1.) $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4 \Rightarrow$

$f_x = 2x + y + 3 = 0 \Rightarrow \boxed{y = -2x - 3}$;

$f_y = x + 2y - 3 = 0 \Rightarrow \boxed{x = 3 - 2y}$;

substitute $\Rightarrow$

$y = -2x - 3 = -2(3 - 2y) - 3 \Rightarrow$

$y = -6 + 4y - 3 \Rightarrow 9 = 3y \Rightarrow$

$y = 3 \Rightarrow x = -3$ so $\boxed{(-3,3)}$ is critical point ; $f_{xx} = 2$, $f_{yy} = 2$, $f_{xy} = 1$; then

$D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (2)(2) - (1)^2 = 3 > 0$,

and $f_{xx} = 2 > 0$, so $(-3,3)$ determines a <u>minimum</u> <u>value</u> of $f(3,-3) = -5$.

3.) $f(x,y) = x^2 + xy + 3x + 2y + 5 \Rightarrow$

$f_x = 2x + y + 3 = 0 \Rightarrow \boxed{y = -2x - 3}$;

$f_y = x + 2 = 0 \Rightarrow \boxed{x = -2}$;

substitute $\Rightarrow$

$y = -2x - 3 = -2(-2) - 3 = 1 \Rightarrow$

$\boxed{(-2,1)}$ is critical point ;

$f_{xx} = 2$, $f_{yy} = 0$, $f_{xy} = 1$; then

$D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (2)(0) - (1)^2 = -1 < 0$, so

$(-2,1)$ determines a <u>saddle</u> <u>point</u>.

5.) $f(x,y) = 2xy - x^2 - 2y^2 + 3x + 4 \Rightarrow$

$f_x = 2y - 2x + 3 = 0 \Rightarrow \boxed{y = x - 3/2}$;

$f_y = 2x - 4y = 0 \Rightarrow \boxed{x = 2y}$ (SUB) $\rightarrow$

$y = (2y) - 3/2 \Rightarrow y = 3/2 \Rightarrow x = 3$ so

$\boxed{(3, 3/2)}$ is critical point ;

$f_{xx} = -2$, $f_{yy} = -4$, $f_{xy} = 2$; then

$$D = (f_{xx})(f_{yy}) - (f_{xy})^2$$

$$= (-2)(-4) - (2)^2 = 4 > 0,$$

and $f_{xx} = -2 < 0$, so $(3, 3/2)$ determines a maximum value of $f(3, 3/2) = 17/2$.

15.) $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy \Rightarrow$

$f_x = 12x - 6x^2 + 6y = 6(2x - x^2 + y) = 0 \Rightarrow$

$2x - x^2 + y = 0 \Rightarrow \boxed{y = x^2 - 2x}$;

$f_y = 6y + 6x = 6(y + x) = 0 \Rightarrow y + x = 0 \Rightarrow$

$\boxed{y = -x}$; substitute $\Rightarrow$

$y = x^2 - 2x \Rightarrow -x = x^2 - 2x \Rightarrow$

$0 = x^2 - x = x(x-1) \Rightarrow \underline{x = 0}$ or $\underline{x = 1}$;

if $x = 0$, then $y = 0$ so $\boxed{(0,0)}$ is critical point ; if $x = 1$, then $y = -1$, so $\boxed{(1,-1)}$ is critical point ;

$f_{xx} = 12 - 12x$, $f_{yy} = 6$, $f_{xy} = 6$;

<u>check $(0,0)$</u> : $D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (12)(6) - (6)^2 = 36 > 0$, and

$f_{xx} = 12 > 0$ so $(0,0)$ determines a <u>minimum</u> <u>value</u> of $f(0,0) = 0$.

<u>check $(1,-1)$</u> : $D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (0)(6) - (6)^2 = -36 < 0$, so

$(1,-1)$ determines a <u>saddle</u> <u>point</u>.

16.) $f(x,y) = x^3 + y^3 + 3x^2 - 3y^2 - 8 \Rightarrow$

$f_x = 3x^2 + 6x = 0 \Rightarrow 3x(x+2) = 0 \Rightarrow$

$\boxed{x = 0}$ or $\boxed{x = -2}$;

$f_y = 3y^2 - 6y = 3y(y - 2) \Rightarrow$

$\boxed{y = 0}$ or $\boxed{y = 2}$; then critical points are

$(0,0),\ (0,2),\ (-2,0),\ (-2,2)$;

$f_{xx} = 6x+6,\ f_{yy} = 6y-6,\ f_{xy} = 0$;

check $(0,0)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (6)(-6) - (0)^2 = -36 < 0$, so

$(0,0)$ determines a saddle point ;

check $(0,2)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (6)(6) - (0)^2 = 36 > 0$, and

$f_{xx} = 6 > 0$ so $(0,2)$ determines a minimum value of $f(0,2) = -12$.

check $(-2,0)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (-6)(-6) - (0)^2 = 36 > 0$, and

$f_{xx} = -6 < 0$ so $(-2,0)$ determines a maximum value of $f(-2,0) = -4$;

check $(-2,2)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (-6)(6) - (0)^2 = -36 < 0$, so

$(-2,2)$ determines a saddle point.

19.) $f(x,y) = 4xy - x^4 - y^4 \Rightarrow$

$f_x = 4y - 4x^3 = 4(y - x^3) = 0 \Rightarrow \boxed{y = x^3}$;

$f_y = 4x - 4y^3 = 4(x - y^3) = 0 \Rightarrow \boxed{x = y^3}$;

substitute $\Rightarrow$

$y = x^3 = (y^3)^3 = y^9 \Rightarrow$

$$0 = y^9 - y = y(y^8-1) = y(y^4-1)(y^4+1)$$
$$= y(y^2-1)(y^2+1)(y^4+1)$$
$$= y(y-1)(y+1)(y^2+1)(y^4+1)$$

$y=0$ or $y=1$ or $y=-1$;

if $y=0$, then $x=0$ so $\boxed{(0,0)}$ is critical point ; if $y=1$, then $x=1$ so $\boxed{(1,1)}$ is critical point ; if $y=-1$, then $x=-1$ so $\boxed{(-1,-1)}$ is critical point ;

$f_{xx} = -12x^2$, $f_{yy} = -12y^2$, $f_{xy} = 4$;

<u>check</u> $(0,0)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (0)(0) - (4)^2 = -16 < 0$, so $(0,0)$ determines a <u>saddle point</u> ;

<u>check</u> $(1,1)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (-12)(-12) - (4)^2 = 128 > 0$, and $f_{xx} = -12 < 0$ so $(1,1)$ determines a <u>maximum value</u> of $f(1,1) = 2$;

<u>check</u> $(-1,-1)$: $D = (f_{xx})(f_{yy}) - (f_{xy})^2$
$= (-12)(-12) - (4)^2 = 128 > 0$, and $f_{xx} = -12 < 0$ so $(-1,-1)$ determines a <u>maximum value</u> of $f(-1,-1) = 2$.

22.) $f(x,y) = \frac{1}{x} + xy + \frac{1}{y} \Rightarrow$

$f_x = \frac{-1}{x^2} + y = 0 \Rightarrow \boxed{y = 1/x^2}$;

$f_y = x - \frac{1}{y^2} = 0 \Rightarrow \boxed{x = \frac{1}{y^2}}$;

substitute $\Rightarrow$

$y = \frac{1}{x^2} = \frac{1}{\left(\frac{1}{y^2}\right)^2} = y^4 \Rightarrow$

$y = y^4 \Rightarrow 0 = y^4 - y = y(y^3 - 1) \Rightarrow$

$y = 0$ or $y = 1$; if $y = 0$, ~~then~~

$x = \frac{1}{y^2}$ (impossible!) ; if $y = 1$,

then $x = 1$ so $\boxed{(1,1)}$ is critical

point ; $f_{xx} = \frac{2}{x^3}$, $f_{yy} = \frac{2}{y^3}$,

$f_{xy} = 1$; ~~then~~

$D = (f_{xx})(f_{yy}) - (f_{xy})^2$

$= (2)(2) - (1)^2 = 3 > 0$, and

$f_{xx} = 2 > 0$, so $(1,1)$ determines

a minimum value of

$f(1,1) = 3$.

26.) $z = e^y - ye^x \Rightarrow$

$z_x = -ye^x = 0 \to \boxed{y=0}$ or $e^x = 0$ (no!);

$z_y = e^y - e^x = 0 \to e^y = e^x \to$

$\ln e^y = \ln e^x \to \boxed{y = x}$ so

$\boxed{(0,0)}$ is critical point; and

$D = z_{xx} \cdot z_{yy} - (z_{xy})^2$

$= (-ye^x)(e^y) - (-e^x)^2$ so at $(0,0)$

$D = (0)(1) - (-1)^2 = -1 < 0$ so

$(0,0)$ determines a <u>saddle point</u>

at $z = 1$.

28.) $z = x^2e^x - y^2e^x \Rightarrow$

$z_x = x^2e^x + 2x \cdot e^x - y^2e^x = 0 \to$

$e^x(x^2 + 2x - y^2) = 0 \to \boxed{x^2 + 2x - y^2 = 0}$,

$z_y = -2ye^x = 0 \to \boxed{y=0} \to$ (SUB) $\to$

$x^2 + 2x - 0 = 0 \to x(x+2) = 0 \to$

$\boxed{x=0}$ or $\boxed{x=-2}$, so critical pts. are

$\boxed{(0,0)}$ and $\boxed{(-2,0)}$; then

$z_{xx} = x^2e^x + 2x \cdot e^x + 2x \cdot e^x + 2 \cdot e^x - y^2e^x$

$= (x^2 + 4x + 2 - y^2)e^x$,

$z_{yy} = -2e^x$, $z_{xy} = -2ye^x$;

<u>TEST (0,0)</u>: $D = z_{xx} \cdot z_{yy} - (z_{xy})^2$

$= (2)(-2) - (0)^2 = -4 < 0$, so $(0,0)$ determines a <u>saddle point</u> at $z = 0$

<u>TEST $(-2,0)$</u>: $D = z_{xx} z_{yy} - (z_{xy})^2$

$= (-2e^{-2})(-2e^{-2}) - (0)^2 = 4e^{-4} > 0$, and $z_{xx} = -2e^{-2} < 0$ so $(-2,0)$ determines a maximum value at $z = 4e^{-2}$

29.) $z = 2\ln x + \ln y - 4x - y \Rightarrow$

$z_x = 2 \cdot \frac{1}{x} - 4 = 0 \rightarrow \frac{2}{x} = 4 \rightarrow \boxed{x = \frac{1}{2}}$,

$z_y = \frac{1}{y} - 1 = 0 \rightarrow \frac{1}{y} = 1 \rightarrow \boxed{y = 1}$, so critical point is $\boxed{(1/2, 1)}$, and

$z_{xx} = \frac{-2}{x^2}$, $z_{yy} = \frac{-1}{y^2}$, $z_{xy} = 0$;

$D = z_{xx} \cdot z_{yy} - z_{xy}^2$

$= (-8)(-1) - (0)^2 = 8 > 0$, and $z_{xx} = -8 < 0$, so $(1/2, 1)$ determines a <u>maximum value</u> at

$z = 2\ln\frac{1}{2} + \ln 1 - 2 - 1 = 2\ln\frac{1}{2} - 3$

30.) $z = \ln(x+y) + x^2 - y \Rightarrow$

$z_x = \boxed{\frac{1}{x+y} + 2x = 0}$,

$z_y = \frac{1}{x+y} - 1 = 0 \rightarrow \frac{1}{x+y} = 1 \rightarrow$

$X+Y=1 \longrightarrow \boxed{Y=1-X} \longrightarrow$ (SUB) $\longrightarrow$

$\frac{1}{X+(1-X)} + 2X = 0 \longrightarrow 1 = -2X \longrightarrow \boxed{X = \frac{-1}{2}}$

$\longrightarrow \boxed{Y = \frac{3}{2}}$, so critical pt. is

$\boxed{\left(-\frac{1}{2}, \frac{3}{2}\right)}$; then

$z_{XX} = \frac{-1}{(X+Y)^2} + 2$, $z_{YY} = \frac{-1}{(X+Y)^2}$,

$z_{XY} = \frac{-1}{(X+Y)^2}$, and

$D = z_{XX} \cdot z_{YY} - (z_{XY})^2$

$= (1)\cdot(-1) - (-1)^2 = -2 < 0$, so

$\left(-\frac{1}{2}, \frac{3}{2}\right)$ determines a <u>saddle pt.</u>

at $z = \ln 1 + \frac{1}{4} - \frac{3}{2} = -5/4$