☒ Review:

Ch 15

- Double Integrals $\iint_R f(x,y)\,dA$ as Riemann Sum
- Iterating double integrals for evaluation
- Fubini's Theorem - "Changing order of integration"
- Polar Coordinates $dxy = r\,dr\,d\theta$
- General change of variables $dx\,dy = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} du\,dv$
- Area, density, center of mass, moment of inertia, kinetic energy of rotation
- Triple Integrals - same thing
- cylindrical coordinates
- spherical coordinates
- General coordinate trans: $dx\,dy\,dz = \begin{vmatrix} - \nabla x - \\ - \nabla y - \\ - \nabla z - \end{vmatrix} du\,dv\,dw$

Ch 13 Theory of curves

- $\vec{r}(t) = (x(t), y(t), z(t))$
- velocity $\vec{v} = \vec{r}\,'(t)$, $\frac{ds}{dt} = \|\vec{v}(t)\|$

  unit tangent $\vec{T} = \frac{\vec{v}}{\|\vec{v}\|}$, $\vec{v} = \frac{ds}{dt}\vec{T}$, $v = \frac{ds}{dt} = \|\vec{v}\|$

  velocity points tangent to curve, length = speed $\frac{ds}{dt}$
- acceleration $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$
- uniform circular motion

  helix
- Derivation of Newton's Gravitational Force Law From Kepler's 3 laws

  $\vec{F} = M_p \vec{a} = -G \frac{M_p M_s}{r^2} \frac{\vec{r}}{r}$, $r = \sqrt{x^2+y^2+z^2}$, $\frac{\partial r}{\partial x} = \frac{x}{r}$ etc
- Projectile Motion
- $\vec{a} = \frac{d^2 s}{dt^2}\vec{T} + \kappa v^2 \vec{N}$ $\quad \vec{N} = \frac{d\vec{T}}{dt} \Big/ \left\|\frac{d\vec{T}}{dt}\right\|$
- $\|\vec{T}(t)\| = \text{const} \Rightarrow \vec{a} \perp \vec{T}$ etc
- arclength: $ds = \|\vec{v}(t)\|\,dt$

Ch 15 Three generalizations of FTC

(1) $\int_{C} \nabla f \cdot \vec{T}\, ds = f(B) - f(A)$ Cons of Energy

(2) $\iint_{S} \text{Curl}\vec{F} \cdot \vec{n}\, dS = \int_{C} \vec{F} \cdot \vec{T}\, ds$ Stokes Thm

(3) $\iiint_{V} \text{Div}\vec{F}\, dV = \iint_{S} \vec{F} \cdot \vec{n}\, dS$ Div Thm

- $\text{Curl}\,F = \nabla \times \vec{F}$, $\text{Div}\,\vec{F} = \nabla \cdot \vec{F}$, $\nabla = (\partial_x, \partial_y, \partial_z)$

- Line Integrals - (Integrals along curves)

$$\int_{C} \vec{F} \cdot \vec{T}\, ds = \int_a^b \vec{F} \cdot \vec{V}(t)\, dt = \int_{C} \vec{F} \cdot d\vec{r} = \int_{C} M\,dx + N\,dy + P\,dz$$

Meaning use Coord Syst = Parameterization to comput 2 equivalent forms

- Liebniz rules of differentials shows they are all equal

- Computing line integrals esp line integrals on circles, helix,

Conservative Vector Fields : $\vec{F} = \nabla f$

- Thm: $\vec{F}$ conservative iff $\oint_C \vec{F}\cdot\vec{T} = 0$

$$f(\underset{\sim}{x}) = \int_A^{\underset{\sim}{x}} \vec{F}\cdot\vec{T}\,ds \quad \text{indept of path}$$

- $\int_C \vec{F}\cdot\vec{T}\,ds = -\int_{-C} \vec{F}\cdot\vec{T}\,ds$ "a curve has an orientation"

- $0 \xrightarrow{\nabla} \mathbb{R}^3 \xrightarrow{\text{Curl}} \mathbb{R}^3 \xrightarrow{\text{Div}} \mathbb{R}$ two in a row make zero

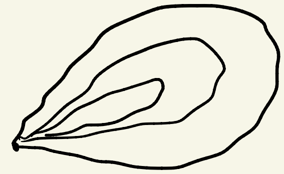
Thm: If $\vec{F}$ conservative then $\text{Curl}\,\vec{F} = 0$

- The other way requires simply connected

Thm: If $\text{Curl}\,\vec{F} = 0$ in a simply connected domain, then $\vec{F}$ is conservative

Defn: Simply connected "every curve contracts to a point...

... so Stokes Thm applies."

p 

$$\iint_S \underbrace{\cancel{\text{Curl}\,\vec{F}}}_{0}\, dS = \int_C \vec{F}\cdot\vec{T}\,ds$$

- Find $\vec{F}$ given $\mathrm{Curl}\,\vec{F} = 0$ in sc domain
  "partial integration"

- Ex: $\vec{F} = \overrightarrow{\left(-\frac{y}{r^2}, \frac{x}{r^2}\right)}$ $\quad$ $\mathrm{Curl}\,\vec{F} = 0$
  $\oint_C \vec{F}\cdot\vec{T}\,ds = 2\pi$

- In Physics $\vec{F} = \nabla f = -\nabla P$ $\quad$ $P = -f$ potential energy for force $\vec{F}$

Conservation of Energy

$$\int_C \vec{F}\cdot\vec{T}\,ds = -(P(B) - P(A))$$

$$\int_C \vec{F}\cdot\vec{T}\,ds = \int_a^b m\,\vec{v}'(t)\cdot\vec{v}(t)\,dt = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

$\vec{F} = m\vec{a}$ is only force acting

Green's Theorem

$$\iint_R N_x - M_y\,dA = \int_C M\,dx + N\,dy$$

- Equiv To Stokes Thm in plane

- Takes form of Div Thm when written in terms of $\vec{F}_\perp$: $\iint_R \mathrm{Div}\,F_\perp\,dA = \int_C \vec{F}_\perp\cdot\vec{n}\,ds$

- Example of $\vec{F}$ st $\oint_C \vec{F}\cdot\vec{T}\,ds$ = Area enclosed by C

Surface Integrals

- Flux Integral:

$\vec{F} = \delta\vec{v}$ = Mass Flux Vector $\quad \iint_S \vec{F}\cdot\vec{n}\,dS = \frac{\text{mass}}{\text{time}}$ thru S

- Parameterization $\equiv$ Coord System on S

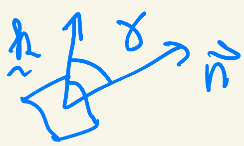
$\vec{r}(u,v) = (x(u,v), y(u,v), z(u,v))$

- $dS = \underbrace{\|\vec{r}_u \times \vec{r}_v\|}_{\text{amplification factor}}\,du\,dv \Rightarrow \iint_S \vec{F}\cdot\vec{n}\,dS = \iint_{R_{uv}} \vec{F}\cdot\vec{r}_u\times\vec{r}_v\,du\,dv$

- Surface Area $= \iint_{R_{uv}} \|\vec{r}_u \times \vec{r}_v\|\,du\,dv$

- $z = f(x,y) \quad \vec{r}(x,y) = (x, y, f(x,y))$

$\|\vec{r}_u \times \vec{r}_v\| = \frac{1}{\vec{n}\cdot\underset{\sim}{k}} = \frac{1}{\cos\gamma}$



Stokes Theorem

- Outer Normal orients closed surface

- Gives Meaning of Curl $\vec{F}$

$$\text{Curl}\,\vec{F} = \lim_{\varepsilon \to 0} \frac{1}{|D_\varepsilon|} \oint_{C_\varepsilon} \vec{F}\cdot\vec{T}\,ds = \frac{\text{Flux}}{\text{Vol}}$$

If $\vec{F} = \vec{V}$, $\text{Curl}\,\vec{F}\cdot\vec{n} = 4\pi w$, $w = \frac{\text{rev}}{\text{sec}}$ around $\vec{n}$

$= \frac{1}{T}$, T = period of 1-rev

Divergence Theorem

- Meaning of Divergence

$$\text{Div}\,\vec{F} = \lim_{\varepsilon \to 0} \frac{1}{|V_\varepsilon|} \iint_{\partial_\varepsilon} \vec{F}\cdot\vec{n}\,dS = \text{Flux}/\text{Vol}$$

- Continuity Equation $\text{Curl}\,\delta\vec{v} = 0 \iff$ Cons of Mass