Section 10.9

1.) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \longrightarrow$

$$e^{-5x} = 1 + (-5x) + \frac{(-5x)^2}{2!} + \frac{(-5x)^3}{3!} + \cdots$$

$$= 1 - 5x + \frac{5^2}{2!}x^2 - \frac{5^3}{3!}x^3 + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{5^n}{n!} x^n$$

4.) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \longrightarrow$

$$\sin\left(\tfrac{\pi}{2}x\right) = \left(\tfrac{\pi}{2}x\right) - \frac{\left(\frac{\pi}{2}x\right)^3}{3!} + \frac{\left(\frac{\pi}{2}x\right)^5}{5!} - \cdots$$

$$= \frac{\pi}{2}x - \frac{\pi^3}{2^3 3!}x^3 + \frac{\pi^5}{2^5 5!}x^5 - \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1} \cdot x^{2n+1}}{2^{2n+1}(2n+1)!}$$

5.) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \longrightarrow$

$$\cos(5x^2) = 1 - \frac{(5x^2)^2}{2!} + \frac{(5x^2)^4}{4!} - \frac{(5x^2)^6}{6!} + \cdots$$

$$= 1 - \frac{5^2x^4}{2!} + \frac{5^4x^8}{4!} - \frac{5^6x^{12}}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{5^{2n}x^{4n}}{(2n)!}$$

6.) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n}}{(2n)!} \longrightarrow$

$$\cos\left(\frac{x^{3/2}}{\sqrt{2}}\right) = 1 - \frac{\left(\frac{x^{3/2}}{\sqrt{2}}\right)^2}{2!} + \frac{\left(\frac{x^{3/2}}{\sqrt{2}}\right)^4}{4!} - \frac{\left(\frac{x^{3/2}}{\sqrt{2}}\right)^6}{6!} + \cdots$$

$$= 1 - \frac{x^3}{2 \cdot 2!} + \frac{x^6}{2^2 \cdot 4!} - \frac{x^9}{2^3 \cdot 6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{3n}}{2^n(2n)!}$$

7.) $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \longrightarrow$

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = 1 + (-x) + (-x)^2 + (-x)^3 + (-x)^4 + \cdots$$
$$= 1 - x + x^2 - x^3 + x^4 - \cdots \longrightarrow$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \cdots \longrightarrow$$

$$\ln(1+x^2) = x^2 - \frac{1}{2}(x^2)^2 + \frac{1}{3}(x^2)^3 - \frac{1}{4}(x^2)^4 + \cdots$$
$$= x^2 - \frac{1}{2}x^4 + \frac{1}{3}x^6 - \frac{1}{4}x^8 + \cdots = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n}}{n}$$

9.) $$\frac{1}{1+\frac{3}{4}x^3} = 1 - \left(\frac{3}{4}x^3\right) + \left(\frac{3}{4}x^3\right)^2 - \left(\frac{3}{4}x^3\right)^3 + \cdots$$
$$= 1 - \frac{3}{4}x^3 + \left(\frac{3}{4}\right)^2 x^6 - \left(\frac{3}{4}\right)^3 x^9 + \cdots = \sum_{n=0}^{\infty} (-1)^n \left(\frac{3}{4}\right)^n x^{3n}$$

10.) $$\frac{1}{2-x} = \frac{1}{2} \cdot \frac{1}{1-\left(\frac{x}{2}\right)} = \frac{1}{2}\left(1 + \left(\frac{x}{2}\right) + \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^3 + \cdots\right)$$
$$= \frac{1}{2} \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n$$

11.) $$xe^x = x \cdot \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)$$
$$= x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

12.) $$x^2 \sin x = x^2 \cdot \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)$$
$$= x^3 - \frac{x^5}{3!} + \frac{x^7}{5!} - \frac{x^9}{7!} + \cdots = \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{x^{2n+1}}{(2n-1)!}$$

16.) $x^2\cos(x^2) = x^2\cdot\left[1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!} - \frac{(x^2)^6}{6!} + \cdots\right]$

$= x^2\cdot\left[1 - \frac{x^4}{2!} + \frac{x^8}{4!} - \frac{x^{12}}{6!} + \cdots\right]$

$= x^2 - \frac{x^6}{2!} + \frac{x^{10}}{4!} - \frac{x^{14}}{6!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{4n+2}}{(2n)!}$

17.) $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$= \frac{1}{2}\left(1 + \left(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \cdots\right)\right.$

$$= \frac{1}{2}\left(2 - \frac{2^2}{2!}x^2 + \frac{2^4}{4!}x^4 - \frac{2^6}{6!}x^6 + \cdots\right)$$

$$= 1 - \frac{2}{2!}x^2 + \frac{2^3}{4!}x^4 - \frac{2^5}{6!}x^6 + \cdots$$

$$= 1 + \sum_{n=1}^{\infty}(-1)^n \cdot \frac{2^{2n-1}}{(2n)!}\, x^{2n} \qquad \text{OR}$$

$$\cos^2 x = \cos x \cdot \cos x$$

$$= \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right)\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right)$$

$$\begin{aligned}
= 1 - \frac{x^2}{2!} &+ \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \\
- \frac{x^2}{2!} &+ \frac{x^4}{4} - \frac{x^6}{48} + \cdots \\
&+ \frac{x^4}{4!} - \frac{x^6}{48} + \cdots \\
&\qquad\quad - \frac{x^6}{6!} + \cdots
\end{aligned}$$

$$= 1 - x^2 + \frac{1}{3}x^4 - \frac{2}{45}x^6 + \cdots$$

19.) $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \longrightarrow$

$$\frac{x^2}{1-2x} = x^2 \cdot \frac{1}{1-(2x)} = x^2 \cdot \left[1 + (2x) + (2x)^2 + (2x)^3 + \cdots\right]$$

$$= x^2 + 2x^3 + 2^2 \cdot x^4 + 2^3 \cdot x^5 + \cdots = \sum_{n=1}^{\infty} 2^{n-1} \cdot x^{n+1}$$

20.) $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \longrightarrow$

$$x \cdot \ln(1+(2x)) = x \cdot \left[(2x) - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \frac{(2x)^4}{4} + \cdots\right]$$

$= x \cdot \left[2x - \frac{2^2}{2}x^2 + \frac{2^3}{3}x^3 - \frac{2^4}{4}x^4 + \cdots \right]$

$= 2x^2 - \frac{2^2}{2}x^3 + \frac{2^3}{3}x^4 - \frac{2^4}{4}x^5 + \cdots$

$= \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{2^n}{n} x^{n+1}$

21.) $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots \xrightarrow{D}$

$D(1-x)^{-1} = D(1 + x + x^2 + x^3 + x^4 + \cdots) \rightarrow$

$-1(1-x)^{-2} \cdot (-1) = 0 + 1 + 2x + 3x^2 + 4x^3 + \cdots \rightarrow$

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots = \sum_{n=1}^{\infty} n x^{n-1}$

OR

$\frac{1}{(1-x)^2} = \frac{1}{1-x} \cdot \frac{1}{1-x}$

$= (1 + x + x^2 + x^3 + \cdots)(1 + x + x^2 + x^3 + \cdots)$

$$
\begin{aligned}
= 1 + x + x^2 + x^3 + \cdots \\
+ x + x^2 + x^3 + \cdots \\
+ x^2 + x^3 + \cdots \\
+ x^3 + \cdots
\end{aligned}
$$

$= 1 + 2x + 3x^2 + 4x^3 + \cdots$

22.) $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \cdots \xrightarrow{D}$

$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \cdots \xrightarrow{D}$

$\frac{2}{(1-x)^3} = 2 + 3 \cdot 2x + 4 \cdot 3x^2 + 5 \cdot 4x^3 + \cdots$

$= \sum_{n=0}^{\infty} (n+2)(n+1) x^n$

24.) $\sin x \cos x = \frac{1}{2}\sin 2x$

$= \frac{1}{2}\left((2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \cdots\right)$

$= \frac{1}{2}\left(2x - \frac{2^3}{3!}x^3 + \frac{2^5}{5!}x^5 - \frac{2^7}{7!}x^7 + \cdots\right)$

$= x - \frac{2^2}{3!}x^3 + \frac{2^4}{5!}x^5 - \frac{2^6}{7!}x^7 + \cdots = \sum_{n=0}^{\infty}(-1)^n \frac{2^{2n}}{(2n+1)!}x^{2n+1}$

OR (much more difficult)

$\sin x \cdot \cos x = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots\right)$

$= x - \frac{1}{2}x^3 + \frac{1}{4!}x^5 - \frac{1}{6!}x^7 + \cdots$

$\quad - \frac{1}{6}x^3 + \frac{1}{12}x^5 - \frac{1}{3!4!}x^7 + \cdots$

$\quad + \frac{1}{5!}x^5 - \frac{1}{5!2!}x^7 + \cdots$

$\quad - \frac{1}{7!}x^7 + \cdots$

HARD WORK

$= \cdots = x - \frac{2^2}{3!}x^3 + \frac{2^4}{5!}x^5 - \frac{2^6}{7!}x^7 + \cdots$

25.) $e^x \cdot \frac{1}{1+x} = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(1 - x + x^2 - x^3 + \cdots\right)$

$$\left.\begin{aligned} = 1 - x + x^2 - x^3 + x^4 - \cdots \\ + x - x^2 + x^3 - x^4 + \cdots \\ + \tfrac{1}{2}x^2 - \tfrac{1}{2}x^3 + \tfrac{1}{2}x^4 - \cdots \\ + \tfrac{1}{6}x^3 - \tfrac{1}{6}x^4 + \cdots \\ + \tfrac{1}{24}x^4 - \cdots \end{aligned}\right\} = 1 + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{3}{8}x^4 + \cdots$$

29.) $e^x \sin x = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)$

$$\left.\begin{aligned}
= \; & x - \tfrac{1}{6}x^3 + \tfrac{1}{120}x^5 - \cdots \\
& + x^2 - \tfrac{1}{6}x^4 + \cdots \\
& + \tfrac{1}{2}x^3 - \tfrac{1}{12}x^5 + \cdots \\
& + \tfrac{1}{6}x^4 - \tfrac{1}{36}x^6 + \cdots \\
& + \tfrac{1}{24}x^5 - \tfrac{1}{144}x^7 + \cdots
\end{aligned}\right\} = x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5 + \cdots$$

31.) $\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - \cdots \longrightarrow$

$$\frac{1}{1+x^2} = 1 - (x^2) + (x^2)^2 - (x^2)^3 + (x^2)^4 - \cdots$$

$$= 1 - x^2 + x^4 - x^6 + x^8 - \cdots \longrightarrow$$

$$\arctan x = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots \longrightarrow$$

$$(\arctan x)^2 = \left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \cdots\right)\left(x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \cdots\right)$$

$$\begin{aligned}
= \; & x^2 - \tfrac{1}{3}x^4 + \tfrac{1}{5}x^6 - \tfrac{1}{7}x^8 + \cdots \\
& - \tfrac{1}{3}x^4 + \tfrac{1}{9}x^6 - \tfrac{1}{15}x^8 + \cdots \\
& + \tfrac{1}{5}x^6 - \tfrac{1}{15}x^8 + \cdots \\
& - \tfrac{1}{7}x^8 + \cdots
\end{aligned}$$

$$= x^2 - \frac{2}{3}x^4 + \frac{23}{45}x^6 - \frac{44}{105}x^8 + \cdots$$

33.) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$,

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \rightarrow$

$e^{\sin x} = 1 + (\sin x) + \frac{(\sin x)^2}{2!} + \frac{(\sin x)^3}{3!} + \frac{(\sin x)^4}{4!} + \cdots$

$$\left\{ (\sin x)^2 = \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right) \right.$$

$$= \left.\begin{matrix} x^2 - \frac{1}{6}x^4 + \cdots \\ - \frac{1}{6}x^4 + \cdots \end{matrix}\right\} = x^2 - \frac{1}{3}x^4 \ ;$$

$$(\sin x)^3 = (\sin x) \cdot (\sin x)^2$$

$$= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots\right)\left(x^2 - \frac{1}{3}x^4 + \cdots\right)$$

$$= \left.\begin{matrix} x^3 - \frac{1}{6}x^5 + \cdots \\ - \frac{1}{3}x^5 + \cdots \end{matrix}\right\} = x^3 - \cdots \ ;$$

$$(\sin x)^4 = (\sin x)^2 (\sin x)^2$$

$$= \left(x^2 - \frac{1}{3}x^4 + \cdots\right)\left(x^2 - \frac{1}{3}x^4 + \cdots\right)$$

$$\left. = \left.\begin{matrix} x^4 - \frac{1}{3}x^6 + \cdots \\ - \frac{1}{3}x^6 + \cdots \end{matrix}\right\} = x^4 - \cdots \right\}$$

$$= 1 + \left(x - \frac{1}{6}x^3 + \cdots\right) + \frac{1}{2}\left(x^2 - \frac{1}{3}x^4 + \cdots\right)$$

$$+ \frac{1}{6}\left(x^3 - \cdots\right) + \frac{1}{24}\left(x^4 - \cdots\right)$$

$$= 1 + x + \frac{1}{2}x^2 - \frac{1}{8}x^4 + \cdots$$

$$R_n(x;a) = \frac{f^{(n+1)}(c_n)}{(n+1)!}(x-a)^{n+1},$$ where c_n is between x and a

37.) $\sin x = \underbrace{x - \frac{x^3}{3!}}_{} + \underbrace{\frac{x^5}{5!} - \frac{x^7}{7!} + \cdots}_{R_4(x;0)}$

!! $\rightarrow$ $P_4(x;0) = P_3(x;0)$

$$|R_4(x;0)| = \left| \frac{f^{(5)}(c_4)\cdot(x-0)^5}{5!} \right|,$$ c_4 is between x and 0

$$= \frac{|\cos(c_4)|\cdot|x|^5}{5!}$$

$$\leq \frac{1\cdot|x|^5}{5!}$$

$$= \frac{|x|^5}{120};$$ require that

$$\frac{|x|^5}{120} \leq 5\cdot 10^{-4} = 0.0005 \longrightarrow$$

$$|x|^5 \leq 0.06 \longrightarrow$$

$$|x| \leq (0.06)^{1/5} \approx 0.5696 \longrightarrow$$

$$-0.5696 \leq x \leq 0.5696$$

40.) $f(x) = (1+x)^{1/2} \xrightarrow{D} f'(x) = \frac{1}{2}(1+x)^{-1/2} \xrightarrow{D}$

$f''(x) = \frac{-1}{2^2}(1+x)^{-3/2} \xrightarrow{D} f'''(x) = \frac{3}{2^3}\cdot(1+x)^{-\frac{5}{2}} \xrightarrow{D}$

$f^{(4)}(x) = -\frac{3\cdot 5}{2^4}\cdot(1+x)^{-7/2} \xrightarrow{D}$

$f^{(5)}(x) = \frac{3\cdot 5\cdot 7}{2^5}(1+x)^{-9/2} \longrightarrow \cdots$

$$f^{(n)}(x) = \frac{(-1)^{n+1} \cdot (1\cdot 3\cdot 5 \cdots (2n-3))}{2^n} (1+x)^{-\frac{(2n-1)}{2}}$$

for $n = 2, 3, 4, 5, \ldots$; then

$$a_0 = \frac{f(0)}{0!} = \frac{1}{1} = 1, \quad a_1 = \frac{f'(0)}{1!} = \frac{\frac{1}{2}}{1} = \frac{1}{2},$$

$$a_2 = \frac{f''(0)}{2!} = \frac{-1}{2^2 \cdot 2!}, \quad a_3 = \frac{f'''(0)}{3!} = \frac{3}{2^3 \cdot 3!},$$

$$a_4 = \frac{f^{(4)}(0)}{4!} = \frac{-3\cdot 5}{2^4 \cdot 4!}, \ldots,$$

$$a_n = \frac{f^{(n)}(0)}{n!} = \frac{(-1)^{n+1}(1\cdot 3\cdot 5 \cdots (2n-3))}{2^n \cdot n!}$$ for $n = 2, 3, 4, \ldots$;

$$(1+x)^{1/2} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3 + \cdots$$

$$= 1 + \frac{1}{2}x + \sum_{n=2}^{\infty} \frac{(-1)^{n+1}(1\cdot 3\cdot 5 \cdots (2n-3))}{2^n \cdot n!} x^n$$

$$(1+x)^{1/2} = \underbrace{1 + \frac{1}{2}x}_{P_1(x;0)} \underbrace{- \frac{1}{8}x^2 + \frac{1}{16}x^3 - \cdots}_{R_1(x;0)}$$

$$|R_1(x;0)| = \left| \frac{f''(c_1)\cdot(x-0)^2}{2!} \right|$$

$$\left(f(x) = (1+x)^{1/2} \xrightarrow{D} f'(x) = \frac{1}{2}(1+x)^{-1/2} \xrightarrow{D} \right.$$
$$\left. f''(x) = \frac{-1}{4}(1+x)^{-3/2} \right)$$

$$= \left| \frac{\frac{-1}{4}(1+c_1)^{-3/2} \cdot x^2}{2!} \right| = \frac{1}{8} \cdot \frac{|x|^2}{|1+c_1|^{3/2}}$$

(where c_1 is between x and 0)

$$\leq \frac{1}{8} \cdot \frac{|0.01|^2}{|1+(-0.01)|^{3/2}} \quad \text{(since } -0.01 < x < 0.01\text{)}$$

$$= \frac{1}{8} \cdot \frac{(0.01)^2}{(0.99)^{3/2}}$$

$$\approx 0.0000127$$

41.) $$e^x = \underbrace{1 + x + \frac{x^2}{2}}_{P_2(x;0)} + \underbrace{\frac{x^3}{3!} + \frac{x^4}{4!} + \cdots}_{R_2(x;0)}$$

$$|R_2(x;0)| = \left| \frac{f^{(3)}(c_2) \cdot (x-0)^3}{3!} \right|, \quad c_2 \text{ is between } x \text{ and } 0$$

$$= \frac{e^{c_2}}{6} \cdot |x|^3$$

$$\leq \frac{e^{0.1}}{6} (0.1)^3 \quad \text{(since } -0.1 < x < 0.1\text{)}$$

$$< \frac{3^{0.1}}{6} (0.1)^3 \approx 0.000186$$

43.) $\sin^2 x = \frac{1}{2}(1-\cos 2x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$

$= \frac{1}{2} - \frac{1}{2}\left(1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \frac{(2x)^6}{6!} - \cdots\right)$

$= \frac{1}{2} - \frac{1}{2} + \frac{2^2x^2}{2\cdot 2!} - \frac{2^4x^4}{2\cdot 4!} + \frac{2^6x^6}{2\cdot 6!} - \cdots$

$= \frac{2}{2!}x^2 - \frac{2^3}{4!}x^4 + \frac{2^5}{6!}x^6 - \frac{2^7}{8!}x^8 + \cdots \xrightarrow{D}$

$D\sin^2 x = 2\sin x\cos x$

$= 2x - \frac{2^3}{4!}\cdot 4x^3 + \frac{2^5}{6!}\cdot 6x^5 - \frac{2^7}{8!}\cdot 8x^7 + \cdots$

$= 2x - \frac{2^3}{3!}x^3 + \frac{2^5}{5!}x^5 - \frac{2^7}{7!}x^7 + \cdots = \sum_{n=0}^{\infty}(-1)^n\frac{2^{2n+1}x^{2n+1}}{(2n+1)!}$

48.) $\frac{1}{1-x} = \underbrace{1+x+x^2+x^3}_{P_3(x;0)} + \underbrace{x^4+x^5+\cdots}_{R_3(x;0)}$

$f(x) = \frac{1}{1-x} = (1-x)^{-1} \xrightarrow{D}$

$f'(x) = -(1-x)^{-2}\cdot(-1) = (1-x)^{-2} \xrightarrow{D}$

$f''(x) = -2(1-x)^{-3}\cdot(-1) = 2(1-x)^{-3} \xrightarrow{D}$

$f'''(x) = -6(1-x)^{-4}\cdot(-1) = 6(1-x)^{-4} \xrightarrow{D}$

$f^{(4)}(x) = -24(1-x)^{-5}(-1) = \frac{24}{(1-x)^5}$; then

$|R_3(x;0)| = \left|\frac{f^{(4)}(c_3)}{4!}\cdot(x-0)^4\right| = \frac{\cancel{24}}{\cancel{24}}\cdot\frac{|x|^4}{|1-c_3|^5}$

$\leq \frac{|0.1|^4}{|1-0.1|^5}$ (since $-0.1 < x < 0.1$ and c_3 is between x and 0)

≈ 0.000169