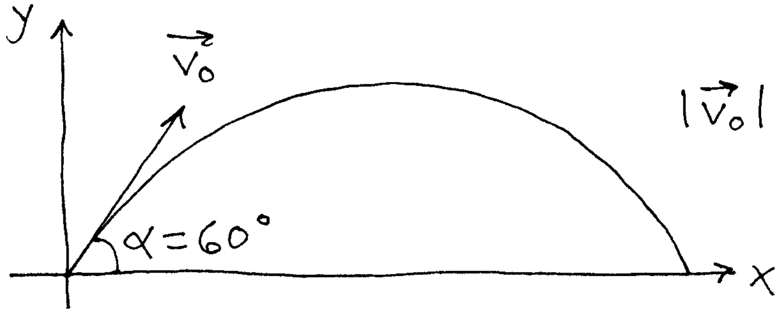
Section 13.2

$x(t) = |\vec{v}_0| \cos\alpha \cdot t$, $y(t) = |\vec{v}_0| \sin\alpha \cdot t - \frac{1}{2} g \cdot t^2$

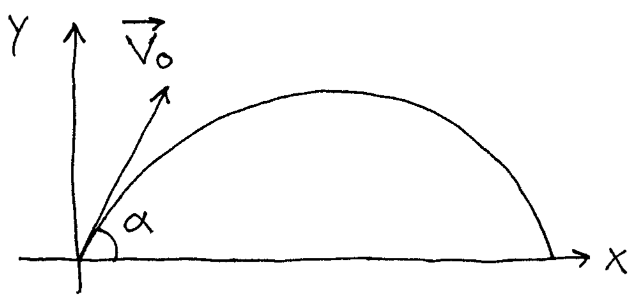
1.)



$|\vec{v}_0| = 840$ m./sec.

$x(t) = (840)(\cos 60°)t = (840)(\frac{1}{2})t = 420t \rightarrow$
$x(t) = 420t$; if $x(t) = 21$ km $= 21{,}000$ m. $\rightarrow$
$420t = 21{,}000 \rightarrow$ $t = 50$ sec.

2.)



$y(t) = |\vec{v}_0| \sin\alpha \cdot t - \frac{1}{2}(9.8)t^2$
$= |\vec{v}_0| \sin\alpha \cdot t - 4.9t^2$; find flight time $\rightarrow$ $y(t) = 0 \rightarrow$ $t(|\vec{v}_0| \sin\alpha - 4.9t) = 0$
$\rightarrow$ $t = \dfrac{|V_0| \sin\alpha}{4.9}$ sec. ; downrange distance is

$x(t) = |\vec{v}_0| \cos\alpha \cdot t = |\vec{v}_0| \cos\alpha \cdot \dfrac{|\vec{v}_0| \sin\alpha}{4.9} \rightarrow$

$x = \dfrac{|\vec{v}_0|^2}{4.9} \sin\alpha \cos\alpha$; find angle α which maximizes x :

$$\frac{dx}{d\alpha} = \frac{|\vec{V}_0|^2}{4.9}(\sin\alpha \cdot (-\sin\alpha) + \cos\alpha \cdot \cos\alpha)$$

$$= \frac{|\vec{V}_0|^2}{4.9}(\cos^2\alpha - \sin^2\alpha) = 0 \rightarrow$$

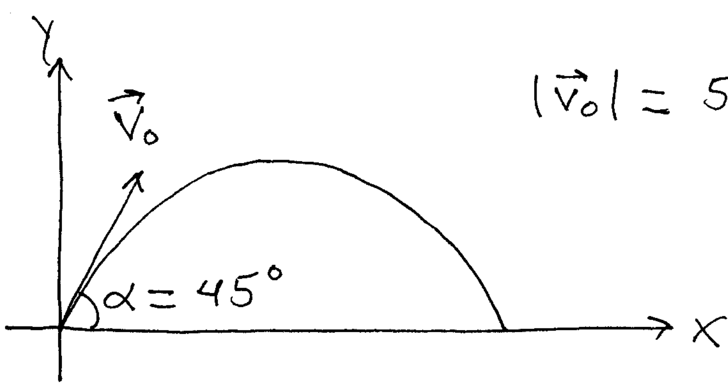
$\cos^2\alpha = \sin^2\alpha \rightarrow \alpha = 45°$, then

$$X = \frac{|\vec{V}_0|^2}{4.9}\sin 45° \cdot \cos 45° = \frac{|\vec{V}_0|}{4.9}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \rightarrow$$

$X = \frac{|\vec{V}_0|^2}{9.8}$; if max. range is 24,500 m then $\frac{|\vec{V}_0|^2}{9.8} = 24{,}500 \rightarrow$

$|\vec{V}_0|^2 = (24{,}500)(9.8) \longrightarrow$

$|\vec{V}_0| = \sqrt{240{,}100} = 490$ m./sec.

3.) 

$|\vec{V}_0| = 500$ m./sec.,

$x(t) = (500)\cos 45° \cdot t = (500)\frac{\sqrt{2}}{2}t = 250\sqrt{2}\,t \rightarrow$

$\underline{x(t) = 250\sqrt{2}\,t}$;

$y(t) = (500)\sin 45° \cdot t - \frac{1}{2}(9.8)t^2$

$= (500)\left(\frac{\sqrt{2}}{2}\right)t - 4.9t^2 = 250\sqrt{2}\,t - 4.9t^2 \rightarrow$

$\underline{y(t) = 250\sqrt{2}\,t - 4.9t^2}$;

a.) find flight time : $y(t) = 0 \rightarrow$

$t(250\sqrt{2} - 4.9t) = 0 \rightarrow t = \frac{250\sqrt{2}}{4.9} \approx 72.15$ sec.;

distance downrange:

$x(72.15) = 250\sqrt{2} \cdot (72.15) \approx 25{,}510.2$ m.

b.) If $x(t) = 5000$ m $\rightarrow 250\sqrt{2}\,t = 5000 \rightarrow$

$t = \dfrac{5000}{250\sqrt{2}} \approx 14.14$ sec., and height is

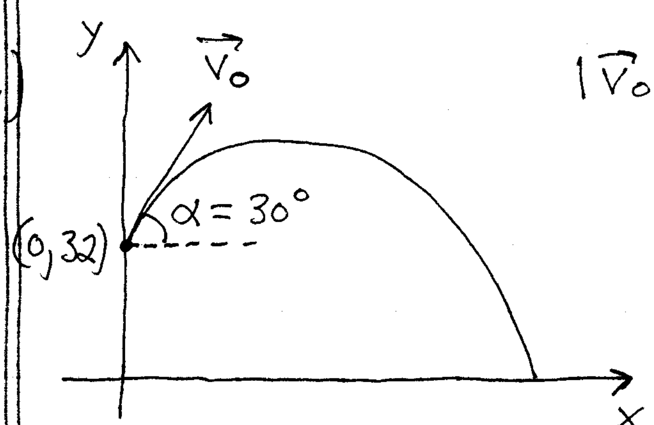
$y(14.14) = 250\sqrt{2}\,(14.14) - 4.9\,(14.14)^2 \approx 4019.5$ m.

c.) maximum $y(t)$: $y'(t) = 0 \longrightarrow$

$250\sqrt{2} - 9.8t = 0 \rightarrow t = \frac{250\sqrt{2}}{9.8} \approx 36.1$ sec.

$\rightarrow y(36.1) = 250\sqrt{2}\,(36.1) - 4.9\,(36.1)^2 \approx 6377.5$ m.

4.)



$|\vec{V_0}| = 32$ ft./sec.

$x(t) = (32) \cdot \cos 30° \cdot t$ and

$y(t) = b + |\vec{V_0}| \sin \alpha \cdot t - \frac{1}{2} g t^2$

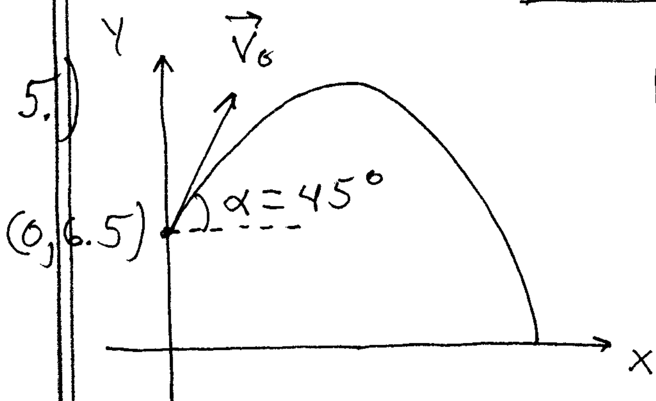
$= 32 + (32) \sin 30° \cdot t - \frac{1}{2}(32) t^2$

$= 32 + (32)(\frac{1}{2}) t - 16 t^2$

$\rightarrow y(t) = 32 + 16t - 16t^2$; flight time $\rightarrow$
$y(t) = 0 \rightarrow 32 + 16t - 16t^2 = 0 \longrightarrow$
$-16(t^2 - t - 2) = -16(t-2)(t+1) = 0 \rightarrow$
$\boxed{t = 2 \text{ sec.}}$; distance downrange is

$x(2) = (32)\cdot\cos 30^\circ \cdot (2) = (32)\left(\frac{\sqrt{3}}{2}\right)(2)$
$= 32\sqrt{3} \approx \boxed{55.4 \text{ ft}}$.

5.)



$|\vec{v}_0| = 44$ ft./sec.

$x(t) = (44)\cos 45^\circ \cdot t$
$= (44)\cdot\frac{\sqrt{2}}{2}\, t \longrightarrow$
$\underline{x(t) = 22\sqrt{2}\, t}$,

$y(t) = 6.5 + (44)\sin 45^\circ \cdot t - \frac{1}{2}(32)t^2$
$= 6.5 + (44)\left(\frac{\sqrt{2}}{2}\right)t - 16t^2 \longrightarrow$
$\underline{y(t) = 6.5 + 22\sqrt{2}\, t - 16t^2}$; flight time

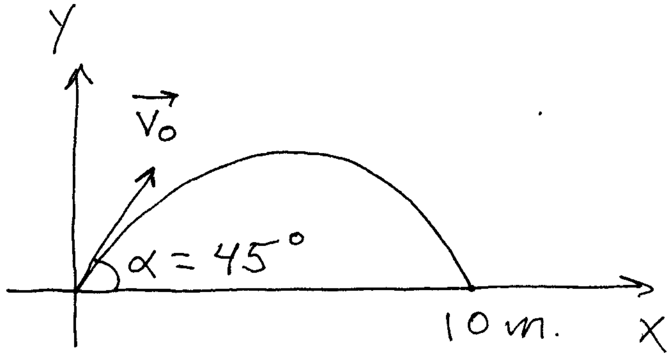
$\rightarrow y(t) = 0 \rightarrow -16t^2 + 22\sqrt{2}\, t + 6.5 = 0 \longrightarrow$

$$t = \frac{-22\sqrt{2} \pm \sqrt{(22\sqrt{2})^2 - 4(-16)(6.5)}}{2(-16)}$$

$$\approx \frac{-22\sqrt{2} \pm 37.2}{-32} \approx 2.135 \text{ sec.}$$ so

$x(2.135) \approx 22\sqrt{2}\,(2.135) \approx \boxed{66.4 \text{ ft.}}$

7.)



a.) Downrange distance :

$$x(t) = \frac{2|\vec{V_0}|^2 \cos\alpha \sin\alpha}{g} \rightarrow$$

$$10 = \frac{2|\vec{V_0}|^2 \cos 45° \sin 45°}{9.8} \rightarrow$$

$$10 = \frac{|\vec{V_0}|^2 (\sqrt{2}/2)(\sqrt{2}/2)}{4.9} \rightarrow$$

$$49 = |\vec{V_0}|^2 \left(\tfrac{1}{2}\right) \rightarrow 98 = |\vec{V_0}|^2 \rightarrow$$

$$|\vec{V_0}| \approx 9.9 \text{ m./sec.}$$

b.) assume that $|\vec{V_0}| = 9.9$ m./sec. and downrange $x(t) = 6$ m. ; then

$$x = \frac{2|\vec{V_0}|^2 \cos\alpha \sin\alpha}{g} \rightarrow$$

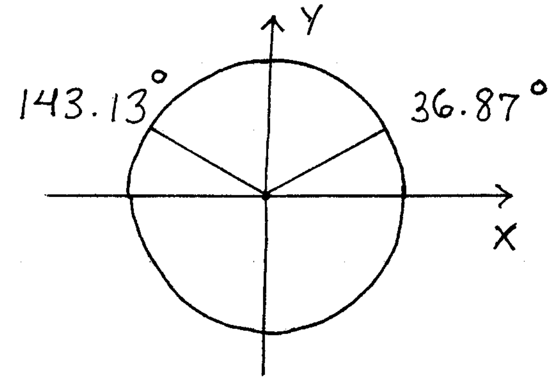
$$6 = \frac{2(9.9)^2}{9.8} \cdot \cos\alpha \sin\alpha \rightarrow$$

$$\frac{(6)(9.8)}{(9.9)^2} = 2\sin\alpha\cos\alpha = \sin 2\alpha \rightarrow$$

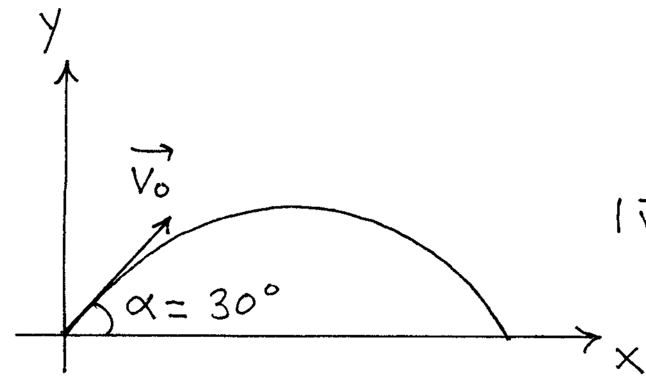
$$\sin 2\alpha \approx 0.6 \rightarrow$$

$$2\alpha \approx 36.87 \text{ or } 143.13 \rightarrow$$

$$\alpha \approx 18.4° \text{ or } \alpha \approx 71.6°$$



11.)



$|\vec{V_0}| = 90$ ft./sec.

$x(t) = |\vec{V_0}| \cos\alpha \cdot t$ and $x(t) = 135$ ft. $\rightarrow$

$135 = (90) \cos 30° \cdot t \rightarrow$

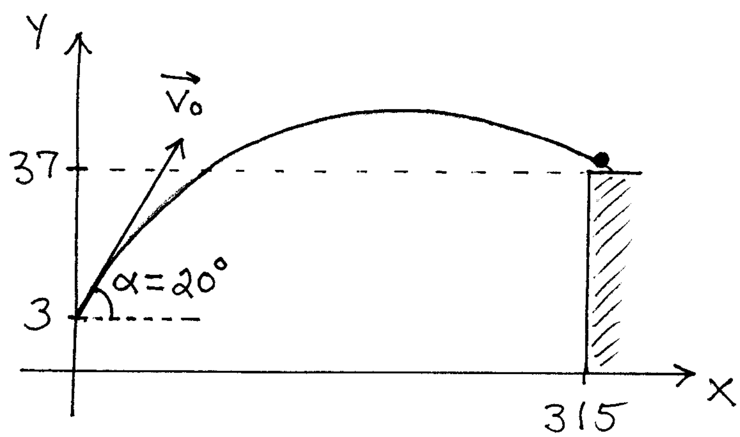
$135 = (90)(\sqrt{3}/2)\, t \rightarrow t = \frac{135}{45\sqrt{3}} \approx 1.732$ sec.;

and $y(t) = |\vec{V_0}| \sin\alpha \cdot t - \frac{1}{2} g t^2$

$\approx (90)(\sin 30°)(1.732) - \frac{1}{2}(32)(1.732)^2$

≈ 29.94 ft., so golf ball hits top of 30-ft. tree!

13.)



$$\begin{cases} x(t) = |\vec{V_0}| \cos 20° \cdot t \\ y(t) = 3 + |\vec{V_0}| \sin 20° \cdot t - \frac{1}{2}(32) t^2 \end{cases} \rightarrow$$

$$\begin{cases} 315 = |\vec{V_0}| \cos 20° \cdot t \\ 37 = 3 + |\vec{V_0}| \sin 20° \cdot t - 16 t^2 \end{cases} \rightarrow$$

$t = \dfrac{315}{|\vec{V_0}| \cos 20°}$ so ~~that~~

$$34 = \cancel{|\vec{V_0}|} \sin 20° \left(\frac{315}{\cancel{|\vec{V_0}|} \cos 20°}\right) - 16\left(\frac{315}{|\vec{V_0}| \cos 20°}\right)^2 \rightarrow$$

$$34 - 315 \frac{\sin 20°}{\cos 20°} = \frac{-16(315)^2}{|\vec{V_0}|^2 \cos^2 20°} \longrightarrow$$

$$|\vec{V_0}|^2 = \frac{\frac{-16(315)^2}{\cos^2 20°}}{34 - 315 \frac{\sin 20°}{\cos 20°}} \cdot \frac{\cos^2 20°}{\cos^2 20°}$$

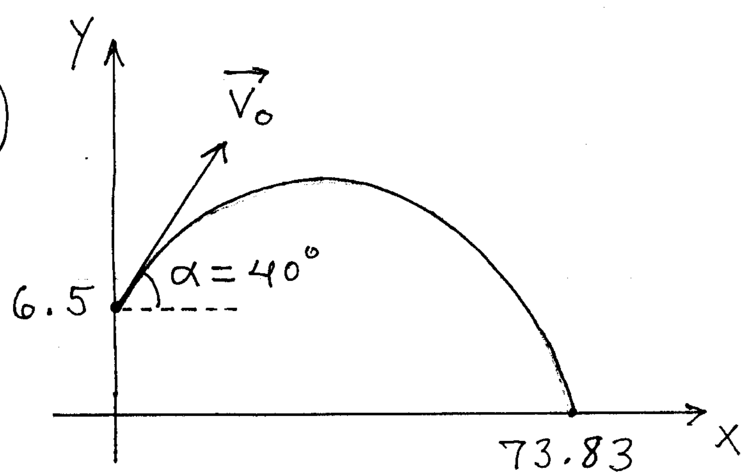
$$= \frac{-16(315)^2}{34 \cos^2 20° - 315 \sin 20° \cos 20°} \approx 22,292.65$$

$$\rightarrow |V_0| \approx \sqrt{22,292.65} \approx \boxed{149.3 \text{ ft./sec.}}$$

and

$$t = \frac{315}{|\vec{V_0}| \cos 20°} \approx \frac{315}{(149.3) \cos 20°} \approx \boxed{2.24 \text{ sec.}}$$

17.)



$$\begin{cases} x(t) = |\vec{V_0}| \cdot \cos 40° \cdot t \\ y(t) = 6.5 + |\vec{V_0}| \sin 40° t - \frac{1}{2}(32) t^2 \end{cases}$$

$$\begin{cases} 73.83 = |\vec{V}_0| \cos 40^\circ \cdot t \\ 0 = 6.5 + |\vec{V}_0| \sin 40^\circ t - 16t^2 \end{cases} \longrightarrow$$

$$t = \frac{73.83}{|\vec{V}_0| \cos 40^\circ} \quad \text{so that}$$

$$0 = 6.5 + \cancel{|\vec{V}_0|} \sin 40^\circ \cdot \left(\frac{73.83}{\cancel{|\vec{V}_0|} \cos 40^\circ}\right) - 16\left(\frac{73.83}{|\vec{V}_0| \cos 40^\circ}\right)^2$$

$$\rightarrow 16 \frac{(73.83)^2}{|\vec{V}_0|^2 \cos^2 40^\circ} = 6.5 + \frac{73.83 \sin 40^\circ}{\cos 40^\circ} \longrightarrow$$

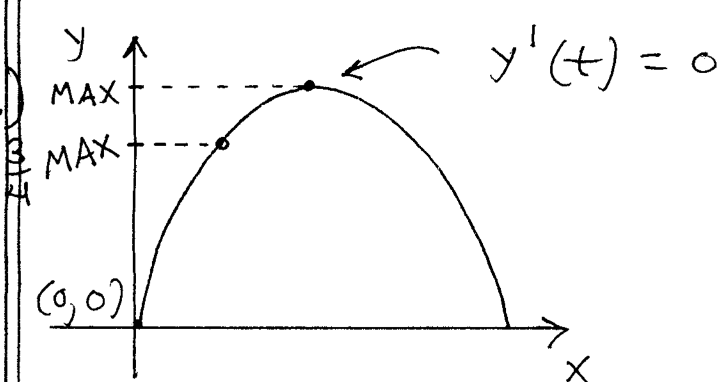
$$|\vec{V}_0|^2 = \frac{\dfrac{16(73.83)^2}{\cos^2 40^\circ}}{6.5 + \dfrac{73.83 \sin 40^\circ}{\cos 40^\circ}} \cdot \frac{\cos^2 40^\circ}{\cos^2 40^\circ} \longrightarrow$$

$$|\vec{V}_0|^2 = \frac{16(73.83)^2}{6.5 \cos^2 40^\circ + 73.83 \sin 40^\circ \cos 40^\circ}$$

$$\approx 2171.2 \quad \text{so}$$

$$|\vec{V}_0| \approx \sqrt{2171.2} \approx 46.6 \text{ ft./sec.}$$

18.)



$$y(t) = |\vec{V}_0| \sin\alpha \cdot t - \tfrac{1}{2} g t^2 \xrightarrow{D}$$

$$y'(t) = |\vec{V}_0| \sin\alpha - gt = 0 \longrightarrow$$

$t_{max} = \dfrac{|\vec{v}_0| \sin\alpha}{g}$ and maximum height is

$$y = |\vec{v}_0| \sin\alpha \cdot \left(\frac{|\vec{v}_0|\sin\alpha}{g}\right) - \frac{1}{2} g \left(\frac{|\vec{v}_0|\sin\alpha}{g}\right)^2$$

$$y_{max} = \boxed{\frac{|\vec{v}_0|^2 \sin^2\alpha}{2g}}$$

; if $t = \frac{1}{2} t_{max} = \dfrac{|\vec{v}_0| \sin\alpha}{2g}$

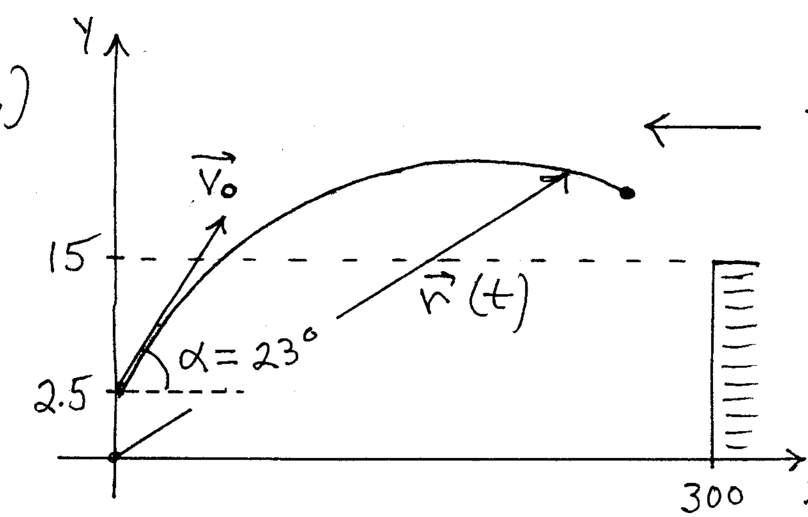
then
$$y = |\vec{v}_0| \sin\alpha \cdot \left(\frac{|\vec{v}_0|\sin\alpha}{2g}\right) - \frac{1}{2} g \left(\frac{|\vec{v}_0|\sin\alpha}{2g}\right)^2$$

$$= \frac{4}{8} \cdot \frac{|\vec{v}_0|^2 \sin^2\alpha}{g} - \frac{1}{8} \cdot \frac{|\vec{v}_0|^2 \sin^2\alpha}{g}$$

$$= \frac{3}{8} \cdot \frac{|\vec{v}_0|^2 \sin^2\alpha}{g} = \frac{3}{4} \cdot \frac{|\vec{v}_0|^2 \sin^2\alpha}{2g}$$

$$= \frac{3}{4} \cdot y_{max}$$

26.)



$-14\vec{i}$ mph when $t=0$

$|\vec{v}_0| = 145$ ft./sec.

$x(t) = (|\vec{v}_0| \cos\alpha - 14)t = (145 \cos 23° - 14)t$,

$$y(t) = 2.5 + |\vec{v}_0| \sin\alpha \cdot t - \tfrac{1}{2} g t^2$$

$$= 2.5 + (145)\sin 23^\circ \cdot t - \tfrac{1}{2}(32)t^2$$

$$= 2.5 + (145)\sin 23^\circ \cdot t - 16t^2 \quad \text{so}$$

a.) $$\vec{r}(t) = (45\cos 23^\circ - 14)t \cdot \vec{i}$$
$$+ \left(2.5 + (145)\sin 23^\circ \cdot t - 16t^2\right) \cdot \vec{j}$$

b.) at max. height $y'(t) = 0$:

$$y'(t) = (145)\sin 23^\circ - 32t = 0 \rightarrow$$

$$t = \frac{(145)\sin 23^\circ}{32} \approx \boxed{1.771 \text{ sec.}} \quad \text{and}$$

$$y = 2.5 + (145)\sin 23^\circ (1.771) - 16(1.771)^2 \rightarrow$$

$$y_{max} \approx \boxed{52.65 \text{ ft.}}$$

c.) Hit ground ; $y(t) = 0 \rightarrow$

$$2.5 + (145)\sin 23^\circ \cdot t - 16t^2 = 0 \rightarrow$$

$$t = \frac{-145\sin 23^\circ \pm \sqrt{145^2 \sin^2 23^\circ - 4(2.5)(-16)}}{2(-16)}$$

$$\approx \frac{-56.656 \pm \sqrt{3369.9}}{-32}$$

$$= \frac{-56.656 \pm 58.051}{-32} \approx \boxed{3.585 \text{ sec.}} \;;$$

downrange dist.

$$x = (145\cos 23^\circ - 14)(3.585) \approx \boxed{428.31 \text{ ft.}}$$

d.) If $y(t) = 20$ ft., then

$2.5 + (145)\sin 23° \cdot t - 16t^2 = 20 \rightarrow$

$0 = 16t^2 + {}^{-}145 \sin 23° \cdot t + 17.5 \rightarrow$

$$t = \frac{145 \sin 23° \pm \sqrt{(-145 \sin 23°)^2 - 4(16)(17.5)}}{2(16)}$$

$$\approx \frac{56.656 \pm \sqrt{2089.904}}{32}$$

$\approx$ 3.2 sec. or 0.342 sec. ;

$X(3.2) = (145 \cos 23° - 14)(3.2) \approx 382.3$ ft.,

$X(0.342) = (145 \cos 23° - 14)(0.342) \approx 40.86$ ft.

e.) Yes, since fence is 15 ft. high and 300 ft. from the batter.

28.) From Projectile Motion handout we know that max. height is

$$y = \frac{|\vec{V}_0|^2 \sin^2\alpha}{2g}$$ and corresponding

$$x = \frac{|\vec{V}_0|^2}{g} \cos\alpha \sin\alpha \ ; \text{ then}$$

$$\sin^2\alpha = \frac{2gy}{|\vec{V}_0|^2} \quad \text{and}$$

$$x^2 = \frac{|\vec{v}_0|^4}{g^2} \cdot \cos^2\alpha \cdot \sin^2\alpha$$

$$= \frac{|\vec{v}_0|^4}{g^2} \cdot (1 - \sin^2\alpha) \cdot \sin^2\alpha$$

$$= \frac{|\vec{v}_0|^4}{g^2} \cdot \left(1 - \frac{2gy}{|\vec{v}_0|^2}\right)\left(\frac{2gy}{|\vec{v}_0|^2}\right)$$

$$= \frac{|\vec{v}_0|^2}{g} \cdot \left(1 - \frac{2gy}{|\vec{v}_0|^2}\right)(2y)$$

$$= \frac{|\vec{v}_0|^2}{g} \cdot \left(2y - \frac{4gy^2}{|\vec{v}_0|^2}\right)$$

$$= \cancel{\frac{|\vec{v}_0|^2}{g}} \cdot \frac{-4\cancel{g}}{\cancel{|\vec{v}_0|^2}} \cdot \left(y^2 - \frac{|\vec{v}_0|^2}{2g} y\right)$$

$$= -4\left(y^2 - \frac{|\vec{v}_0|^2}{2g} y + \frac{|\vec{v}_0|^4}{16g^2}\right) + \frac{|\vec{v}_0|^4}{4g^2} \longrightarrow$$

$$x^2 + 4\left(y - \frac{|\vec{v}_0|^2}{4g}\right)^2 = \frac{|\vec{v}_0|^4}{4g^2}$$