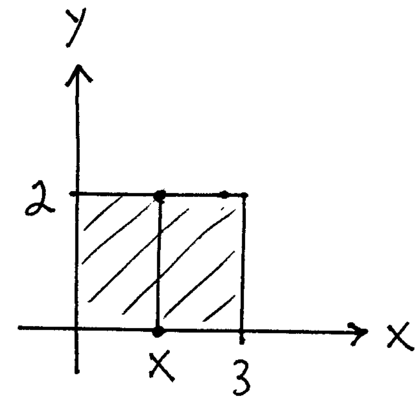
Section 15.1

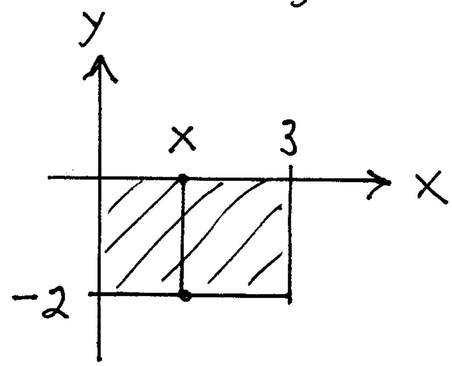
1.) $\int_0^3 \int_0^2 (4-y^2)\, dy\, dx$

$= \int_0^3 (4y - \frac{1}{3}y^3)\Big|_0^2 \, dx$

$= \int_0^3 (8 - \frac{8}{3})\, dx = \int_0^3 \frac{16}{3}\, dx$

$= \frac{16}{3} x \Big|_0^3 = 16$



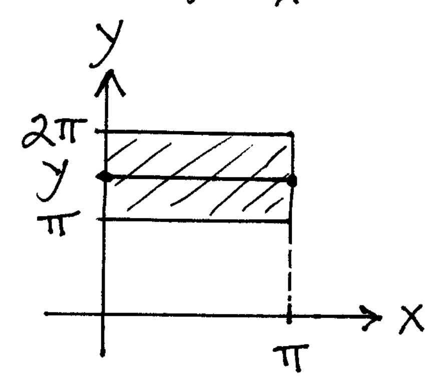


2.) $\int_0^3 \int_{-2}^0 (x^2 y - 2xy)\, dy\, dx$

$= \int_0^3 (x^2 \cdot \frac{1}{2} y^2 - x y^2)\Big|_{y=-2}^{y=0} \, dx$

$= \int_0^3 [0 - (2x^2 - 4x)]\, dx = \int_0^3 (4x - 2x^2)\, dx$

$= (2x^2 - \frac{2}{3}x^3)\Big|_0^3 = 18 - 18 = 0$



4.) $\int_\pi^{2\pi} \int_0^\pi (\sin x + \cos y)\, dx\, dy$

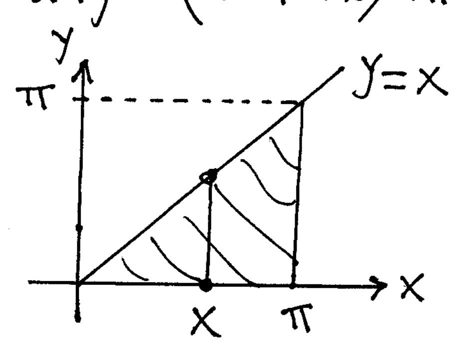
$= \int_\pi^{2\pi} (-\cos x + x \cos y)\Big|_{x=0}^{x=\pi} \, dy$

$= \int_\pi^{2\pi} [(-\overset{-1}{\cos \pi} + \pi \cos y) - (-\overset{1}{\cos 0} + 0)]\, dy$

$= \int_\pi^{2\pi} (1 + \pi \cos y + 1)\, dy = \int_\pi^{2\pi} (2 + \pi \cos y)\, dy$

$= (2y + \pi \cdot \sin y)\Big|_\pi^{2\pi} = (4\pi + \pi \overset{0}{\sin 2\pi}) - (2\pi + \pi \overset{0}{\sin \pi})$

$= 2\pi$



5.) $\int_0^\pi \int_0^x x \sin y\, dy\, dx$

$= \int_0^\pi [x \cdot -\cos y \Big|_{y=0}^{y=x}]\, dx$

$$= \int_0^{\pi} (-x\cos x - {}^{-}x \cdot \overset{1}{\cancel{\cos} 0})\,dx$$

$$= \int_0^{\pi} (x - x\cos x)\,dx = \int_0^{\pi} x\,dx - \int_0^{\pi} x\cos x\,dx$$

$$= \frac{1}{2}x^2\Big|_0^{\pi}$$

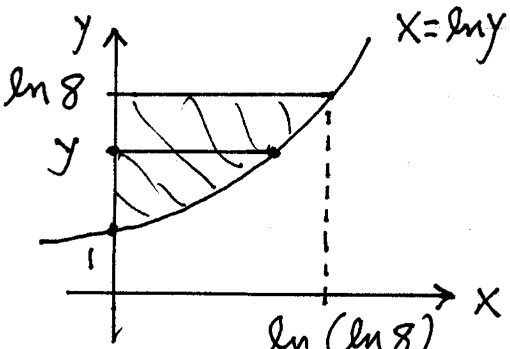
$$- \left[x\sin x\Big|_0^{\pi} - \int_0^{\pi}\sin x\,dx\right]$$

(Let $u = x$, $dv = \cos x\,dx \rightarrow du = dx$, $v = \sin x$)

$$= \frac{1}{2}\pi^2 - \left[(\pi \overset{0}{\cancel{\sin}\pi} - 0\,\overset{0}{\cancel{\sin} 0}) - {}^{-}\cos x\Big|_0^{\pi}\right]$$

$$= \frac{1}{2}\pi^2 - (\overset{-1}{\cancel{\cos}\pi} - \overset{+1}{\cancel{\cos} 0}) = \frac{1}{2}\pi^2 + 2$$

7.) $\displaystyle\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\,dx\,dy$



$$= \int_1^{\ln 8}\left(e^{x+y}\Big|_{x=0}^{x=\ln y}\right)dy$$

$$= \int_1^{\ln 8}\left(e^{\ln y + y} - e^{y}\right)dy$$

$$= \int_1^{\ln 8} e^{\ln y}\cdot e^{y}\,dy - \int_1^{\ln 8} e^{y}\,dy$$

$$= \int_1^{\ln 8} y\cdot e^{y}\,dy - e^{y}\Big|_1^{\ln 8}$$

(Let $u = y$, $dv = e^{y}dy$ $\rightarrow du = dy$, $v = e^{y}$)

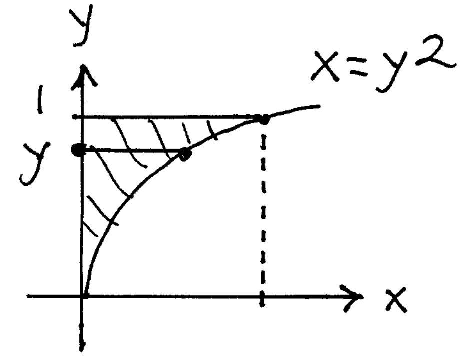
$$= \left(ye^{y}\Big|_1^{\ln 8} - \int_1^{\ln 8} e^{y}dy\right) - (e^{\ln 8} - e)$$

$$= (\ln 8 \cdot e^{\ln 8} - e) - e^{y}\Big|_1^{\ln 8} - 8 + e$$

$$= 8\ln 8 - \cancel{e} - (e^{\ln 8} - e) - 8 + \cancel{e}$$

$$= 8\ln 8 - 8 - 8 + e = 8\ln 8 - 16 + e$$

9.) $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$

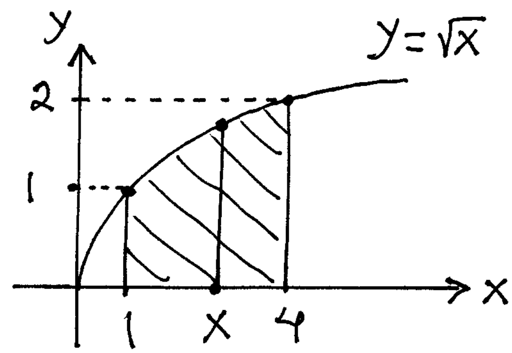


$= \int_0^1 \left(3y^2 e^{xy} \Big|_{x=0}^{x=y^2} \right) dy$

$= \int_0^1 (3y^2 e^{y^3} - 3y^2)\, dy = (e^{y^3} - y^3) \Big|_0^1$

$= (e-1) - (1-0) = e-2$

10.) $\int_1^4 \int_0^{\sqrt{x}} \frac{3}{2} e^{y/\sqrt{x}}\, dy\, dx$



$= \int_1^4 \left(\frac{3}{2} \cdot \sqrt{x} \cdot e^{y/\sqrt{x}} \Big|_{y=0}^{y=\sqrt{x}} \right) dx$

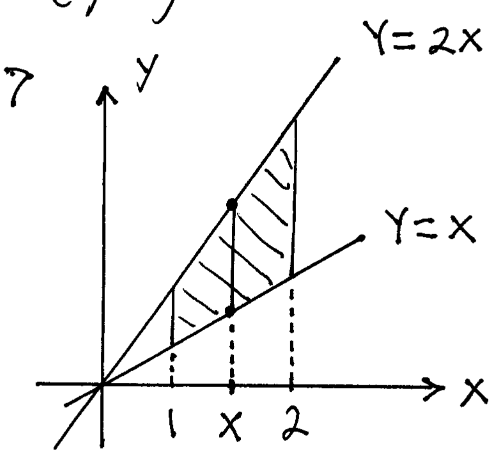
$= \int_1^4 \left(\frac{3}{2}\sqrt{x} \cdot e - \frac{3}{2}\sqrt{x} \right) dx$

$= \left(\cancel{\frac{3}{2}} \cdot e \cdot \cancel{\frac{2}{3}} x^{3/2} - \cancel{\frac{3}{2}} \cdot \cancel{\frac{2}{3}} x^{3/2} \right) \Big|_1^4$

$= \left(e(4)^{3/2} - (4)^{3/2} \right) - \left(e(1)^{3/2} - (1)^{3/2} \right)$

$= 8e - 8 - e + 1 = 7e - 7$

11.) $\iint_R \frac{x}{y}\, dA = \int_1^2 \int_x^{2x} \frac{x}{y}\, dy\, dx$



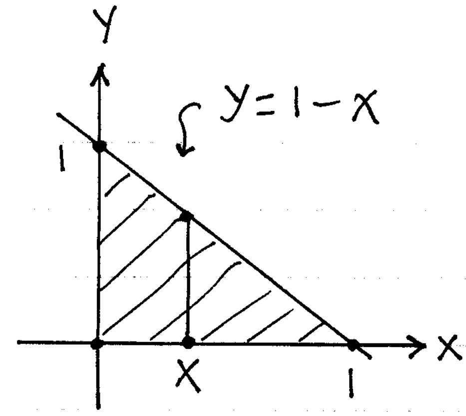
$= \int_1^2 \left(x \ln y \Big|_{Y=X}^{Y=2X} \right) dx$

$= \int_1^2 (x \ln 2x - x \ln x)\, dx$

$= \int_1^2 \left(x(\ln 2 + \ln x) - x \ln x \right) dx = \int_1^2 \ln 2 \cdot x\, dx$

$= \ln 2 \cdot \frac{1}{2} x^2 \Big|_1^2 = \ln 2 \cdot \left(2 - \frac{1}{2}\right) = \frac{3}{2} \ln 2$

13.) $\iint\limits_R (x^2+y^2)\, dA = \int_0^1 \int_0^{1-x} (x^2+y^2)\,dy\,dx$



$= \int_0^1 \left(x^2y + \frac{1}{3}y^3\right)\Big|_{y=0}^{y=1-x} dx$

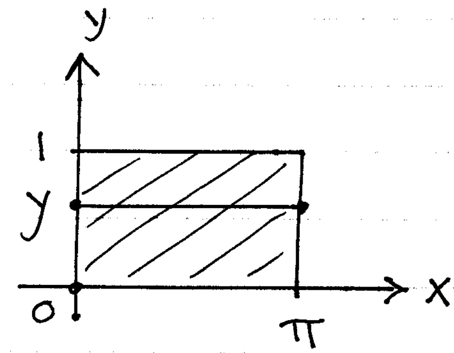
$= \int_0^1 \left[x^2(1-x) + \frac{1}{3}(1-x)^3\right] dx$

$= \int_0^1 \left[x^2 - x^3 + \frac{1}{3}(1-x)^3\right] dx$

$= \left(\frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{-1}{12}(1-x)^4\right)\Big|_0^1$

$= \left(\frac{1}{3} - \frac{1}{4} + 0\right) - \left(0 - 0 - \frac{1}{12}\right)$

$= \frac{4}{12} - \frac{3}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$



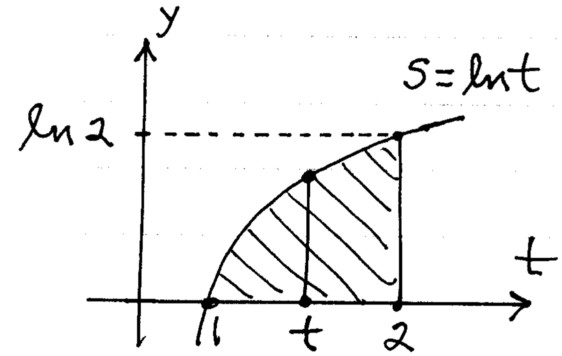
14.) $\iint\limits_R y\cos xy\, dA$

$= \int_0^1 \int_0^{\pi} y\cos xy\, dx\,dy$

$= \int_0^1 \left(\sin xy\, \Big|_{x=0}^{x=\pi}\right) dy = \int_0^1 (\sin \pi y - \overset{0}{\cancel{\sin 0}})\, dy$

$= \frac{-1}{\pi}\cos \pi y\,\Big|_0^1 = \frac{-1}{\pi}\overset{-1}{\cancel{\cos \pi}} - \frac{-1}{\pi}\overset{1}{\cancel{\cos 0}}$

$= \frac{1}{\pi} + \frac{1}{\pi} = \frac{2}{\pi}$



16.) $\iint\limits_R e^s \ln t\, dA$

$= \int_1^2 \int_0^{\ln t} e^s \ln t\, ds\,dt$

$$= \int_1^2 \left(e^s \ln t \Big|_{s=0}^{s=\ln t}\right) dt$$

$$= \int_1^2 \left[e^{\ln t}\cdot \ln t - e^0 \ln t\right] dt$$

$$= \int_1^2 (t\ln t - \ln t)\,dt = \int_1^2 (t-1)\ln t\,dt$$

(Let $u = \ln t$, $dv = (t-1)\,dt \to$

$du = \frac{1}{t}\,dt$, $v = \frac{1}{2}t^2 - t$)

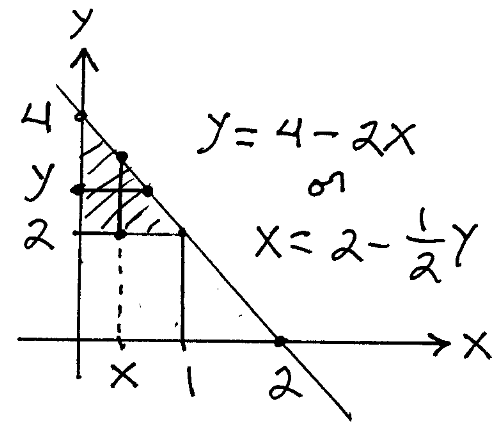
$$= \left(\tfrac{1}{2}t^2 - t\right)\ln t \Big|_1^2 - \int_1^2 \left(\tfrac{1}{2}t - 1\right) dt$$

$$= (0-0) - \left(\tfrac{1}{4}t^2 - t\right)\Big|_1^2$$

$$= -(-1) - {}^{-}\left(\tfrac{1}{4} - 1\right) = 1 + {}^{-}\tfrac{3}{4} = \tfrac{1}{4}$$

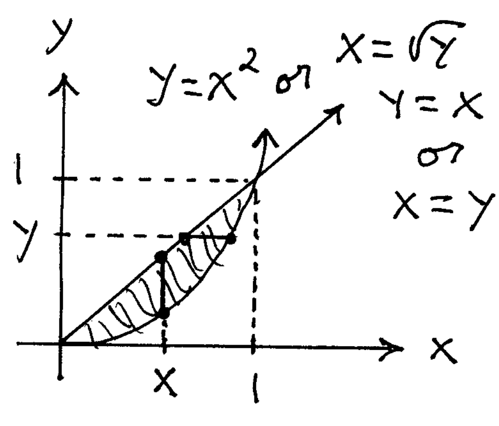
21.) $\int_0^1 \int_2^{4-2x} dy\,dx$

$$= \int_2^4 \int_0^{2-\frac{1}{2}y} dx\,dy$$



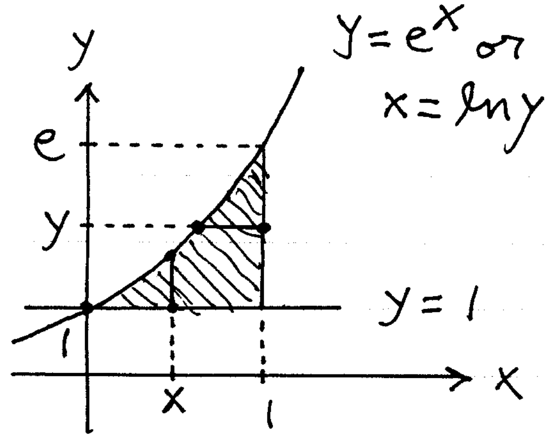
23.) $\int_0^1 \int_y^{\sqrt{y}} dx\,dy$

$$= \int_0^1 \int_{x^2}^{x} dy\,dx$$



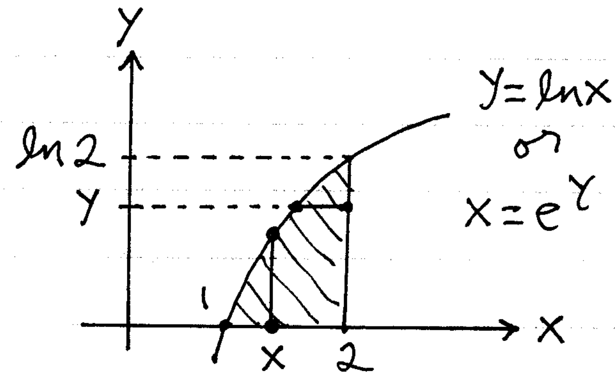
25.) $\int_0^1 \int_1^{e^x} dy\, dx$

$= \int_1^e \int_{\ln y}^1 dx\, dy$



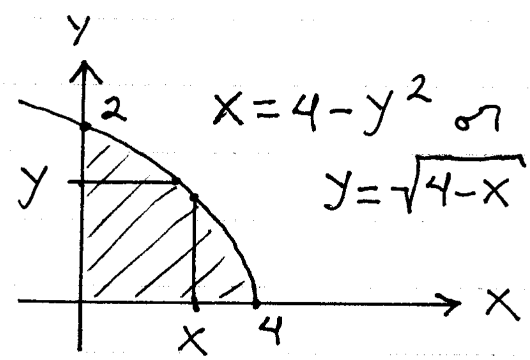
26.) $\int_0^{\ln 2} \int_{e^y}^2 dx\, dy$

$= \int_1^2 \int_0^{\ln x} dy\, dx$



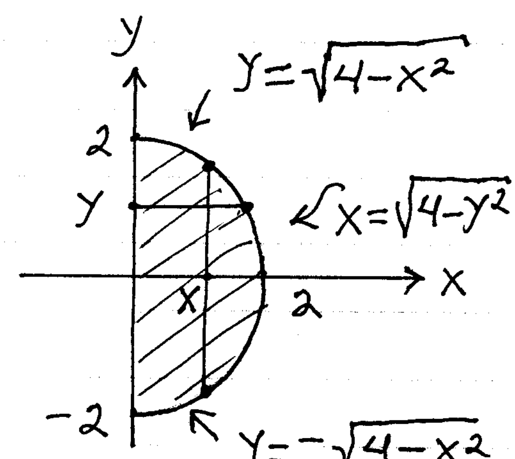
28.) $\int_0^2 \int_0^{4-y^2} y\, dx\, dy$

$= \int_0^4 \int_0^{\sqrt{4-x}} y\, dy\, dx$



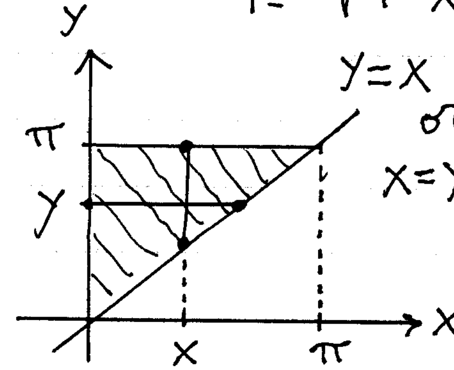
30.) $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 6x\, dy\, dx$

$= \int_{-2}^2 \int_0^{\sqrt{4-y^2}} 6x\, dx\, dy$



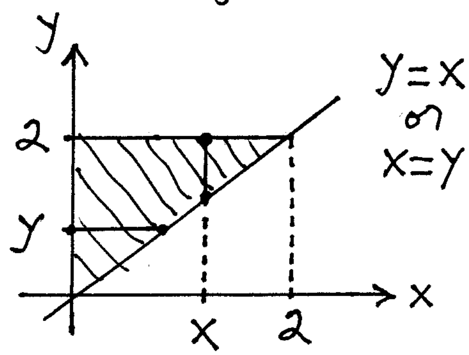
31.) $\int_0^\pi \int_x^\pi \frac{\sin y}{y}\, dy\, dx$

$= \int_0^\pi \int_0^y \frac{\sin y}{y}\, dx\, dy$



$$= \int_0^\pi \left(\frac{\sin y}{y}\cdot x \Big|_{x=0}^{x=y}\right) dy = \int_0^\pi (\sin y - 0)\, dy$$

$$= -\cos y \Big|_0^\pi = -\cos\pi - -\cos 0 = -(-1)+1 = 2$$



32.) $\int_0^2 \int_x^2 2y^2 \sin xy \, dy\, dx$

$$= \int_0^2 \int_0^y 2y^2 \sin xy \, dx\, dy$$

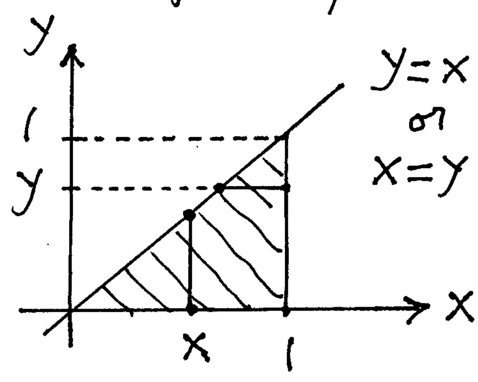
$$= \int_0^2 \left(2y^2 \cdot \frac{-1}{y} \cos xy \Big|_{x=0}^{x=y}\right) dy$$

$$= \int_0^2 \left(-2y\cos y^2 - -2y\,\overset{1}{\cancel{\cos 0}}\right) dy$$

$$= \int_0^2 \left(-2y\cos y^2 + 2y\right) dy$$

$$= \left(-\sin y^2 + y^2\right)\Big|_0^2 = (-\sin 4 + 4) - (-\overset{0}{\cancel{\sin 0}} + 0)$$

$$= 4 - \sin 4$$



33.) $\int_0^1 \int_y^1 x^2 e^{xy}\, dx\, dy$

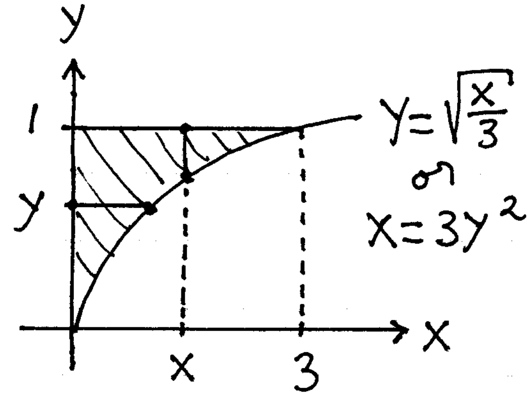
$$= \int_0^1 \int_0^x x^2 e^{xy}\, dy\, dx$$

$$= \int_0^1 \left(x^2 \cdot \frac{1}{x} e^{xy} \Big|_{y=0}^{y=x}\right) dx$$

$$= \int_0^1 \left(x e^{x^2} - x\cdot \overset{1}{\cancel{e^0}}\right) dx = \left(\frac{1}{2}e^{x^2} - \frac{1}{2}x^2\right)\Big|_0^1$$

$$= \left(\frac{1}{2}e - \frac{1}{2}\right) - \left(\frac{1}{2}e^0 - 0\right) = \frac{1}{2}e - 1$$

36.) $\int_0^3 \int_{\sqrt{\frac{x}{3}}}^1 e^{y^3}\,dy\,dx$

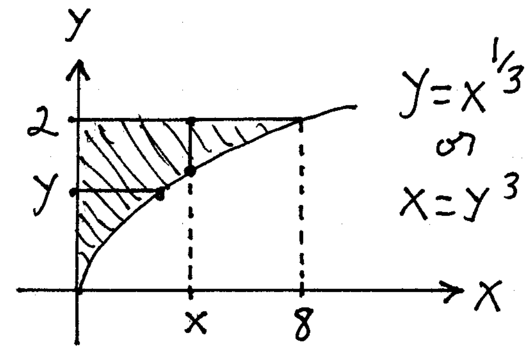


$= \int_0^1 \int_0^{3y^2} e^{y^3}\,dx\,dy$

$= \int_0^1 \left(e^{y^3} \cdot x \Big|_{x=0}^{x=3y^2}\right) dy = \int_0^1 3y^2 e^{y^3}\,dy$

$= e^{y^3}\Big|_0^1 = e^1 - e^0 = e - 1$

38.) $\int_0^8 \int_{x^{1/3}}^2 \frac{1}{y^4+1}\,dy\,dx$



$= \int_0^2 \int_0^{y^3} \frac{1}{y^4+1}\,dx\,dy$

$= \int_0^2 \left(\frac{1}{y^4+1} \cdot x \Big|_{x=0}^{x=y^3}\right) dy$

$= \int_0^2 \frac{y^3}{y^4+1}\,dy = \frac{1}{4}\ln|y^4+1|\Big|_0^2$

$= \frac{1}{4}\ln 17 - \frac{1}{4}\cancel{\ln 1}^{\,0} = \frac{1}{4}\ln 17$

39.) $|x|+|y|=1$ (graph it)

case 1: $x \ge 0,\ y \ge 0 \rightarrow x+y=1 \rightarrow y=1-x$

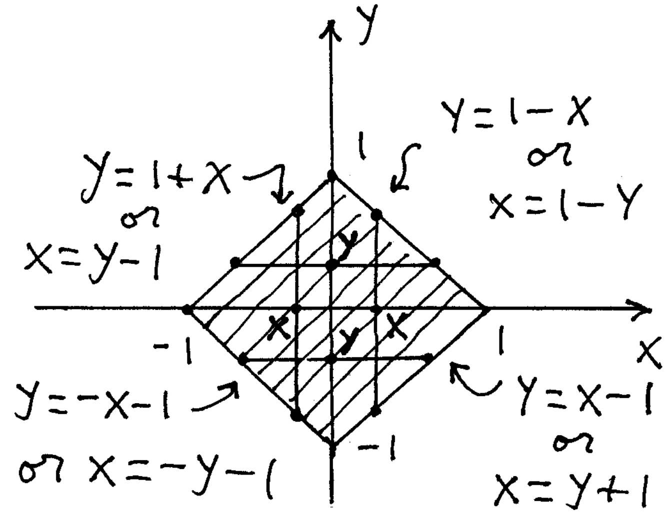
case 2: $x \le 0,\ y \ge 0 \rightarrow -x+y=1 \rightarrow y=1+x$

case 3: $x \le 0,\ y \le 0 \rightarrow -x+{}^-y=1 \rightarrow y=-x-1$

case 4: $x \ge 0,\ y \le 0 \rightarrow x+{}^-y=1 \rightarrow y=x-1$

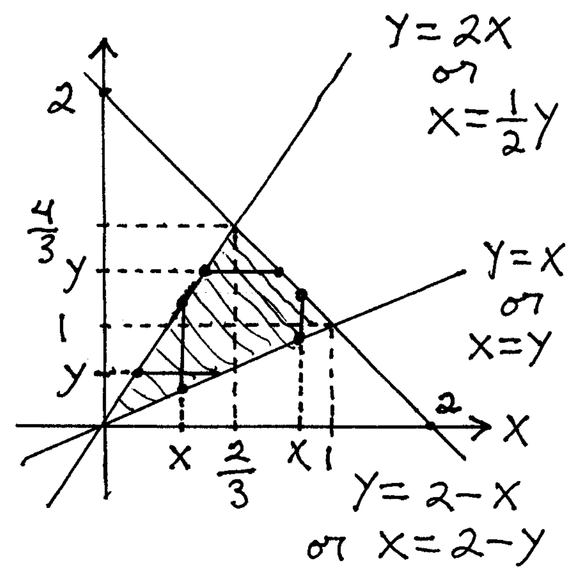
$\iint_R (y-2x^2)\,dA = \int_{-1}^{0}\int_{-x-1}^{x+1} (y-2x^2)\,dy\,dx \ +$

$$\int_0^1 \int_{x-1}^{1-x} (y-2x^2)\,dy\,dx$$



$$= \int_{-1}^{0} \int_{-y-1}^{y+1} (y-2x^2)\,dx\,dy$$

$$+ \int_0^1 \int_{y-1}^{1-y} (y-2x^2)\,dx\,dy$$



40.) $\iint_R xy\,dA$

$$= \int_0^{2/3} \int_x^{2x} xy\,dy\,dx$$

$$+ \int_{2/3}^{1} \int_x^{2-x} xy\,dy\,dx$$

$$= \int_0^1 \int_{\frac{1}{2}y}^{y} xy\,dx\,dy + \int_1^{\frac{4}{3}} \int_{\frac{1}{2}y}^{2-y} xy\,dx\,dy$$