Section 14.2

1.) $\lim_{(x,y)\to(0,0)} \frac{3x^2-y^2+5}{x^2+y^2+2} = \frac{0-0+5}{0+0+2} = \frac{5}{2}$

4.) $\lim_{(x,y)\to(2,-3)} \left(\frac{1}{x}+\frac{1}{y}\right)^2 = \left(\frac{1}{2}+\frac{-1}{3}\right)^2 = \frac{1}{36}$

9.) $\lim_{(x,y)\to(0,0)} \frac{e^y \cdot \sin x}{x} = \lim_{(x,y)\to(0,0)} e^y \cdot \left(\frac{\sin x}{x}\right)$

$= e^0 \cdot (1) = (1)(1) = 1$

12.) $\lim_{(x,y)\to(\frac{\pi}{2},0)} \frac{\cos y+1}{y-\sin x} = \frac{\cos 0+1}{0-\sin\frac{\pi}{2}} = \frac{1+1}{-1} = -2$

14.) $\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(x,y)\to(1,1)} \frac{\cancel{(x-y)}(x+y)}{\cancel{(x-y)}} = 1+1 = 2$

16.) $\lim_{(x,y)\to(2,-4)} \frac{y+4}{x^2y-xy+4x^2-4x}$

$= \lim_{(x,y)\to(2,-4)} \frac{y+4}{xy(x-1)+4x(x-1)}$

$= \lim_{(x,y)\to(2,-4)} \frac{y+4}{(x-1)[xy+4x]}$

$= \lim_{(x,y)\to(2,-4)} \frac{\cancel{y+4}}{x(x-1)[\cancel{y+4}]} = \frac{1}{2(1)} = \frac{1}{2}$

20.) $\lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(x,y)\to(4,3)} \frac{\sqrt{x}-\sqrt{y+1}}{x-y-1} \cdot \frac{\sqrt{x}+\sqrt{y+1}}{\sqrt{x}+\sqrt{y+1}}$

$$= \lim_{(x,y)\to(4,3)} \frac{x-(y+1)}{(x-y-1)(\sqrt{x}+\sqrt{y+1})} = \lim_{(x,y)\to(4,3)} \frac{\cancel{x-y-1}}{\cancel{(x-y-1)}(\sqrt{x}+\sqrt{y+1})}$$

$$= \frac{1}{2+2} = \frac{1}{4}$$

21.) $$\lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2} = 1$$

(Since $\lim_{h\to 0} \frac{\sin h}{h} = 1$)

22.) $$\lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{xy} = \frac{1-\cos 0}{0} = \frac{1-1}{0} = \frac{0}{0}$$

$$= \lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{xy} \cdot \frac{1+\cos(xy)}{1+\cos(xy)}$$

$$= \lim_{(x,y)\to(0,0)} \frac{1-\cos^2(xy)}{xy(1+\cos(xy))}$$

$$= \lim_{(x,y)\to(0,0)} \frac{\sin^2(xy)}{xy(1+\cos(xy))}$$

$$= \lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{xy} \cdot \frac{\sin(xy)}{1+\cos(xy)}$$

$$= (1)\cdot\frac{\sin 0}{1+\cos 0} = (1)\cdot\frac{(0)}{1+(1)} = 0$$

23.) $$\lim_{(x,y)\to(1,-1)} \frac{x^3+y^3}{x+y} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(x,y)\to(1,-1)} \frac{\cancel{(x+y)}(x^2-xy+y^2)}{\cancel{x+y}}$$

$$= (1)^2 - (1)(-1) + (-1)^2 = 1+1+1 = 3$$

24.) $$\lim_{(x,y)\to(2,2)} \frac{x-y}{x^4-y^4} \overset{"\frac{0}{0}"}{=} \lim_{(x,y)\to(2,2)} \frac{x-y}{(x^2-y^2)(x^2+y^2)}$$

$$= \lim_{(x,y)\to(2,2)} \frac{\cancel{x-y}}{\cancel{(x-y)}(x+y)(x^2+y^2)}$$

$$= \frac{1}{(2+2)(4+4)} = \frac{1}{32}$$

41.) $$\lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2+y^2}}$$ DNE since

<u>along path $y=0$</u>:

$$\lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2+y^2}} = \lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2}}$$

$$= \lim_{x\to 0} \frac{-x}{|x|} = \begin{cases} \lim_{x\to 0} \frac{-x}{x} = \lim_{x\to 0} -1 = -1, & \text{if } x>0 \\ \lim_{x\to 0} \frac{-x}{-x} = \lim_{x\to 0} 1 = 1, & \text{if } x<0 \end{cases}$$

so $\lim_{(x,y)\to(0,0)} \frac{-x}{\sqrt{x^2+y^2}}$ DNE along path $y=0$.

42.) $$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2}$$ DNE since

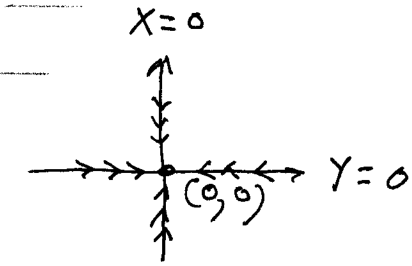
<u>along path $y=0$</u>:

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4} = \lim_{x\to 0} 1 = 1;$$

along path $x=0$:

$$\lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{0}{0+y^2} = \lim_{y\to 0} 0 = 0.$$

43.) $\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$ DNE since



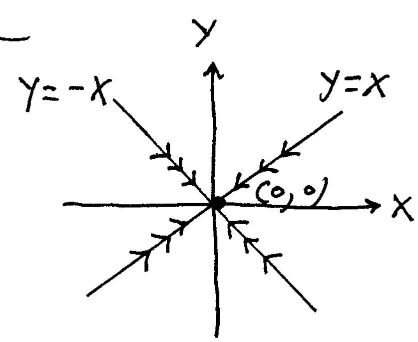
along path $y=0$:

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4} = \lim_{x\to 0} 1 = 1 \;;$$

along path $x=0$:

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{-y^2}{y^2} = \lim_{y\to 0} -1 = -1$$

44.) $\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|}$ DNE since



along path $y=x$:

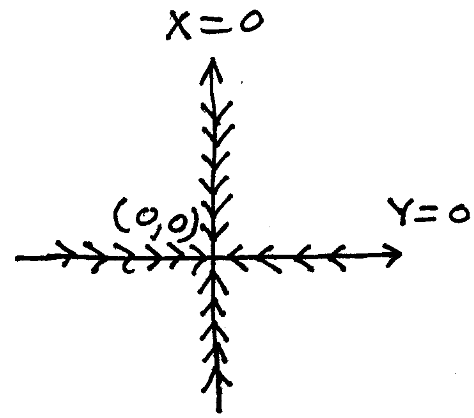
$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{(x,y)\to(0,0)} \frac{x^2}{|x^2|}$$

$$= \lim_{(x,y)\to(0,0)} \frac{x^2}{x^2} = \lim_{(x,y)\to(0,0)} 1 = 1 \;;$$

along path $y=-x$:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{|xy|} = \lim_{(x,y)\to(0,0)} \frac{-x^2}{x^2} = \lim_{(x,y)\to(0,0)} -1 = -1 .$$

$$45.)\ \lim_{(x,y)\to(0,0)} \frac{x-y}{x+y} \text{ D.N.E. since}$$



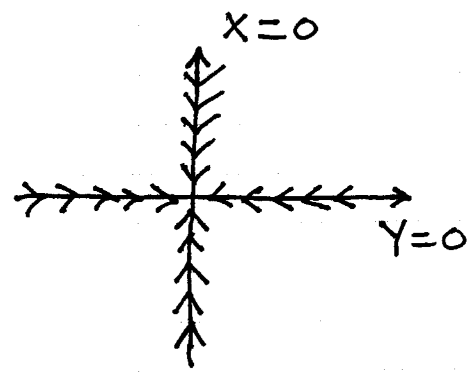
<u>along path $x=0$:</u>

$$\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y} = \lim_{(x,y)\to(0,0)} \frac{-y}{y} = \lim_{(x,y)\to(0,0)} -1 = -1$$

<u>along path $y=0$:</u>

$$\lim_{(x,y)\to(0,0)} \frac{x-y}{x+y} = \lim_{(x,y)\to(0,0)} \frac{x}{x} = \lim_{(x,y)\to(0,0)} 1 = 1$$

$$46.)\ \lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y} \text{ D.N.E. since}$$



<u>along path $y=0$:</u>

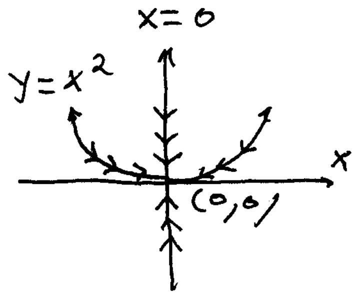
$$\lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y} = \lim_{(x,y)\to(0,0)} \frac{x^2}{x}$$

$$= \lim_{(x,y)\to(0,0)} x = 0$$

<u>along path $x=0$:</u>

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y}{x-y} = \lim_{(x,y)\to(0,0)} \frac{-y}{-y} = \lim_{(x,y)\to(0,0)} 1 = 1$$

47.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y}$ DNE since



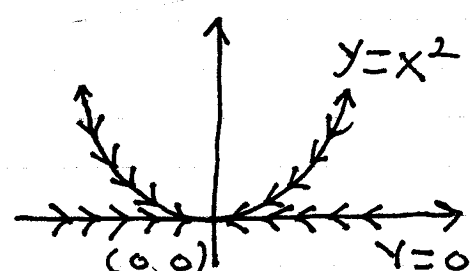
along path $x=0$:

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y} = \lim_{(x,y)\to(0,0)} \frac{y}{y} = \lim_{y\to 0} 1 = 1 ;$$

along path $y=x^2$:

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y}{y} = \lim_{(x,y)\to(0,0)} \frac{2x^2}{x^2} = \lim_{x\to 0} 2 = 2 .$$

48.) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ DNE since



along path $y=0$:

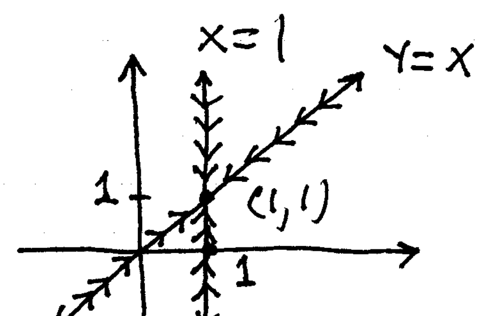
$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{0}{y^2} = \lim_{(x,y)\to(0,0)} 0 = 0$$

along path $y=x^2$:

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{x^4}{x^4+x^4}$$

$$= \lim_{(x,y)\to(0,0)} \frac{x^4}{2x^4} = \lim_{(x,y)\to(0,0)} \frac{1}{2} = \frac{1}{2}$$

49.) $\displaystyle\lim_{(x,y)\to(1,1)} \frac{xy^2-1}{y-1}$



along path $x=1$:

$$\lim_{(x,y)\to(1,1)} \frac{xy^2-1}{y-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(x,y)\to(1,1)} \frac{y^2-1}{y-1}$$

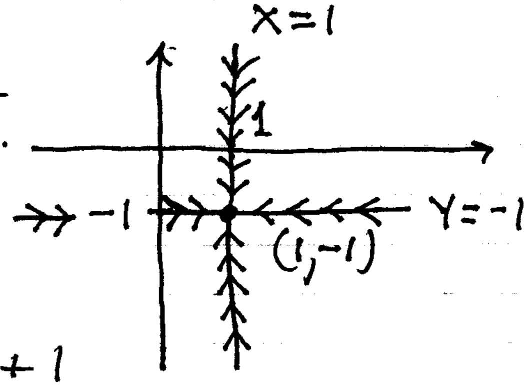
$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(X,Y)\to(1,1)} \frac{\cancel{(Y-1)}(Y+1)}{\cancel{Y-1}} = (1)+1 = 2$$

along path $Y=X$:

$$\lim_{(X,Y)\to(1,1)} \frac{XY^2-1}{Y-1} = \lim_{(X,Y)\to(1,1)} \frac{X^3-1}{X-1}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(X,Y)\to(1,1)} \frac{\cancel{(X-1)}(X^2+X+1)}{\cancel{X-1}} = (1)^2+(1)+1 = 3$$

50.) $\displaystyle\lim_{(X,Y)\to(1,-1)} \frac{XY+1}{X^2-Y^2}$ D.N.E. since



along path $X=1$:

$$\lim_{(X,Y)\to(1,-1)} \frac{XY+1}{X^2-Y^2} = \lim_{(X,Y)\to(1,-1)} \frac{Y+1}{1-Y^2}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(X,Y)\to(1,-1)} \frac{\cancel{1+Y}}{(1-Y)\cancel{(1+Y)}} = \frac{1}{1-(-1)} = \frac{1}{2}$$

along path $Y=-1$:

$$\lim_{(X,Y)\to(1,-1)} \frac{XY+1}{X^2-Y^2} = \lim_{(X,Y)\to(1,-1)} \frac{1-X}{X^2-1}$$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{(X,Y)\to(1,-1)} \frac{-\cancel{(X-1)}}{\cancel{(X-1)}(X+1)} = \frac{-1}{(1)+1} = \frac{-1}{2}$$

Chapter 14
Practice Exercises

12.) $\lim\limits_{(x,y)\to(1,1)} \dfrac{x^3y^3-1}{xy-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim\limits_{(x,y)\to(1,1)} \dfrac{(xy)^3-(1)^3}{xy-1}$

$= \lim\limits_{(x,y)\to(1,1)} \dfrac{\cancel{(xy-1)}((xy)^2+(xy)+1)}{\cancel{xy-1}}$

$= 1+1+1 = 3$

16.) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{xy}$ DNE since

<u>along path $y=x$</u>:

$\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{xy} = \lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2}{x^2} = \lim\limits_{x\to 0} 2 = 2$;

<u>along path $y=-x$</u>:

$\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{xy} = \lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2}{-x^2} = \lim\limits_{x\to 0} -2 = -2$.