Section 13.4

2.) $\vec{r}(t) = (\ln \sec t)\vec{i} + t\vec{j}$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$ $\xrightarrow{D}$

$$\vec{v}(t) = \frac{\cancel{\sec t}\tan t}{\cancel{\sec t}}\vec{i} + (1)\vec{j}$$

$$= (\tan t)\vec{i} + (1)\vec{j}, \text{ then}$$

$$|\vec{v}(t)| = \sqrt{(\tan t)^2 + (1)^2} = \sqrt{\sec^2 t} = \sec t;$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\tan t}{\sec t}\cdot\vec{i} + \frac{1}{\sec t}\cdot\vec{j} \rightarrow$$

$$\underline{\vec{T}(t) = (\sin t)\vec{i} + (\cos t)\vec{j}} \;; \xrightarrow{D}$$

$$\vec{T}'(t) = (\cos t)\vec{i} + (-\sin t)\vec{j} \text{ and}$$

$$|\vec{T}'(t)| = \sqrt{(\cos t)^2 + (-\sin t)^2}$$

$$= \sqrt{\cos^2 t + \sin^2 t} = \sqrt{1} = 1, \text{ so}$$

$$\underline{\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = (\cos t)\vec{i} + (-\sin t)\vec{j}} \;;$$

$$K = \frac{1}{|\vec{v}(t)|}\cdot|\vec{T}'(t)| = \frac{1}{\sec t}(1) = \cos t$$

3.) $\vec{r}(t) = (2t+3)\vec{i} + (5-t^2)\vec{j}$ $\xrightarrow{D}$

$$\vec{v}(t) = (2)\vec{i} + (-2t)\vec{j} \quad \text{so that}$$

$$|\vec{v}(t)| = \sqrt{(2)^2 + (-2t)^2} = \sqrt{4+4t^2} = 2\sqrt{1+t^2};$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{2}{2\sqrt{1+t^2}}\,\vec{i} + \frac{-2t}{2\sqrt{1+t^2}}\,\vec{j} \longrightarrow$$

$$\vec{T}(t) = \frac{1}{\sqrt{1+t^2}}\,\vec{i} + \frac{-t}{\sqrt{1+t^2}}\,\vec{j} \quad ;$$

$$\vec{T}(t) = (1+t^2)^{-1/2}\,\vec{i} - t(1+t^2)^{-1/2}\,\vec{j} \quad \xrightarrow{D}$$

$$\vec{T}'(t) = \frac{-1}{\cancel{2}}(1+t^2)^{-3/2}\cdot(\cancel{2}t)\,\vec{i}$$

$$- \left[t\cdot\frac{-1}{\cancel{2}}(1+t^2)^{-3/2}(\cancel{2}t) + (1)(1+t^2)^{-1/2}\right]\vec{j}$$

$$= \frac{-t}{(1+t^2)^{3/2}}\,\vec{i} + \left(\frac{t^2}{(1+t^2)^{3/2}} - \frac{1}{(1+t^2)^{1/2}}\right)\vec{j}$$

$$= \frac{-t}{(1+t^2)^{3/2}}\,\vec{i} + \left(\frac{t^2}{(1+t^2)^{3/2}} - \frac{1+t^2}{(1+t^2)^{3/2}}\right)\vec{j} \longrightarrow$$

$$\vec{T}'(t) = \frac{-t}{(1+t^2)^{3/2}}\,\vec{i} + \frac{-1}{(1+t^2)^{3/2}}\,\vec{j} \quad ; \text{ and}$$

$$|\vec{T}'(t)| = \sqrt{\left(\frac{-t}{(1+t^2)^{3/2}}\right)^2 + \left(\frac{-1}{(1+t^2)^{3/2}}\right)^2}$$

$$= \sqrt{\frac{t^2}{(1+t^2)^3} + \frac{1}{(1+t^2)^3}}$$

$$= \sqrt{\frac{1+t^2}{(1+t^2)^3}} = \sqrt{\frac{1}{(1+t^2)^2}} = \frac{1}{1+t^2} \quad ; \text{ then}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \frac{\vec{T}'(t)}{\frac{1}{1+t^2}} = (1+t^2)\cdot\vec{T}'(t)$$

$$= \frac{-t(1+t^2)}{(1+t^2)^{3/2}} \cdot \vec{i} + \frac{-(1+t^2)}{(1+t^2)^{3/2}} \cdot \vec{j} \longrightarrow$$

$$N(t) = \frac{-t}{\sqrt{1+t^2}}\,\vec{i} + \frac{-1}{\sqrt{1+t^2}}\,\vec{j} \quad ;$$

$$K = \frac{1}{|\vec{v}(t)|} \cdot |\vec{T}\,'(t)| = \frac{1}{2(1+t^2)^{1/2}} \cdot \frac{1}{1+t^2} \longrightarrow$$

$$K = \frac{1}{2(1+t^2)^{3/2}}$$

4.) $\vec{r}(t) = (\cos t + t\sin t)\,\vec{i} + (\sin t - t\cos t)\,\vec{j} \xrightarrow{D}$ for $t > 0$

$$\vec{v}(t) = (\cancel{-\sin t} + t\cos t + \cancel{\sin t})\,\vec{i}$$
$$+ (\cancel{\cos t} - (-t\sin t + \cancel{\cos t}))\,\vec{j}$$
$$= (t\cos t)\,\vec{i} + (t\sin t)\,\vec{j}, \text{ then}$$

$$|\vec{v}(t)| = \sqrt{(t\cos t)^2 + (t\sin t)^2}$$
$$= \sqrt{t^2(\cos^2 t + \sin^2 t)} = \sqrt{t^2(1)} = t;$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\vec{v}(t)}{t} = \frac{\cancel{t}\cos t}{\cancel{t}}\,\vec{i} + \frac{\cancel{t}\sin t}{\cancel{t}}\,\vec{j}$$

$$\rightarrow \vec{T}(t) = (\cos t)\,\vec{i} + (\sin t)\,\vec{j}; \text{ and}$$

$$\xrightarrow{D} \vec{T}\,'(t) = (-\sin t)\,\vec{i} + (\cos t)\,\vec{j} \text{ with}$$

$$|\vec{T}\,'(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2}$$
$$= \sqrt{\sin^2 t + \cos^2 t} = \sqrt{1} = 1, \text{ so that}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \frac{\vec{T}'(t)}{1} = (-\sin t)\,\vec{i} + (\cos t)\,\vec{j}\,;$$

$$K = \frac{1}{|\vec{v}(t)|}\,|\vec{T}'(t)| = \frac{1}{t}(1) = \frac{1}{t}$$

5.) $\vec{r}(x) = x \cdot \vec{i} + f(x) \cdot \vec{j} \xrightarrow{D}$

$$\vec{v}(x) = 1 \cdot \vec{i} + f'(x) \cdot \vec{j} \longrightarrow$$

$$|\vec{v}(x)| = \sqrt{(1)^2 + (f'(x))^2} = \sqrt{1 + (f'(x))^2}\,;$$

$$\vec{T}(t) = \frac{\vec{v}(x)}{|\vec{v}(x)|} = \frac{\vec{v}(x)}{(1+(f'(x))^2)^{1/2}}\,;$$

$$= (1+(f'(x))^2)^{-1/2}\,\vec{i} + \frac{f'(x)}{(1+(f'(x))^2)^{1/2}}\,\vec{j} \longrightarrow$$

$$\vec{T}'(t) = -\frac{1}{\cancel{2}}(1+(f'(x))^2)^{-3/2} \cdot \cancel{2} f'(x) \cdot f''(x) \cdot \vec{i}$$

$$+ \frac{(1+(f'(x))^2)^{1/2} \cdot f''(x) - f'(x)\,\frac{1}{\cancel{2}}(1+(f'(x))^2)^{-1/2} \cdot \cancel{2} f'(x)\, f''(x)}{1+(f'(x))^2}\,\vec{j}$$

$$= \frac{-f'(x)\,f''(x)}{(1+(f'(x))^2)^{3/2}}\,\vec{i} + \frac{\dfrac{(1+(f'(x))^2)^{1/2} f''(x)}{1} - \dfrac{(f'(x))^2 \cdot f''(x)}{(1+(f'(x))^2)^{1/2}}}{\dfrac{1+(f'(x))^2}{1}}\,\vec{j}$$

$$= \frac{-f'(x)f''(x)}{(1+(f'(x))^2)^{3/2}}\,\vec{i}$$

$$+\ \frac{(1+(f'(x))^2)f''(x) - (f'(x))^2 \cdot f''(x)}{(1+(f'(x))^2)^{1/2}\cdot(1+(f'(x))^2)}\,\vec{j}$$

$$\vec{T}'(t) = \frac{-f'(x)f''(x)}{(1+(f'(x))^2)^{3/2}}\,\vec{i} + \frac{f''(x)}{(1+(f'(x))^2)^{3/2}}\,\vec{j}\ ;\ \text{~~then~~}$$

$$|\vec{T}'(t)| = \sqrt{\left[\frac{-f'(x)f''(x)}{(1+(f'(x))^2)^{3/2}}\right]^2 + \left[\frac{f''(x)}{(1+(f'(x))^2)^{3/2}}\right]^2}$$

$$= \sqrt{\frac{(f'(x))^2(f''(x))^2}{(1+(f'(x))^2)^3} + \frac{(f''(x))^2}{(1+(f'(x))^2)^3}}$$

$$= \sqrt{\frac{(f''(x))^2\cdot(1+(f'(x))^2)}{(1+(f'(x))^2)^3}}$$

$$= \frac{\sqrt{(f''(x))^2}}{\sqrt{(1+(f'(x))^2)^2}} = \frac{|f''(x)|}{1+(f'(x))^2}\ ;\ \text{then}$$

$$K = \frac{1}{|\vec{v}(x)|}\cdot|\vec{T}'(x)| = \frac{1}{(1+(f'(x))^2)^{1/2}}\cdot\frac{|f''(x)|}{(1+(f'(x))^2)}$$

$$\rightarrow K = \frac{|f''(x)|}{(1+(f'(x))^2)^{3/2}}.$$

9.) $\vec{r}(t) = (3\sin t)\,\vec{i} + (3\cos t)\,\vec{j} + (4t)\,\vec{k} \xrightarrow{D}$

$\vec{v}(t) = (3\cos t)\,\vec{i} + (-3\sin t)\,\vec{j} + (4)\,\vec{k}$ and

$$|\vec{v}(t)| = \sqrt{(3\cos t)^2 + (-3\sin t)^2 + (4)^2}$$

$$= \sqrt{9\cos^2 t + 9\sin^2 t + 16}$$

$$= \sqrt{9(\underbrace{\cos^2 t + \sin^2 t}_{1}) + 16}$$

$$= \sqrt{25} = 5 \;; \text{ then}$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\vec{v}(t)}{5} =$$

$$= \frac{3}{5}\cos t \cdot \vec{i} + \frac{-3}{5}\sin t \cdot \vec{j} + \frac{4}{5}\,\vec{k} \xrightarrow{D}$$

$$\vec{T}'(t) = \frac{-3}{5}\sin t \cdot \vec{i} + \frac{-3}{5}\cos t \cdot \vec{j} + 0 \cdot \vec{k},$$

then $|\vec{T}'(t)| = \sqrt{\left(\frac{-3}{5}\sin t\right)^2 + \left(\frac{-3}{5}\cos t\right)^2 + (0)^2}$

$$= \sqrt{\frac{9}{25}\sin^2 t + \frac{9}{25}\cos^2 t} = \sqrt{\frac{9}{25}(\underbrace{\sin^2 t + \cos^2 t}_{1})}$$

$$= 3/5 \longrightarrow$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \frac{\vec{T}'(t)}{3/5}$$

$$= \frac{-\cancel{3/5}\sin t}{\cancel{3/5}} \cdot \vec{i} + \frac{-\cancel{3/5}\cos t}{\cancel{3/5}} \vec{j} \longrightarrow$$

$$\vec{N}(t) = (-\sin t)\,\vec{i} + (-\cos t)\,\vec{j} \text{ ; now}$$

$$K = \frac{1}{|\vec{v}(t)|} \cdot |\vec{T}'(t)| = \frac{1}{5} \cdot \frac{3}{5} = \frac{3}{25}.$$

11.) $\vec{r}(t) = (e^t \cos t)\,\vec{i} + (e^t \sin t)\,\vec{j} + 2\,\vec{k} \xrightarrow{D}$

$$\vec{v}(t) = (-e^t \sin t + e^t \cos t)\,\vec{i}$$
$$+ (e^t \cos t + e^t \sin t)\,\vec{j} + (0)\,\vec{k} \text{, then}$$

$$|\vec{v}| = \sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \cos t + e^t \sin t)^2}$$

$$= \sqrt{e^{2t}\cos^2 t - \cancel{2e^{2t}\cos t \sin t} + e^{2t}\sin^2 t + e^{2t}\cos^2 t + \cancel{2e^{2t}\cos t \sin t} + e^{2t}\sin^2 t}$$

$$= \sqrt{2e^{2t}\cos^2 t + 2e^{2t}\sin^2 t}$$

$$= \sqrt{2e^{2t}\,(\underbrace{\cos^2 t + \sin^2 t}_{1})} = \sqrt{2} \cdot e^t \text{ ;}$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\vec{v}(t)}{\sqrt{2} \cdot e^t}$$

$$= \frac{\cancel{e^t}(\cos t - \sin t)}{\sqrt{2} \cdot \cancel{e^t}}\,\vec{i} + \frac{\cancel{e^t}(\cos t - \sin t)}{\sqrt{2} \cdot \cancel{e^t}}\,\vec{j} \longrightarrow$$

$$\vec{T}(t) = \frac{1}{\sqrt{2}}(\cos t - \sin t)\,\vec{i} + \frac{1}{\sqrt{2}}(\cos t + \sin t)\,\vec{j} \xrightarrow{D}$$

$$\vec{T}'(t) = \frac{1}{\sqrt{2}}(-\sin t - \cos t)\,\vec{i} + \frac{1}{\sqrt{2}}(-\sin t + \cos t)\,\vec{j} \longrightarrow$$

$$|\vec{T}'(t)| = \sqrt{\left(\frac{1}{\sqrt{2}}(-\sin t - \cos t)\right)^2 + \left(\frac{1}{\sqrt{2}}(-\sin t + \cos t)\right)^2}$$

$$= \sqrt{\frac{1}{2}(\sin^2 t + 2\cancel{\sin t}\cos t + \cos^2 t) + \frac{1}{2}(\sin^2 t - 2\cancel{\sin t}\cos t + \cos^2 t)}$$

$$= \sqrt{\sin^2 t + \cos^2 t} = \sqrt{1} = 1, \text{ so that}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \vec{T}'(t)$$

$$= \frac{1}{\sqrt{2}}(-\sin t - \cos t)\,\vec{i} + \frac{1}{\sqrt{2}}(-\sin t + \cos t)\,\vec{j}\ ;$$

$$\kappa = \frac{1}{|\vec{v}(t)|} \cdot |\vec{T}'(t)| = \frac{1}{\sqrt{2}\cdot e^t}(1) = \frac{1}{\sqrt{2}\cdot e^t}.$$

13.) $\vec{r}(t) = \frac{1}{3}t^3 \cdot \vec{i} + \frac{1}{2}t^2 \cdot \vec{j} \xrightarrow{D}$ (for $t > 0$)

$\vec{v}(t) = t^2 \cdot \vec{i} + t \cdot \vec{j}$ and

$$|\vec{v}(t)| = \sqrt{(t^2)^2 + (t)^2} = \sqrt{t^4 + t^2}$$

$$= \sqrt{t^2(t^2+1)} = t\sqrt{t^2+1}\ ; \text{ then}$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\vec{v}(t)}{t\sqrt{t^2+1}} = \frac{t^2}{t\sqrt{t^2+1}}\,\vec{i} + \frac{t}{t\sqrt{t^2+1}}\,\vec{j}$$

$$\rightarrow \vec{T}(t) = \frac{t}{\sqrt{t^2+1}}\vec{i} + \frac{1}{\sqrt{t^2+1}}\vec{j} \quad \text{and } \xrightarrow{D}$$

$$\vec{T}'(t) = \frac{(t^2+1)^{1/2}\cdot(1) - t\cdot\frac{1}{\cancel{2}}(t^2+1)^{-1/2}\cdot\cancel{2}t}{t^2+1}\vec{i}$$

$$+ \frac{-1}{\cancel{2}}(t^2+1)^{-3/2}(\cancel{2}t)\,\vec{j}$$

$$= \frac{\frac{(t^2+1)^{1/2}}{1} - \frac{t^2}{(t^2+1)^{1/2}}}{\frac{t^2+1}{1}}\vec{i} + \frac{-t}{(t^2+1)^{3/2}}\vec{j}$$

$$= \frac{(t^2+1)-t^2}{(t^2+1)^{1/2}(t^2+1)}\vec{i} + \frac{-t}{(t^2+1)^{3/2}}\vec{j} \longrightarrow$$

$$\vec{T}'(t) = \frac{1}{(t^2+1)^{3/2}}\vec{i} + \frac{-t}{(t^2+1)^{3/2}}\vec{j} \text{ ; then}$$

$$|\vec{T}'(t)| = \sqrt{\left(\frac{1}{(t^2+1)^{3/2}}\right)^2 + \left(\frac{-t}{(t^2+1)^{3/2}}\right)^2}$$

$$= \sqrt{\frac{1}{(t^2+1)^3} + \frac{t^2}{(t^2+1)^3}} = \sqrt{\frac{(1+t^2)}{(1+t^2)^3}}$$

$$= \sqrt{\frac{1}{(t^2+1)^2}} = \frac{1}{t^2+1} \text{ ;}$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \frac{\vec{T}'(t)}{\frac{1}{t^2+1}} = (t^2+1)\,\vec{T}'(t)$$

$$= \frac{(t^2+1)}{(t^2+1)^{3/2}}\vec{i} + \frac{-t(t^2+1)}{(t^2+1)^{3/2}}\vec{j} \longrightarrow$$

$$\vec{N}(t) = \frac{1}{(t^2+1)^{1/2}}\,\vec{i} + \frac{-t}{(t^2+1)^{1/2}}\,\vec{j} \quad ;$$

$$K = \frac{1}{|\vec{v}(t)|} \cdot |\vec{T}'(t)| = \frac{1}{t\sqrt{t^2+1}} \cdot \frac{1}{t^2+1} = \frac{1}{t(t^2+1)^{3/2}} .$$

14.) $\vec{r}(t) = (\cos^3 t)\,\vec{i} + (\sin^3 t)\,\vec{j} \;,\; 0 < t < \frac{\pi}{2} \xrightarrow{D}$

$$\vec{v}(t) = -3\cos^2 t \cdot \sin t \cdot \vec{i} + 3\sin^2 t \cdot \cos t \cdot \vec{j} \longrightarrow$$

$$|\vec{v}(t)| = \sqrt{(-3\cos^2 t \cdot \sin t)^2 + (3\sin^2 t \cos t)^2}$$

$$= \sqrt{9\cos^4 t \cdot \sin^2 t + 9\sin^4 t \cos^2 t}$$

$$= \sqrt{9\cos^2 t \sin^2 t \cdot (\underbrace{\cos^2 t + \sin^2 t}_{1})}$$

$$= 3\cos t \sin t \quad ; \quad \text{then}$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\vec{v}(t)}{3\cos t \sin t}$$

$$= \frac{-\cancel{3}\cos^{\cancel{2}} t \,\cancel{\sin t}}{\cancel{3}\,\cancel{\cos t}\,\cancel{\sin t}}\,\vec{i} + \frac{\cancel{3}\sin^{\cancel{2}} t\,\cancel{\cos t}}{\cancel{3}\,\cancel{\cos t}\,\cancel{\sin t}}\,\vec{j} \longrightarrow$$

$$\vec{T}(t) = (-\cos t)\,\vec{i} + (\sin t)\,\vec{j} \xrightarrow{D}$$

$$\vec{T}'(t) = (\sin t)\,\vec{i} + (\cos t)\,\vec{j} \quad \text{and}$$

$$|\vec{T}'(t)| = \sqrt{(\sin t)^2 + (\cos t)^2} = \sqrt{1} = 1 \longrightarrow$$

$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \vec{T}'(t) = (\sin t)\,\vec{i} + (\cos t)\,\vec{j} \;;$$

$$K = \frac{1}{|\vec{v}(t)|} \cdot |\vec{T}\,'(t)| = \frac{1}{3\cos t \sin t} \cdot (1) \longrightarrow$$

$$K = \frac{1}{3\cos t \sin t}.$$

17.) $y = ax^2 \xrightarrow{D} y' = 2ax \xrightarrow{D} y'' = 2a$,

then $K = \dfrac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}$

$$= \frac{2a}{[1+(2ax)^2]^{3/2}} = \frac{2a}{[1+4a^2x^2]^{3/2}};$$

K will be largest when $1+4a^2x^2$ is smallest, i.e., when $x=0$ (vertex of parabola);

$\lim\limits_{x \to \pm\infty} \dfrac{2a}{[1+4a^2x^2]^{3/2}} = 0$, but

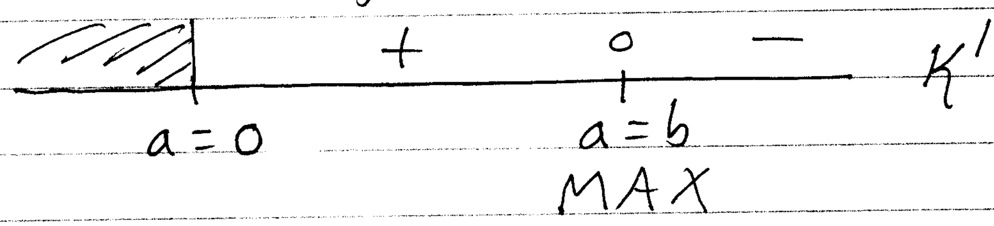
$\dfrac{2a}{[1+4a^2x^2]^{3/2}} > 0$, so no minimum curvature occurs.

19.) $K = K(a) = \dfrac{a}{a^2+b^2} \xrightarrow{D}$

$$K'(a) = \frac{(a^2+b^2)(1)-(a)(2a)}{(a^2+b^2)^2}$$

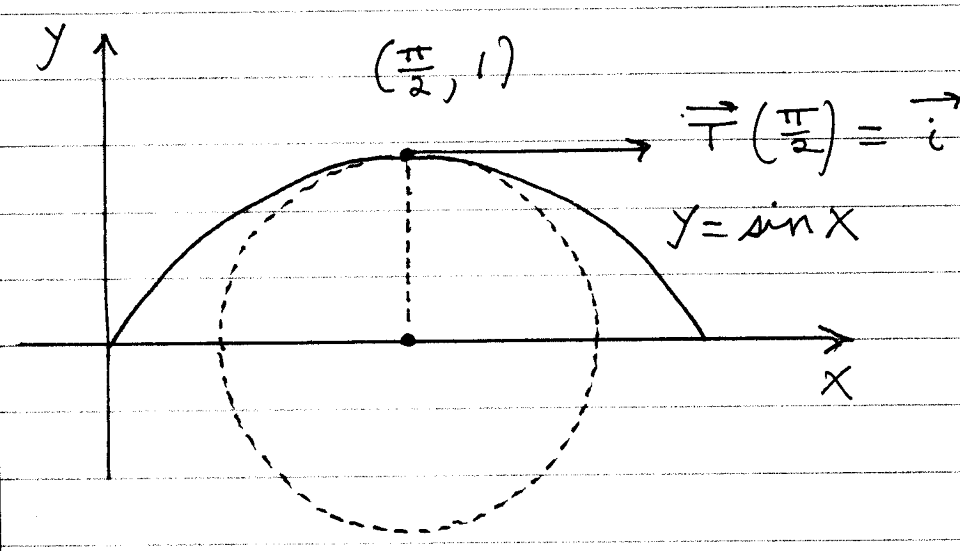
$$= \frac{a^2+b^2-2a^2}{(a^2+b^2)^2} = \frac{b^2-a^2}{(a^2+b^2)^2}$$

$$= \frac{(b-a)(b+a)}{(a^2+b^2)^2} = 0 \longrightarrow a=b \text{ or } \cancel{a=-b} :$$



so MAX $K = \frac{b}{b^2+b^2} = \frac{b}{2b^2} = \frac{1}{2b}$.

21.)



$$\vec{r}(t) = (t)\,\vec{i} + (\sin t)\,\vec{j} \quad \xrightarrow{D}$$

$$\vec{v}(t) = (1)\,\vec{i} + (\cos t)\,\vec{j} \quad \longrightarrow$$

$$|\vec{v}(t)| = \sqrt{(1)^2 + (\cos t)^2} = \sqrt{1+\cos^2 t} \;;$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = (1+\cos^2 t)^{-1/2}\,\vec{i} + \frac{\cos t}{(1+\cos^2 t)^{1/2}}\,\vec{j} \xrightarrow{D}$$

$$\vec{T}'(t) = -\tfrac{1}{2}(1+\cos^2 t)^{-3/2}\cdot -2\cos t \sin t \cdot \vec{i}$$

$$+ \frac{(1+\cos^2 t)^{\frac{1}{2}} \cdot -\sin t - \cos t \cdot \frac{1}{2}(1+\cos^2 t)^{-1/2} \cdot -2\cos t \sin t}{(1+\cos^2 t)}\,\vec{j}\;;$$

and $t = \frac{\pi}{2}$ so

$|\vec{v}(\frac{\pi}{2})| = \sqrt{1+\cos^2(\frac{\pi}{2})} = \sqrt{1+0} = 1$,

$\vec{T}(\frac{\pi}{2}) = \vec{i}$ so radius of circle is $\perp$ to $\vec{i}$ (parallel to Y-axis);

$\vec{T}'(\frac{\pi}{2}) = -\vec{j}$ and $|\vec{T}'(\frac{\pi}{2})| = 1$ so curvature at $(\frac{\pi}{2}, 1)$ is

$$K = \frac{1}{|\vec{v}(\frac{\pi}{2})|} \cdot |\vec{T}'(\frac{\pi}{2})| = \frac{1}{1}(1) = 1, \text{ and}$$

curvature of circle of radius a is $K = \frac{1}{a} \rightarrow \frac{1}{a} = 1 \rightarrow a = 1$;

and center of circle is $(\frac{\pi}{2}, 0)$, so circle of curvature is

$$\left(x - \frac{\pi}{2}\right)^2 + (y-0)^2 = 1^2.$$

22.) $\vec{r}(t) = (2\ln t)\,\vec{i} + \left(t + \frac{1}{t}\right)\vec{j} \xrightarrow{D}$

$$\vec{v}(t) = \left(\frac{2}{t}\right)\vec{i} + \left(1 + \frac{-1}{t^2}\right)\vec{j}$$

$$= \left(\frac{2}{t}\right)\vec{i} + \left(\frac{1}{t^2} - 1\right)\vec{j}$$

$$= \left(\frac{2}{t}\right)\vec{i} + \left(\frac{1-t^2}{t^2}\right)\vec{j} \quad , \text{ and}$$

$$|\vec{v}(t)| = \sqrt{\left(\frac{2}{t}\right)^2 + \left(\frac{1}{t^2} - 1\right)^2}$$

$$= \sqrt{\frac{4}{t^2} + \frac{1}{t^4} - \frac{2}{t^2} + 1}$$

$$= \sqrt{\frac{1}{t^4} + \frac{2}{t^2} + 1}$$

$$= \sqrt{\left(\frac{1}{t^2} + 1\right)^2} = \frac{1}{t^2} + 1 = \frac{1+t^2}{t^2} \ ;$$

$$\vec{T}(t) = \frac{\vec{v}(t)}{|\vec{v}(t)|} = \frac{\frac{2}{t}}{\frac{1+t^2}{t^2}}\,\vec{i} + \frac{\frac{1-t^2}{t^2}}{\frac{1+t^2}{t^2}}\,\vec{j} \longrightarrow$$

$$\vec{T}(t) = \frac{2t}{1+t^2}\,\vec{i} + \frac{1-t^2}{1+t^2}\,\vec{j} \xrightarrow{D}$$

$$\vec{T}'(t) = \frac{(1+t^2)(2) - (2t)(2t)}{(1+t^2)^2}\,\vec{i}$$

$$+ \frac{(1+t^2)(-2t) - (1-t^2)(2t)}{(1+t^2)^2}\,\vec{j}$$

$$= \frac{2+2t^2-4t^2}{(1+t^2)^2}\vec{i} + \frac{-2t-\cancel{2t^3}-2t+\cancel{2t^3}}{(1+t^2)^2}\vec{j} \longrightarrow$$

$\vec{T}'(t) = \frac{2-2t^2}{(1+t^2)^2}\vec{i} + \frac{-4t}{(1+t^2)^2}\vec{j}$; then at point $(0,-2)$ for $t=1$:

$|\vec{v}(1)| = 2$, $\vec{T}'(1) = 0\cdot\vec{i} + (-1)\vec{j}$ and $|\vec{T}'(1)| = \sqrt{0^2+(-1)^2} = 1$; then curvature is

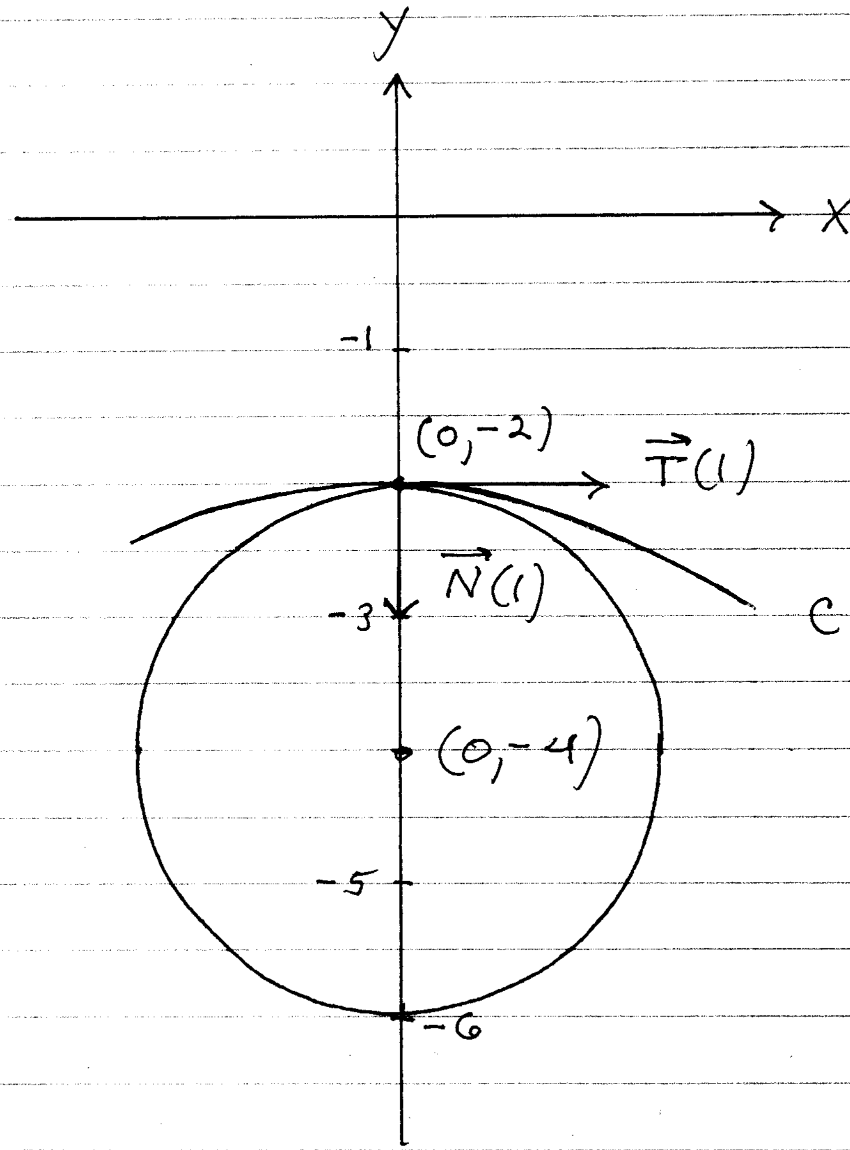
$K = \frac{1}{|\vec{v}(1)|}\cdot|\vec{T}'(1)| = \frac{1}{2}(1) = \frac{1}{2}$ so associated circle of curvature has radius a determined by $K = \frac{1}{2} = \frac{1}{a} \longrightarrow a = 2$; now

$\vec{T}(1) = \vec{i}$ and $\vec{N}(1) = \frac{\vec{T}'(1)}{|\vec{T}'(1)|} = \frac{-\vec{j}}{1} = -\vec{j}$

direction of motion

direction of turn

so circle of curvature has center on the y-axis at $(0,-4)$.



the circle of curvature is

$$(x-0)^2+(y+4)^2=4$$