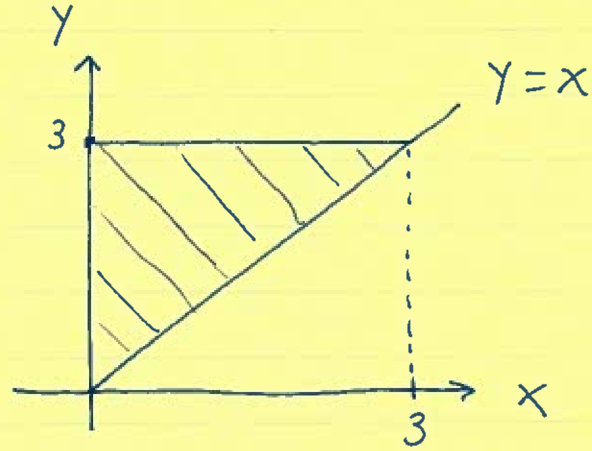
31.)



$f(x,y) = 2x^2 - 4x + y^2 - 4y + 1 \Rightarrow$
$f_x = 4x - 4 = 4(x-1) = 0 \Rightarrow x = 1$;
$f_y = 2y - 4 = 2(y-2) = 0 \Rightarrow y = 2$,
so $\boxed{(1,2)}$ is critical point ;
corners are $\boxed{(0,0)}$, $\boxed{(3,3)}$, and $\boxed{(0,3)}$;

<u>along path $x=0$</u>: $z = y^2 - 4y + 1 \Rightarrow$
$z' = 2y - 4 = 2(y-2) = 0 \Rightarrow y = 2$ so
$\boxed{(0,2)}$ is critical point ;

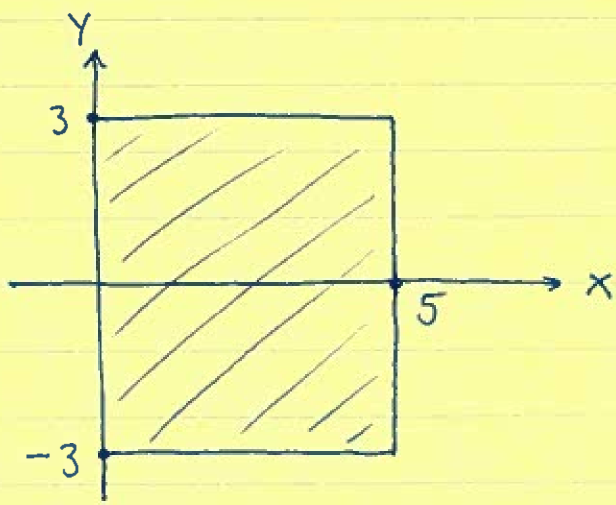
<u>along path $y=3$</u>: $z = 2x^2 - 4x - 2 \Rightarrow$
$z' = 4x - 4 = 4(x-1) = 0 \Rightarrow x = 1$ so
$\boxed{(1,3)}$ is critical point ;

<u>along path $y=x$</u>:
$z = 2x^2 - 4x + x^2 - 4x + 1 \Rightarrow$
$z = 3x^2 - 8x + 1 \Rightarrow$
$z' = 6x - 8 = 0 \Rightarrow x = 4/3$, so
$\boxed{(4/3, 4/3)}$ is critical point :

compare function values:

critical points and corners	function values	
$(1,2)$	$f(1,2) = -5$	
$(0,0)$	$f(0,0) = 1$	
$(3,3)$	$f(3,3) = 4$	MAX
$(0,3)$	$f(0,3) = -2$	
$(0,2)$	$f(0,2) = -3$	
$(1,3)$	$f(1,3) = -12$	MIN
$(4/3, 4/3)$	$f(\frac{4}{3}, 4/3) = -37/9$	

34.)



$T(x,y) = x^2 + xy + y^2 - 6x \Rightarrow$

$T_x = 2x + y - 6 = 0 \Rightarrow \underline{y = -2x + 6}$;

$T_y = x + 2y = 0 \Rightarrow \underline{x = -2y}$; substitute

$\Rightarrow y = -2x + 6 = -2(-2y) + 6 = 4y + 6 \Rightarrow$

$0 = 3y + 6 \Rightarrow y = -2 \Rightarrow x = 4$ so

$\boxed{(4,-2)}$ is critical point ;

corners are $(0,3)$, $(0,-3)$, $(5,3)$, $(5,-3)$;

along path $X=0$: $Z = Y^2 \Rightarrow Z' = 2Y = 0 \Rightarrow$
$Y = 0$ so $\boxed{(0,0)}$ is critical point ;

along path $X=5$: $Z = Y^2 + 5Y - 5 \Rightarrow$
$Z' = 2Y + 5 = 0 \Rightarrow Y = -5/2$, so
$\boxed{(5, -5/2)}$ is critical point ;

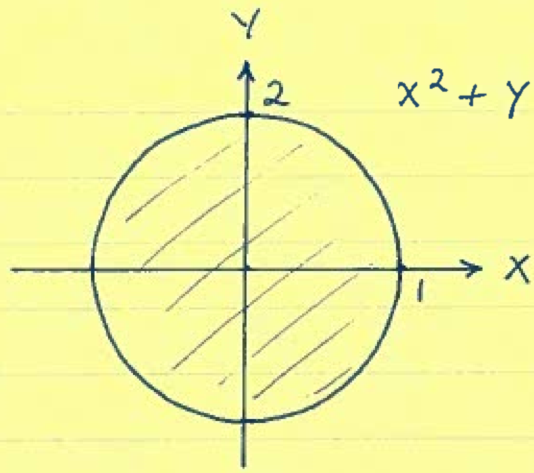
along path $Y=3$: $Z = X^2 - 3X + 9 \Rightarrow$
$Z' = 2X - 3 = 0 \Rightarrow X = 3/2$, so
$\boxed{(3/2, 3)}$ is critical point ;

along path $Y=-3$: $Z = X^2 - 9X + 9 \Rightarrow$
$Z' = 2X - 9 = 0 \Rightarrow X = 9/2$, so
$\boxed{(9/2, 3)}$ is critical point ;

compare function values:

critical points and corners	function values	
$(4,-2)$	$T(4,-2) = -12$	MIN
$(0,3)$	$T(0,3) = 9$	
$(0,-3)$	$T(0,-3) = 9$	
$(5,3)$	$T(5,3) = 19$	MAX
$(5,-3)$	$T(5,-3) = -11$	
$(0,0)$	$T(0,0) = 0$	
$(5,-5/2)$	$T(5,-5/2) = -45/4$	
$(3/2,3)$	$T(3/2,3) = 27/4$	
$(9/2,3)$	$T(9/2,3) = 63/4$	

41.)



$x^2 + y^2 = 1$

$T(x,y) = x^2 + 2y^2 - x \Rightarrow$

$T_x = 2x - 1 = 0 \Rightarrow x = 1/2$;

$T_y = 4y = 0 \Rightarrow y = 0$, so

$\boxed{(1/2, 0)}$ is critical point ;

along path $x^2 + y^2 = 1$: $\begin{cases} x = \cos t \\ y = \sin t \end{cases}$

for $0 \le t \le 2\pi$:

$z = (\cos t)^2 + 2(\sin t)^2 - \cos t$

$= \underline{\cos^2 t + \sin^2 t} + \sin^2 t - \cos t$

$= 1 + \sin^2 t - \cos t \Rightarrow$

$z' = 2 \sin t \cos t + \sin t$

$= \sin t\,(2\cos t + 1) = 0 \Rightarrow$

$\sin t = 0 \Rightarrow \underline{t = 0°}$ or $\underline{t = 180°}$ OR

$\cos t = -1/2 \Rightarrow \underline{t = 120°}$ or $\underline{t = 240°}$;

so critical points are :

$t = 0°$: $(1, 0)$

$t = 180°$: $(-1, 0)$

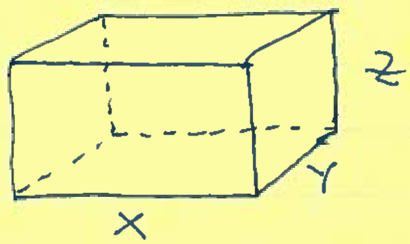
$t = 120°$: $(-1/2, \sqrt{3}/2)$

$t = 240°$: $(-1/2, -\sqrt{3}/2)$

compare function values :

critical points	function values	
$(\frac{1}{2}, 0)$	$T(\frac{1}{2}, 0) = \boxed{-\frac{1}{4}\,^\circ F}$	MIN
$(1, 0)$	$T(1, 0) = 0^\circ F$	
$(-1, 0)$	$T(-1, 0) = 2^\circ F$	
$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$	$T(-\frac{1}{2}, \frac{\sqrt{3}}{2}) = \boxed{2\frac{1}{4}\,^\circ F}$	MAX
$(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$	$T(-\frac{1}{2}, -\frac{\sqrt{3}}{2}) = \boxed{2\frac{1}{4}\,^\circ F}$	MAX

I.)



Volume

$V = XYZ = 1$ ~~so~~

$\boxed{Z = \frac{1}{XY}}$;

minimize cost (¢)

$$C = C_{top} + C_{bottom} + C_{sides}$$

$$= 3XY + 3XY + 2(2XZ + 2YZ)$$

$$= 6XY + 4(X+Y) \cdot Z$$

$$= 6XY + 4(X+Y) \cdot \frac{1}{XY} \Rightarrow$$

$$\boxed{C = 6XY + \frac{4}{Y} + \frac{4}{X}}$$; ~~then~~

$C_X = 6Y - \frac{4}{X^2} = 0 \Rightarrow \underline{Y = \frac{2}{3X^2}}$;

$C_Y = 6X - \frac{4}{Y^2} = 0 \Rightarrow \underline{X = \frac{2}{3Y^2}}$;

substitute $\Rightarrow$

$$Y = \frac{2}{3X^2} = \frac{2}{3\left(\frac{2}{3Y^2}\right)^2} = \frac{2}{\frac{4}{3Y^4}} = 2 \cdot \frac{3}{4} Y^4 \Rightarrow$$

$$Y = \frac{3}{2}Y^4 \Rightarrow 0 = \frac{3}{2}Y^4 - Y = Y\left(\frac{3}{2}Y^3 - 1\right)$$

$$\Rightarrow Y = 0 \text{ (NO!) OR } Y = \underline{\underline{\left(\frac{2}{3}\right)^{1/3}}} \text{ ft.} \Rightarrow$$

$$X = \frac{2}{3\left(\frac{2}{3}\right)^{2/3}} = \frac{2}{3 \cdot \frac{2^{2/3}}{3^{2/3}}} = \underline{\underline{\left(\frac{2}{3}\right)^{1/3}}} \text{ ft.} \Rightarrow$$

$$Z = \frac{1}{XY} = \frac{1}{\left(\frac{2}{3}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3}} = \frac{1}{\left(\frac{2}{3}\right)^{2/3}} = \underline{\underline{\left(\frac{3}{2}\right)^{2/3}}} \text{ ft.}$$

and minimum cost is

$$C = 6XY + \frac{4}{Y} + \frac{4}{X}$$

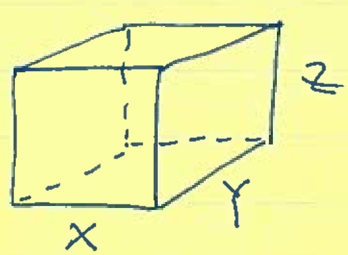
$$= 6\left(\frac{2}{3}\right)^{1/3}\left(\frac{2}{3}\right)^{1/3} + \frac{4}{\left(\frac{2}{3}\right)^{1/3}} + \frac{4}{\left(\frac{2}{3}\right)^{1/3}}$$

$$= 6\left(\frac{2}{3}\right)^{2/3} + 8 \cdot \left(\frac{3}{2}\right)^{1/3}$$

$$= 2^{5/3}3^{1/3} + 2^{5/3} \cdot 3^{1/3}$$

$$= \underline{\underline{2^{8/3}3^{1/3}}} \approx \underline{\underline{9.16}}\ ¢$$

II.)



Surface area

$$S = 2XY + 2XZ + 2YZ = 12$$

$$\Rightarrow XY + XZ + YZ = 6$$

$$\Rightarrow XY + (X+Y)Z = 6$$

$$\Rightarrow \boxed{Z = \frac{6 - XY}{X+Y}} \quad ;$$

maximize volume

$$V = XYZ = XY \cdot \frac{6 - XY}{X+Y} = \frac{6XY - X^2Y^2}{X+Y} \Rightarrow$$

$$\boxed{V = \frac{6XY - X^2Y^2}{X+Y}} \text{ ; then}$$

$$V_X = \frac{(X+Y)(6Y - 2XY^2) - (6XY - X^2Y^2)}{(X+Y)^2} = 0 \Rightarrow$$

$$Y\left[(X+Y)\cdot(6 - 2XY) - (6X - X^2Y)\right] = 0 \Rightarrow$$

$$Y = 0 \text{ (NO!) OR } \cancel{6X} + 6Y - 2X^2Y - 2XY^2 - \cancel{6X} + X^2Y = 0 \Rightarrow$$

$$Y \cdot \left[6 - 2X^2 - 2XY + X^2\right] = 0 \Rightarrow Y = 0 \text{ (NO!)}$$

$$\text{OR } 6 - X^2 - 2XY = 0 \Rightarrow$$

$$2XY = 6 - X^2 \Rightarrow \boxed{Y = \frac{6 - X^2}{2X}} \text{ ; and}$$

$$V_Y = \frac{(X+Y)(6X - 2X^2Y) - (6XY - X^2Y^2)}{(X+Y)^2} = 0 \Rightarrow$$

$$X\left[(X+Y)(6 - 2XY) - (6Y - XY^2)\right] = 0 \Rightarrow$$

$$X = 0 \text{ (NO!) OR } 6X + \cancel{6Y} - 2X^2Y - 2XY^2 - \cancel{6Y} + XY^2 = 0 \Rightarrow$$

$$X \cdot \left[6 - 2XY - 2Y^2 + Y^2\right] = 0 \Rightarrow X = 0 \text{ (NO!) OR}$$

$$6 - 2XY - Y^2 = 0 \Rightarrow 2XY = 6 - Y^2 \Rightarrow$$

$$\boxed{X = \frac{6 - Y^2}{2Y}} \text{ ; substitute } \Rightarrow$$

$$X = \frac{6 - Y^2}{2Y} = \frac{6 - \left(\frac{6 - X^2}{2X}\right)^2}{2\left(\frac{6 - X^2}{2X}\right)} \cdot \frac{(2X)^2}{(2X)^2} \Rightarrow$$

$$X = \frac{6(2X)^2 - (6 - X^2)^2}{2(2X)(6 - X^2)} \Rightarrow$$

$$4x^2(6-x^2) = 24x^2 - (36-12x^2+x^4) \Rightarrow$$

$$\cancel{24x^2} - 4x^4 = \cancel{24x^2} - 36 + 12x^2 - x^4 \Rightarrow$$

$$\begin{aligned} 0 &= 3x^4 + 12x^2 - 36 \\ &= 3\left((x^2)^2 + 4(x^2) - 12\right) \\ &= 3(x^2-2)(x^2+6) \Rightarrow \end{aligned}$$

$x^2 - 2 = 0 \Rightarrow \boxed{x = \sqrt{2}\text{ m.}}$ or $x = -\sqrt{2}$ (NO!);

if $x = \sqrt{2}$, then

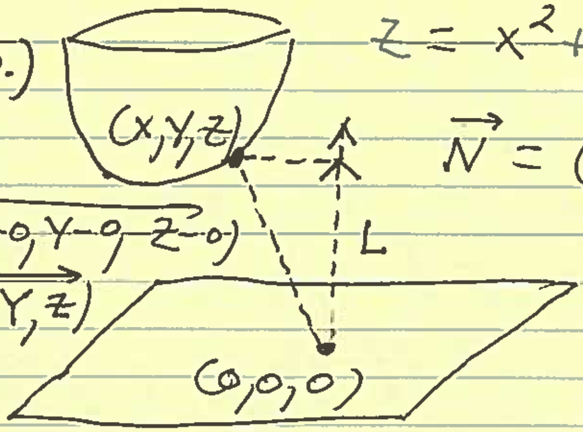
$$Y = \frac{6-(\sqrt{2})^2}{2(\sqrt{2})} = \frac{6-2}{2\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \Rightarrow$$

$\boxed{Y = \sqrt{2}\text{ m.}}$, then $z = \dfrac{6-(\sqrt{2})(\sqrt{2})}{\sqrt{2}+\sqrt{2}} = \dfrac{4}{2\sqrt{2}}$

$\Rightarrow \boxed{z = \sqrt{2}\text{ m.}}$; and max. volume is $V = (\sqrt{2})^3 \Rightarrow$

$\boxed{V = 2\sqrt{2}\text{ m.}^3}$.

50.)



$z = x^2 + y^2 + 10$

$\vec{N} = \overrightarrow{(1,2,-1)}$

$\vec{A} = \overrightarrow{(x-0, y-0, z-0)}$
$= \overrightarrow{(x, y, z)}$

$x + 2y - z = 0$

Distance from (x,y,z) to plane is

$$L = |\text{proj}_{\vec{N}} \vec{A}| = \frac{|\vec{A} \cdot \vec{N}|}{|\vec{N}|}$$

$$= \frac{|x + 2y - z|}{\sqrt{6}}$$

$$= \frac{|x + 2y - (x^2 + y^2 + 10)|}{\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}} |x^2 - x + y^2 - 2y + 10| \qquad (*)$$

$$= \frac{1}{\sqrt{6}} \left| x^2 - x + \tfrac{1}{4} + y^2 - 2y + 1 + 10 - 1 - \tfrac{1}{4} \right|$$

$$= \frac{1}{\sqrt{6}} \left| \underbrace{\left(x - \tfrac{1}{2}\right)^2 + (y-1)^2 + \tfrac{35}{4}}_{>0} \right| , \text{ so}$$

Minimize

$$\boxed{L = \frac{1}{\sqrt{6}} (x^2 - x + y^2 - 2y + 10)} \rightarrow$$

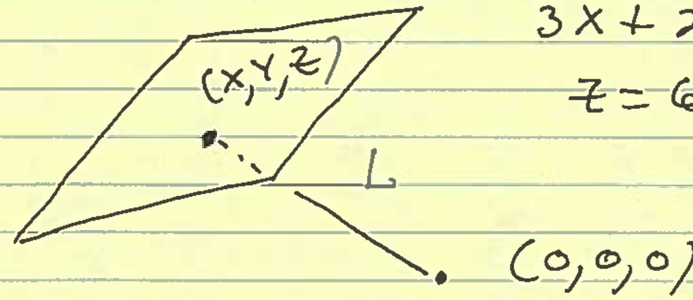
$$L_x = \frac{1}{\sqrt{6}} (2x - 1) = 0 \rightarrow x = \frac{1}{2},$$

$$L_y = \frac{1}{\sqrt{6}} (2y - 2) = 0 \rightarrow y = 1 \rightarrow$$

$$z = \left(\tfrac{1}{2}\right)^2 + (1)^2 + 10 = \tfrac{1}{4} + 11 = 45/4, \text{ so}$$

point $(x, y, z) = \left(\frac{1}{2}, 1, \frac{45}{4}\right)$.

51.)



$3x + 2y + z = 6 \rightarrow$

$z = 6 - 3x - 2y$

Minimize

$$L = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$$

$$= \sqrt{x^2 + y^2 + z^2} \longrightarrow$$

$$\boxed{L = \sqrt{x^2 + y^2 + (6 - 3x - 2y)^2}} \longrightarrow$$

$$L_x = \frac{1}{\cancel{2}}(\sim)^{-1/2}\left[\cancel{2}x + \cancel{2}(6 - 3x - 2y)\cdot(-3)\right] = 0 \rightarrow$$

$x - 18 + 9x + 6y = 0 \rightarrow 10x + 6y = 18 \rightarrow$

$\boxed{5x + 3y = 9}$;

$$L_y = \frac{1}{\cancel{2}}(\sim)^{-1/2}\left[\cancel{2}y + \cancel{2}(6 - 3x - 2y)\cdot(-2)\right] = 0 \rightarrow$$

$y - 12 + 6x + 4y = 0 \rightarrow \boxed{6x + 5y = 12} \rightarrow$

$$\begin{cases} -25x - 15y = -45 \\ 18x + 15y = 36 \end{cases} \rightarrow -7x = -9 \rightarrow$$

$x = \frac{9}{7} \rightarrow \quad 5\left(\frac{9}{7}\right) + 3y = 9 \rightarrow$

$3y = \frac{63}{7} - \frac{45}{7} = \frac{18}{7} \rightarrow \quad y = \frac{6}{7} \longrightarrow$

$z = 6 - 3\left(\frac{9}{7}\right) - 2\left(\frac{6}{7}\right) = \frac{42}{7} - \frac{27}{7} - \frac{12}{7} = \frac{3}{7}$

so ~~pt.~~ $(x, y, z) = \left(\frac{9}{7}, \frac{6}{7}, \frac{3}{7}\right)$

53.) Let x, y, z be #'s, $x+y+z=9$ and $\hookrightarrow z = 9-x-y$

minimize $S = x^2+y^2+z^2$

$\rightarrow \boxed{S = x^2+y^2+(9-x-y)^2}$

$\rightarrow S_x = 2x + 2(9-x-y)(-1)$

$= 2x - 18 + 2x + 2y$

$= 4x + 2y - 18 = 0 \rightarrow$

$2y = 18 - 4x \rightarrow \boxed{y = 9-2x}$;

$S_y = 2y + 2(9-x-y)(-1)$

$= 2y - 18 + 2x + 2y$

$= 4y + 2x - 18 = 0 \rightarrow$

$2x = 18 - 4y \rightarrow \boxed{x = 9-2y} \rightarrow$ (SUB) $\rightarrow$

$y = 9 - 2(9-2y) = 9 - 18 + 4y \rightarrow$

$3y = 9 \rightarrow \boxed{y=3} \rightarrow \boxed{x=3} \rightarrow \boxed{z=3}$.

54.) Let x, y, z be + #'s, $x+y+z = 3 \rightarrow$

$z = 3-x-y$ and

minimize

$P = xyz \rightarrow P = xy(3-x-y) \rightarrow$

$\boxed{P = 3xy - x^2y - xy^2} \rightarrow$

$P_x = 3y - 2xy - y^2 = y(3-2x-y) = 0$

$\rightarrow y = 0$ (NO) or $3-2x-y=0$

$\rightarrow \boxed{y = 3-2x}$,

$P_Y = 3X - X^2 - 2XY = X(3 - X - 2Y) = 0 \rightarrow$

$X = 0$ (NO) or $3 - X - 2Y = 0 \rightarrow$

$\boxed{X = 3 - 2Y} \rightarrow$ (SUB) $\rightarrow$

$Y = 3 - 2(3 - 2Y) = 3 - 6 + 4Y \rightarrow$

$3Y = 3 \rightarrow \boxed{Y = 1} \rightarrow \boxed{X = 1} \rightarrow \boxed{Z = 1}$

55.) $X + Y + Z = 6 \rightarrow Z = 6 - X - Y$ and maximize $S = XY + YZ + XZ \rightarrow$ (SUB) $\rightarrow$

$S = XY + Y(6 - X - Y) + X(6 - X - Y)$

$= \cancel{XY} + 6Y - \cancel{XY} - Y^2 + 6X - X^2 - XY \rightarrow$

$\boxed{S = 6X + 6Y - X^2 - Y^2 - XY} \rightarrow$

$S_X = 6 - 2X - Y = 0 \rightarrow \boxed{Y = 6 - 2X}$,

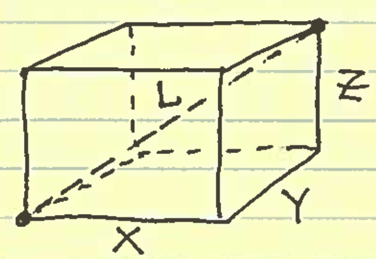
$S_Y = 6 - 2Y - X = 0 \rightarrow \boxed{X = 6 - 2Y}$

$\rightarrow$ (SUB) $\rightarrow Y = 6 - 2(6 - 2Y) = -6 + 4Y$

$\rightarrow 3Y = 6 \rightarrow \boxed{Y = 2} \rightarrow \boxed{X = 2} \rightarrow$

$\boxed{Z = 2}$

57.)



Length of diagonal is diameter of sphere so

$X^2 + Y^2 + Z^2 = L^2 = 4^2$

$\rightarrow X^2 + Y^2 + Z^2 = 16$ and maximize

$\hookrightarrow Z = \sqrt{16 - X^2 - Y^2}$

volume $V = XYZ \longrightarrow$

$$\boxed{V = XY \cdot \sqrt{16 - X^2 - Y^2}} \longrightarrow$$

$$V_X = XY \cdot \frac{1}{\cancel{2}}(16 - X^2 - Y^2)^{-1/2} \cdot (-\cancel{2}X)$$
$$+ Y\sqrt{16 - X^2 - Y^2}$$

$$= \frac{-X^2Y}{\sqrt{16 - X^2 - Y^2}} + \frac{Y\sqrt{16 - X^2 - Y^2}}{1}$$

$$= \frac{-X^2Y + Y(16 - X^2 - Y^2)}{\sqrt{16 - X^2 - Y^2}} = 0 \longrightarrow$$

$$Y \cdot [-X^2 + (16 - X^2 - Y^2)] = 0 \longrightarrow$$

$$Y = 0 \text{ (NO) or } 16 - 2X^2 - Y^2 = 0 \longrightarrow$$

$$\boxed{2X^2 + Y^2 = 16},$$

$$V_Y = XY \cdot \frac{1}{\cancel{2}}(16 - X^2 - Y^2)^{-1/2} \cdot (-\cancel{2}Y)$$
$$+ X\sqrt{16 - X^2 - Y^2}$$

$$= \frac{-XY^2}{\sqrt{16 - X^2 - Y^2}} + \frac{X\sqrt{16 - X^2 - Y^2}}{1}$$

$$= \frac{-XY^2 + X(16 - X^2 - Y^2)}{\sqrt{16 - X^2 - Y^2}} = 0 \longrightarrow$$

$$X[-Y^2 + (16 - X^2 - Y^2)] = 0 \longrightarrow$$

$x=0$ (NO) or $16-x^2-2y^2=0 \longrightarrow$

$\boxed{x^2+2y^2=16} \longrightarrow x^2=16-2y^2$

$\longrightarrow$ (SUB) $\longrightarrow 2(16-2y^2)+y^2=16$

$\longrightarrow 32-4y^2+y^2=16$

$\longrightarrow 3y^2=16 \longrightarrow y^2=\frac{16}{3} \longrightarrow y=\pm\sqrt{\frac{16}{3}}$

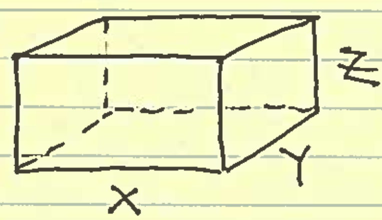
$\longrightarrow \boxed{y=\frac{4}{\sqrt{3}}} \longrightarrow x^2=16-2\left(\frac{16}{3}\right)$

$\longrightarrow x^2=\frac{48}{3}-\frac{32}{3}=\frac{16}{3} \longrightarrow \boxed{x=\frac{4}{\sqrt{3}}}$

$\longrightarrow z=\sqrt{16-\left(\frac{4}{\sqrt{3}}\right)^2-\left(\frac{4}{\sqrt{3}}\right)^2}=\sqrt{\frac{48}{3}-\frac{16}{3}-\frac{16}{3}}$

$\longrightarrow \boxed{z=\frac{4}{\sqrt{3}}}$

58.)



Volume

$xyz=27\text{ cm.}^3 \longrightarrow$

$z=\frac{27}{xy}$ and

minimize surface area

$S=2xy+2xz+2yz$

$=2xy+2x\cdot\left(\frac{27}{xy}\right)+2y\left(\frac{27}{xy}\right) \longrightarrow$

$\boxed{S=2xy+\frac{54}{y}+\frac{54}{x}} \longrightarrow$

$$S_X = 2Y - \frac{54}{X^2} = 0 \to Y = \frac{27}{X^2},$$

$$S_Y = 2X - \frac{54}{Y^2} = 0 \to X = \frac{27}{Y^2} \to$$

(SUB) $\to X = \dfrac{27}{\left(\dfrac{27}{X^2}\right)^2} = \dfrac{1}{27}X^4 \to$

$$27X = X^4 \to X^4 - 27X = 0 \to$$

$X(X^3 - 27) = 0 \to X = 0$ (NO) or

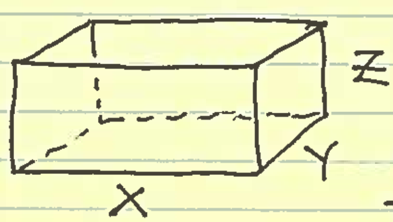
$$X^3 - 27 = 0 \to \boxed{X = 3 \text{ cm.}} \to$$

$$Y = \frac{27}{(3)^2} \to \boxed{Y = 3 \text{ cm.}} \to$$

$\boxed{Z = 3 \text{ cm.}}$ and min. surface area $S = 18 + 18 + 18 \to$

$$\boxed{S = 54 \text{ cm}^2}$$

59.)



Surface area

$XY + 2XZ + 2YZ = 12$ ft^2

$$\to XY + Z(2X + 2Y) = 12$$

$$\to Z(2X + 2Y) = 12 - XY \to$$

$\boxed{Z = \dfrac{12 - XY}{2X + 2Y}}$, and maximize volume

$$V = XYZ = XY \cdot \frac{12 - XY}{2X + 2Y} \longrightarrow$$

$$\boxed{V = \frac{12XY - X^2Y^2}{2X + 2Y}} \longrightarrow$$

$$V_X = \frac{(2X+2Y)(12Y - 2XY^2) - (12XY - X^2Y^2)(2)}{(2X+2Y)^2}$$

$$= \frac{\cancel{24XY} - 4X^2Y^2 + 24Y^2 - 4XY^3 - \cancel{24XY} + 2X^2Y^2}{(2X+2Y)^2}$$

$$= \frac{24Y^2 - 4XY^3 - 2X^2Y^2}{(2X+2Y)^2} = 0 \longrightarrow$$

$$2Y^2(12 - 2XY - X^2) = 0 \rightarrow Y = 0 \text{ (NO)}$$

$$\text{or } 12 - 2XY - X^2 = 0 \longrightarrow$$

$$2XY = 12 - X^2 \rightarrow \boxed{Y = \frac{12 - X^2}{2X}} \text{, and}$$

$$V_Y = \frac{(2X+2Y)(12X - 2X^2Y) - (12XY - X^2Y^2)(2)}{(2X+2Y)^2}$$

$$= \frac{24X^2 - 4X^3Y + \cancel{24XY} - 4X^2Y^2 - \cancel{24XY} + 2X^2Y^2}{(2X+2Y)^2}$$

$$= \frac{24X^2 - 4X^3Y - 2X^2Y^2}{(2X+2Y)^2} = 0 \longrightarrow$$

$$X^2[24 - 4XY - 2Y^2] = 0 \longrightarrow$$

$x = 0$ (NO) or $24 - 4xy - 2y^2 = 0$

$\rightarrow \boxed{12 - 2xy - y^2 = 0} \rightarrow$ (SUB) $\rightarrow$

$$12 - \cancel{2x} \cdot \frac{12 - x^2}{\cancel{2x}} - \frac{(12 - x^2)^2}{(2x)^2} = 0 \rightarrow$$

$$\cancel{12} - \cancel{12} + x^2 - \frac{x^4 - 24x^2 + 144}{4x^2} = 0$$

$\rightarrow 4x^4 - x^4 + 24x^2 - 144 = 0$

$\rightarrow 3x^4 + 24x^2 - 144 = 0$

$\rightarrow 3(x^4 + 8x^2 - 48) = 0$

$\rightarrow 3(x^2 - 4)(x^2 + 12) = 0$

$\rightarrow 3(x - 2)(x + 2)(x^2 + 12) = 0$

$\rightarrow \boxed{x = 2 \text{ ft.}} \rightarrow \boxed{y = 2 \text{ ft.}} \rightarrow$

$\boxed{z = 1 \text{ ft.}}$