Section 10.2

3.) $S_n = 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + (-1)^{n-1}\frac{1}{2^{n-1}}$

$$= 1 + \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right)^2 + \left(\frac{-1}{2}\right)^3 + \cdots + \left(\frac{-1}{2}\right)^{n-1}$$

$$= \frac{1 - \left(\frac{-1}{2}\right)^{(n-1)+1}}{1 - \left(\frac{-1}{2}\right)} = \frac{1 - \left(\frac{-1}{2}\right)^n}{3/2} \longrightarrow$$

$S_n = \frac{2}{3}\left(1 - \left(\frac{-1}{2}\right)^n\right)$; then sum of series is

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{2}{3}\left(1 - \left(\frac{-1}{2}\right)^n\right) = \frac{2}{3}\left(1 - (0)\right) = \frac{2}{3}$$

5.) (HINT: Use partial fractions)

$$\frac{1}{(n+1)(n+2)} = \frac{A}{n+1} + \frac{B}{n+2} \longrightarrow A(n+2) + B(n+1) = 1$$

Let $n = -2$: $-B = 1 \longrightarrow B = -1$

Let $n = -1$: $A = 1$; then parial sum

$$S_n = \sum_{k=1}^{n} \frac{1}{(k+1)(k+2)} = \sum_{k=1}^{n} \left(\frac{1}{k+1} + \frac{-1}{k+2}\right)$$

$$= \left(\frac{1}{2} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{5}}\right) + \cdots + \left(\cancel{\frac{1}{n}} - \cancel{\frac{1}{n+1}}\right)$$

$$+ \left(\cancel{\frac{1}{n+1}} - \frac{1}{n+2}\right) \longrightarrow$$

$S_n = \frac{1}{2} - \frac{1}{n+2}$; then sum of series is

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(\frac{1}{2} - \frac{1}{n+2}\right) = \frac{1}{2} - 0 = \frac{1}{2}$$

7.) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n} = 1 + \frac{-1}{4} + \frac{1}{4^2} + \frac{-1}{4^3} + \frac{1}{4^4} + \frac{-1}{4^5} + \cdots$

$= 1 + \left(\frac{-1}{4}\right) + \left(\frac{-1}{4}\right)^2 + \left(\frac{-1}{4}\right)^3 + \left(\frac{-1}{4}\right)^4 + \cdots$

$= \frac{1}{1-\left(\frac{-1}{4}\right)} = \frac{1}{5/4} = \frac{4}{5}$

8.) $\sum_{n=2}^{\infty} \frac{1}{4^n} = \frac{1}{4^2} + \frac{1}{4^3} + \frac{1}{4^4} + \frac{1}{4^5} + \cdots$

$= \frac{1}{4^2} \cdot \left[1 + \left(\frac{1}{4}\right) + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots\right]$

$= \frac{1}{16} \cdot \frac{1}{1-\left(\frac{1}{4}\right)} = \frac{1}{16} \cdot \frac{4}{3} = \frac{1}{12}$

12.) $\sum_{n=0}^{\infty} \left(\frac{5}{2^n} - \frac{1}{3^n}\right) = 5 \sum_{n=0}^{\infty} \frac{1}{2^n} - \sum_{n=0}^{\infty} \frac{1}{3^n}$

$= 5\left(1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \cdots\right) - \left(1 + \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right)^2 + \cdots\right)$

$= 5 \cdot \frac{1}{1-\left(\frac{1}{2}\right)} - \frac{1}{1-\left(\frac{1}{3}\right)}$

$= 5 \cdot (2) - \frac{3}{2} = \frac{17}{2}$

14.) $\sum_{n=0}^{\infty} \left(\frac{2^{n+1}}{5^n}\right) = \sum_{n=0}^{\infty} 2 \cdot \frac{2^n}{5^n}$

$= 2 \cdot \left[1 + \left(\frac{2}{5}\right) + \left(\frac{2}{5}\right)^2 + \left(\frac{2}{5}\right)^3 + \cdots\right]$

$= 2 \cdot \frac{1}{1-\frac{2}{5}} = 2 \cdot \frac{5}{3} = \frac{10}{3}$

16.) $r = -3 \le -1$, so series diverges

17.) $r = \frac{1}{8}$, $-1 < r < 1$, so series converges

18.) $r = \frac{-2}{3}$, $-1 < r < 1$, so series converges

20.) $0.234234234\cdots$

$$= \frac{234}{1000} + \frac{234}{1{,}000{,}000} + \frac{234}{1{,}000{,}000{,}000} + \cdots$$

$$= 234\left(\frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \frac{1}{10^{12}} + \cdots\right)$$

$$= (234)\left(\frac{1}{10^3}\right)\cdot\left[1 + \frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \cdots\right]$$

$$= \frac{234}{1000}\cdot\left[1 + \left(\frac{1}{1000}\right) + \left(\frac{1}{1000}\right)^2 + \left(\frac{1}{1000}\right)^3 + \cdots\right]$$

$$= \frac{234}{1000}\cdot\frac{1}{1 - \frac{1}{1000}} = \frac{234}{999} = \frac{26}{111}$$

21.) $0.7777\cdots$

$$= \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \frac{7}{10{,}000} + \cdots$$

$$= \frac{7}{10}\cdot\left[1 + \left(\frac{1}{10}\right) + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \cdots\right]$$

$$= \frac{7}{10}\cdot\frac{1}{1 - \frac{1}{10}} = \frac{7}{10}\cdot\frac{10}{9} = \frac{7}{9}$$

28.) $\lim\limits_{n\to\infty} \dfrac{n(n+1)}{(n+2)(n+3)} = \lim\limits_{n\to\infty} \dfrac{n^2+n}{n^2+5n+6} \cdot \dfrac{\frac{1}{n^2}}{\frac{1}{n^2}}$

$= \lim\limits_{n\to\infty} \dfrac{1+\frac{1}{n}}{1+\frac{5}{n}+\frac{6}{n^2}} = \dfrac{1+0}{1+0+0} = 1 \neq 0,$

so series diverges

30.) $\lim\limits_{n\to\infty} \dfrac{n}{n^2+3} \cdot \dfrac{\frac{1}{n^2}}{\frac{1}{n^2}} = \lim\limits_{n\to\infty} \dfrac{1/n}{1+3/n^2} = \dfrac{0}{1+0} = 0$

so nth-term test is inconclusive

31.) $\lim\limits_{n\to\infty} \cos\frac{1}{n} = \cos 0 = 1 \neq 0,$

so series diverges

32.) $\lim\limits_{n\to\infty} \dfrac{e^n}{e^n+n} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{n\to\infty} \dfrac{e^n}{e^n+1} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim\limits_{n\to\infty} \dfrac{e^n}{e^n} = 1 \neq 0$

~~so series diverges.~~ 0

33.) $\lim\limits_{n\to\infty} \ln\frac{1}{n} = \lim\limits_{n\to\infty} (\ln 1 - \ln n)$

$= \lim\limits_{n\to\infty} (-\ln n) = -\infty \neq 0,$

so series diverges

34.) $\lim\limits_{n\to\infty} \cos n\pi$ DNE so $\neq 0$

since $\cos n\pi$: $\cos\pi, \cos 2\pi, \cos 3\pi, \ldots$

$\rightarrow -1, 1, -1, 1, -1, 1, \ldots$; so

series diverges

35.) $S_n = \left(1 - \cancel{\frac{1}{2}}\right) + \left(\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \cdots$
$+ \left(\cancel{\frac{1}{n-1}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+1}\right)$, i.e.,
$S_n = 1 - \frac{1}{n+1}$; then $\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(1 - \frac{1}{n+1}\right)$
$= 1 - (0) = 1$, so series converges with sum of 1

39.) $S_n = \left(\arccos\frac{1}{2} - \cancel{\arccos\frac{1}{3}}\right)$
$+ \left(\cancel{\arccos\frac{1}{3}} - \cancel{\arccos\frac{1}{4}}\right) + \left(\cancel{\arccos\frac{1}{4}} - \cancel{\arccos\frac{1}{5}}\right) + \cdots$
$+ \left(\cancel{\arccos\frac{1}{n}} - \cancel{\arccos\frac{1}{n+1}}\right) + \left(\cancel{\arccos\frac{1}{n+1}} - \arccos\frac{1}{n+2}\right)$, i.e.,
$S_n = \arccos\frac{1}{2} - \arccos\frac{1}{n+2} = \frac{\pi}{3} - \arccos\frac{1}{n+2}$;
then $\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{\pi}{3} - \arccos\frac{1}{n+2}\right)$
$= \frac{\pi}{3} - \arccos 0 = \frac{\pi}{3} - \frac{\pi}{2} = \frac{2\pi}{6} - \frac{3\pi}{6} = \frac{-\pi}{6}$,
so series converges with sum of $\frac{-\pi}{6}$.

40.) $S_n = \left(\cancel{\sqrt{5}} - \sqrt{4}\right) + \left(\cancel{\sqrt{6}} - \cancel{\sqrt{5}}\right) + \left(\cancel{\sqrt{7}} - \cancel{\sqrt{6}}\right) + \cdots$
$+ \left(\cancel{\sqrt{n+3}} - \cancel{\sqrt{n+2}}\right) + \left(\sqrt{n+4} - \cancel{\sqrt{n+3}}\right)$, i.e.,
$S_n = \sqrt{n+4} - \sqrt{4} = \sqrt{n+4} - 2$; then
$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\sqrt{n+4} - 2\right) = \infty - 2 = \infty$,
so series diverges

42.) (HINT: use partial fractions)

$$\frac{6}{(2n-1)(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1}$$

$$= \frac{A(2n+1) + B(2n-1)}{(2n-1)(2n+1)} \longrightarrow$$

$A(2n+1) + B(2n-1) = 6$;

Let $x = \frac{1}{2}$: $2A = 6 \rightarrow A = 3$,

Let $x = \frac{-1}{2}$: $-2B = 6 \rightarrow B = -3$; ~~then~~

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \sum_{n=1}^{\infty} \left(\frac{3}{2n-1} + \frac{-3}{2n+1} \right) \quad ;$$

$S_1 = 3 + (-1) = 2$,

$S_2 = (3 + \cancel{(-1)}) + (\cancel{1} + (-3/5)) = 3 - 3/5$,

$S_3 = (3 + \cancel{(-1)}) + (\cancel{1} + \cancel{(\frac{-3}{5})}) + (\cancel{\frac{3}{5}} + (\frac{-3}{7})) = 3 - \frac{3}{7}$,

$S_4 = (3 + \cancel{(-1)}) + (\cancel{1} + \cancel{(\frac{-3}{5})}) + (\cancel{\frac{3}{5}} + \cancel{(\frac{-3}{7})}) + (\cancel{\frac{3}{7}} + (\frac{-3}{9})) = 3 - \frac{3}{9}$,

$\vdots$

$S_n = 3 - \frac{3}{2n+1}$; and by sequence of partial sums series <u>converges</u> since

$$\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} = \lim_{n \to \infty} \left(3 - \frac{3}{2n+1} \right) = 3 - 0 = 3$$

45.) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$;

$S_1 = 1 - \frac{1}{\sqrt{2}}$,

$S_2 = (1 - \cancel{\frac{1}{\sqrt{2}}}) + (\cancel{\frac{1}{\sqrt{2}}} - \frac{1}{\sqrt{3}}) = 1 - \frac{1}{\sqrt{3}}$,

$S_3 = (1 - \cancel{\frac{1}{\sqrt{2}}}) + (\cancel{\frac{1}{\sqrt{2}}} - \cancel{\frac{1}{\sqrt{3}}}) + (\cancel{\frac{1}{\sqrt{3}}} - \frac{1}{\sqrt{4}}) = 1 - \frac{1}{\sqrt{4}}$,

$\vdots$

$s_n = 1 - \frac{1}{\sqrt{n+1}}$; and by sequence of partial sums series <u>converges</u> since

$$\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) = \lim_{n \to \infty} \left(1 - \frac{1}{\sqrt{n+1}} \right)$$

$$= 1 - 0 = 1 .$$

50.) $\sum_{n=0}^{\infty} (\sqrt{2})^n$; $\lim_{n \to \infty} (\sqrt{2})^n = \infty \neq 0$

so series <u>diverges</u> by nth-term test.

51.) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{3}{2^n} = \sum_{n=1}^{\infty} (-1)^n \cdot (-1) \cdot \frac{3}{2^n}$

$= -3 \cdot \sum_{n=1}^{\infty} \left(\frac{-1}{2} \right)^n$ so series <u>converges</u>

by geometric series test since $r = -\frac{1}{2}$ and $-1 < r < 1$ with value

$$-3 \sum_{n=1}^{\infty} \left(\frac{-1}{2} \right)^n = -3 \cdot \left[\left(\frac{-1}{2} \right) + \left(\frac{-1}{2} \right)^2 + \left(\frac{-1}{2} \right)^3 + \cdots \right]$$

$$= (-3)\left(\frac{-1}{2} \right) \cdot \left[1 + \left(\frac{-1}{2} \right) + \left(\frac{-1}{2} \right)^2 + \left(\frac{-1}{2} \right)^3 + \cdots \right]$$

$$= \frac{3}{2} \cdot \frac{1}{1 - \left(\frac{-1}{2} \right)} = \frac{3}{2} \cdot \frac{2}{3} = 1$$

52.) $\sum_{n=1}^{\infty} (-1)^{n+1} \cdot n$; $\lim_{n \to \infty} (-1)^{n+1} \cdot n \neq 0$

so series <u>diverges</u> by nth-term test.

54.) $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n} = \frac{\cos 0}{1} + \frac{\cos \pi}{5} + \frac{\cos 2\pi}{5^2} + \cdots$

$= 1 - \frac{1}{5} + \frac{1}{5^2} - \frac{1}{5^3} + \cdots$

$= 1 + \left(\frac{-1}{5}\right) + \left(\frac{-1}{5}\right)^2 + \left(\frac{-1}{5}\right)^3 + \cdots$

$= \frac{1}{1-(-1/5)} = \frac{5}{6}$ so series converges by geometric series test since $r = -1/5$ and $-1 < r < 1$.

59.) $\sum_{n=0}^{\infty} \frac{2^n - 1}{3^n} = \sum_{n=0}^{\infty} \left[\left(\frac{2}{3}\right)^n - \left(\frac{1}{3}\right)^n\right]$ (converges) (geo. series)

$= \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n - \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n = \frac{1}{1-2/3} - \frac{1}{1-1/3} = 3 - \frac{3}{2} = \frac{3}{2}$

60.) $\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^n$; $\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n$

$= \lim_{n\to\infty} \left(1 + \frac{(-1)}{n}\right)^n = e^{-1} \neq 0$ so series diverges by nth-term test

62.) $\sum_{n=1}^{\infty} \frac{n^n}{n!} = \frac{1}{1} + \frac{2\cdot 2}{2\cdot 1} + \frac{3\cdot 3\cdot 3}{3\cdot 2\cdot 1} + \frac{4\cdot 4\cdot 4\cdot 4}{4\cdot 3\cdot 2\cdot 1} + \cdots$

$> 1 + 1 + 1 + 1 + \cdots$ so series diverges by comparison to a divergent series

64.) $\lim_{n\to\infty} \frac{2^n + 4^n}{3^n + 4^n} \cdot \frac{1/4^n}{1/4^n} = \lim_{n\to\infty} \frac{(1/2)^n + 1}{(3/4)^n + 1} = \frac{0+1}{0+1} = 1 \neq 0$

so $\sum_{n=1}^{\infty} \frac{2^n + 4^n}{3^n + 4^n}$ diverges by nth-term test

65.) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \sum_{n=1}^{\infty} (\ln n - \ln(n+1))$;

$S_1 = \cancel{\ln 1} - \ln 2 = -\ln 2$,

$S_2 = (\cancel{\ln 1} - \cancel{\ln 2}) + (\cancel{\ln 2} - \ln 3) = -\ln 3$,

$S_3 = (\cancel{\ln 1} - \cancel{\ln 2}) + (\cancel{\ln 2} - \cancel{\ln 3}) + (\cancel{\ln 3} - \ln 4)$

$= -\ln 4$,

$\vdots$

$S_n = -\ln(n+1)$; and by sequence of partial sums series <u>diverges</u> since

$$\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right) = \lim_{n\to\infty} -\ln(n+1) = -\infty$$

67.) $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n = 1 + \left(\frac{e}{\pi}\right) + \left(\frac{e}{\pi}\right)^2 + \left(\frac{e}{\pi}\right)^3 + \cdots$

$= \dfrac{1}{1-\left(\frac{e}{\pi}\right)} = \dfrac{\pi}{\pi - e}$ so series

<u>converges</u> by geometric series test since $r = \frac{e}{\pi}$ and $-1 < r < 1$.

83.) $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ and $\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$ both converge

BUT $\sum_{n=0}^{\infty} \dfrac{(1/2)^n}{(1/3)^n} = \sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$ diverges

84.) $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = \dfrac{1}{1-1/2} = \dfrac{1}{1/2} = 2$ and

$$\sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n = \frac{1}{1-1/3} = \frac{1}{2/3} = 3/2 \text{, BUT}$$

$$\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n\left(\frac{1}{3}\right)^n = \sum_{n=0}^{\infty}\left(\frac{1}{6}\right)^n = \frac{1}{1-1/6} = \frac{1}{5/6}$$

$$= \frac{6}{5} \neq (2)\left(\frac{3}{2}\right) = 3 .$$

85.) SEE examples in problem 84.)

Then $$\sum_{n=0}^{\infty}\frac{(1/3)^n}{\left(\frac{1}{2}\right)^n} = \sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n = \frac{1}{1-2/3}$$

$$= \frac{1}{1/3} = 3 \neq \frac{3/2}{2} = 3/4$$

86.) If Σa_n converges and $a_n > 0$, then $\lim_{n\to\infty} a_n = 0$. Thus,

$$\lim_{n\to\infty}\frac{1}{a_n} = \text{"}\frac{1}{0+}\text{"} = +\infty \neq 0 \text{ so}$$

$\Sigma \frac{1}{a_n}$ diverges by the nth-term test.

87.) If $\sum_{n=1}^{\infty} a_n = \infty$ and $\sum_{n=1}^{K} b_n = b_1 + b_2 + \cdots + b_K = L$

then $\sum_{n=1}^{\infty} a_n \pm \sum_{n=1}^{K} b_K = \infty \pm L = \infty$, so new series still diverges

88.) If $\Sigma a_n = L$ and $\Sigma b_n = \infty$, then $\Sigma (a_n + b_n) = \infty$ (diverges); if $\Sigma (a_n + b_n) = M$ (finite), ~~then~~

$$\Sigma b_n = \Sigma [(a_n + b_n) - a_n]$$

$$= \Sigma (a_n + b_n) - \Sigma a_n = M - L \text{ (finite)}$$

would be a contradiction

89.) a.) $\sum_{n=1}^{\infty} 2r^{n-1} = 2(1 + r + r^2 + r^3 + \cdots)$

$$= 2 \cdot \frac{1}{1-r} = 5 \rightarrow \frac{2}{5} = 1 - r \rightarrow$$

$$r = 1 - \frac{2}{5} = \frac{3}{5} \rightarrow \sum_{n=1}^{\infty} 2\left(\frac{3}{5}\right)^{n-1}$$

90.) $1 + e^b + e^{2b} + e^{3b} + \cdots = 9 \rightarrow$

$$1 + (e^b) + (e^b)^2 + (e^b)^3 + \cdots = 9 \rightarrow$$

$$\frac{1}{1-e^b} = 9 \rightarrow \frac{1}{9} = 1 - e^b \rightarrow$$

$$e^b = \frac{8}{9} \rightarrow b = \ln(8/9).$$

91.) $1+2r+r^2+2r^3+r^4+2r^5+\cdots$

$$= 1+r+r^2+r^3+r^4+r^5+ \\ + r + r^3 + r^5 + \cdots$$

$$= \frac{1}{1-r} + r(1+r^2+r^4+r^6+\cdots)$$

for $-1<r<1$

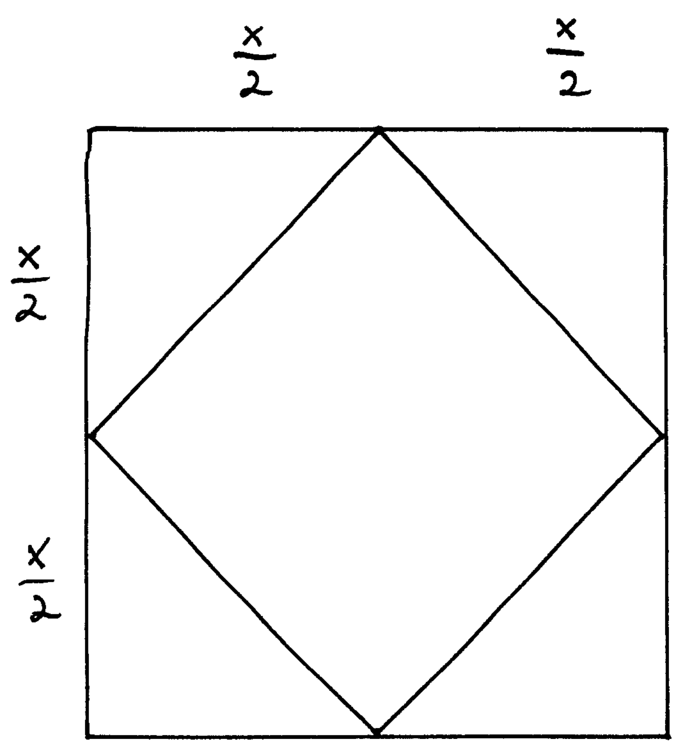
$$= \frac{1}{1-r} + r\left(1+(r^2)+(r^2)^2+(r^2)^3+\cdots\right)$$

$$= \frac{1}{1-r} + r\cdot\frac{1}{1-r^2} \quad \text{for } -1<r<1$$

$$= \frac{1}{1-r}\cdot\frac{1+r}{1+r} + \frac{r}{1-r^2}$$

$$= \frac{1+r}{1-r^2} + \frac{r}{1-r^2} = \frac{1+2r}{1-r^2} \quad \text{for } -1<r<1$$

92.)



assume big square is x by x, then area of inscribed square is

$$\text{Area} = x^2 - 4\left(\frac{x}{2}\right)^2\cdot\frac{1}{2} = \frac{1}{2}x^2 ;$$

inscribed square is $\frac{1}{\sqrt{2}}x$ by $\frac{1}{\sqrt{2}}x$;

then sum of areas of all squares is

$$S = (2)^2 + \left(\frac{1}{\sqrt{2}}\cdot 2\right)^2 + \left(\left(\frac{1}{\sqrt{2}}\right)^2 \cdot 2\right)^2$$

$$+ \left(\left(\frac{1}{\sqrt{2}}\right)^3 \cdot 2\right)^2 + \left(\left(\frac{1}{\sqrt{2}}\right)^4 \cdot 2\right)^2 + \cdots$$

$$= 4 + \left(\frac{1}{\sqrt{2}}\right)^2 \cdot (4) + \left(\frac{1}{\sqrt{2}}\right)^4 \cdot (4) + \left(\frac{1}{\sqrt{2}}\right)^6 \cdot (4) + \cdots$$

$$= 4 \cdot \left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots\right]$$

$$= 4 \cdot \left[1 + \left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots\right]$$

$$= 4 \cdot \frac{1}{1 - \left(\frac{1}{2}\right)} = 8 \ m.^2$$