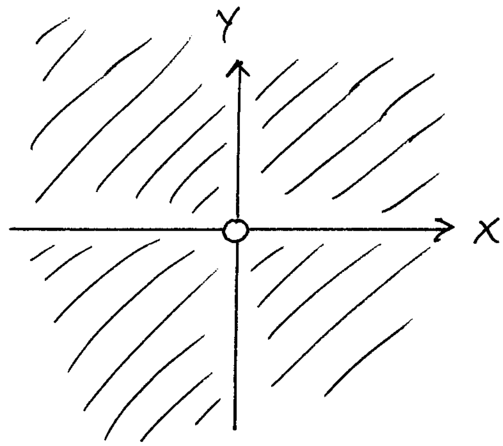


a.) <u>Domain</u>: $9 - x^2 - y^2 \ge 0$ $\rightarrow$ $x^2 + y^2 \le 9$ so domain is set of all pts. (x,y) <u>on</u> <u>or</u> <u>inside</u> the circle $x^2 + y^2 = 3^2$

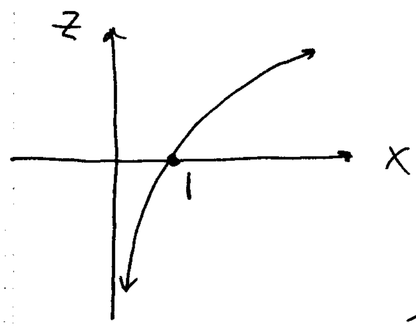
b.) Consider that $z = \sqrt{9-x^2-y^2}$ $\rightarrow$ $z^2 = 9 - x^2 - y^2 \rightarrow x^2 + y^2 + z^2 = 3^2$ is a sphere of radius 3 centered at $(0,0,0)$; the $z = \sqrt{9-x^2-y^2}$ is the top half of the sphere, so the <u>Range</u> of f is $0 \le z \le 3$

25.) $f(x,y) = \ln(x^2 + y^2)$

a.) <u>Domain</u>: $x^2 + y^2 > 0$ so domain is set of all pts. (x,y) <u>except</u> $(0,0)$;



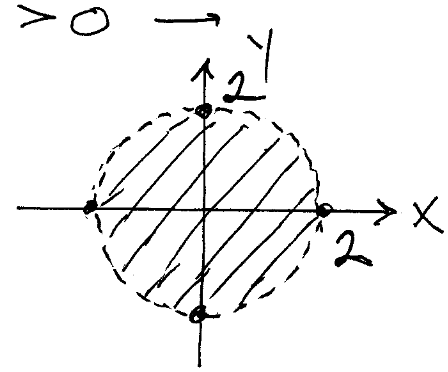
b.) Consider all pts. $(x, 0)$, where $0 < x < \infty$; for these pts. the z-value is $z = \ln x^2 \rightarrow z = 2\ln x$;



these z-values range from $-\infty$ to $+\infty$; it follows that the Range of f is $-\infty < z < \infty$.

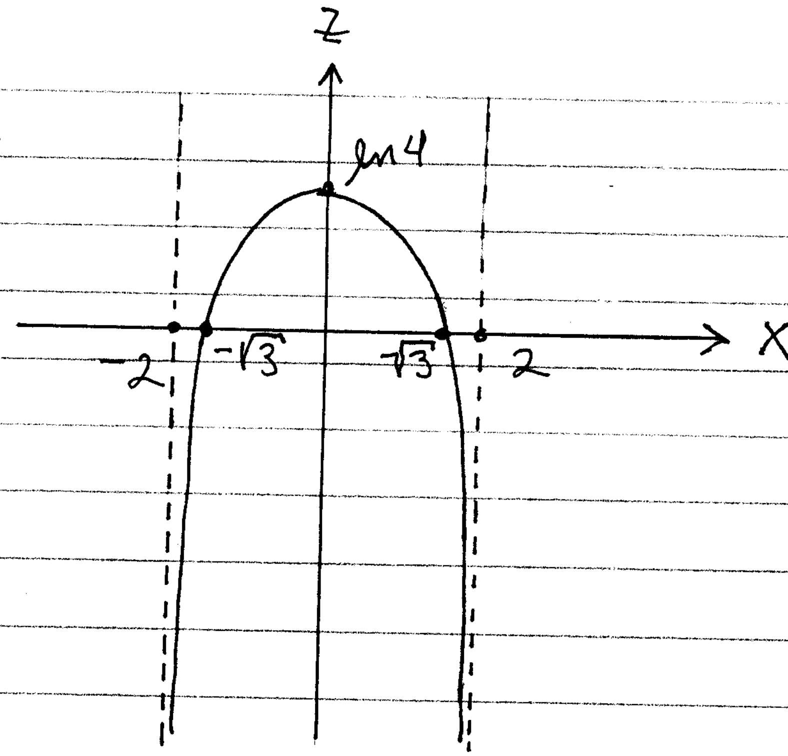
Ex: $f(x, y) = \ln(4 - x^2 - y^2)$

a.) Domain: $4 - x^2 - y^2 > 0 \rightarrow$ $x^2 + y^2 < 4$ so domain is set of all pts. (x, y) inside the circle $x^2 + y^2 = 4$.



b.) Range: Consider xz-trace of surface $z = \ln(4 - x^2 - y^2) \rightarrow$ $y = 0 \rightarrow$

$z = \ln(4 - x^2)$;



if $z = 0$:

$0 = \ln(4 - x^2)$

$\rightarrow 4 - x^2 = 1$

$\rightarrow x^2 = 3$

$\rightarrow x = \pm\sqrt{3}$;

if $x = 0$:

$z = \ln 4$;

so Range is all values of z

satisfying $-\infty < z \leq \ln 4$