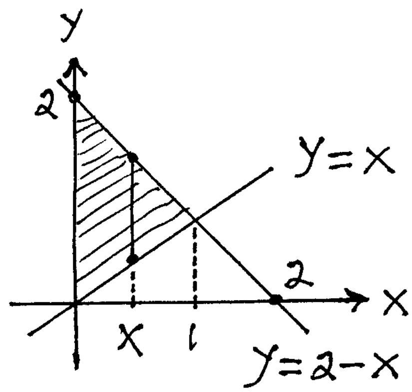
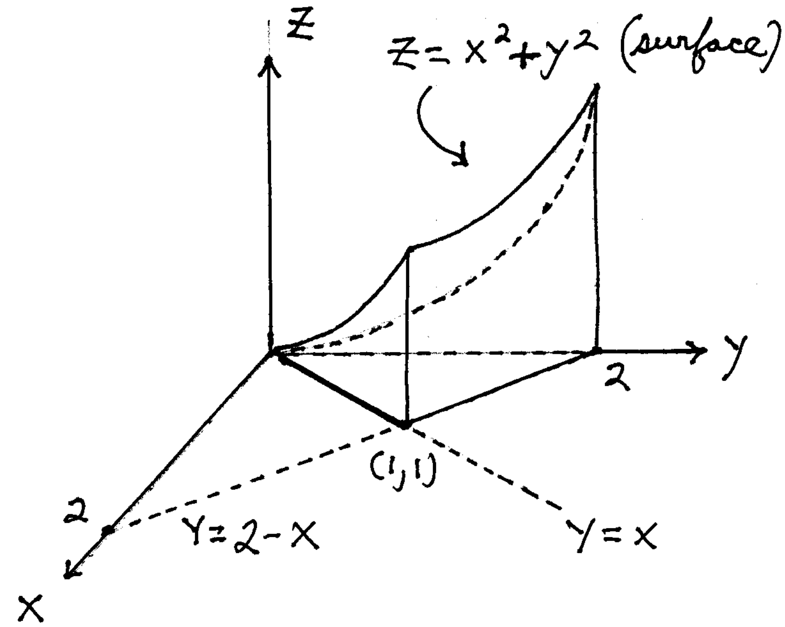
Section 15.1

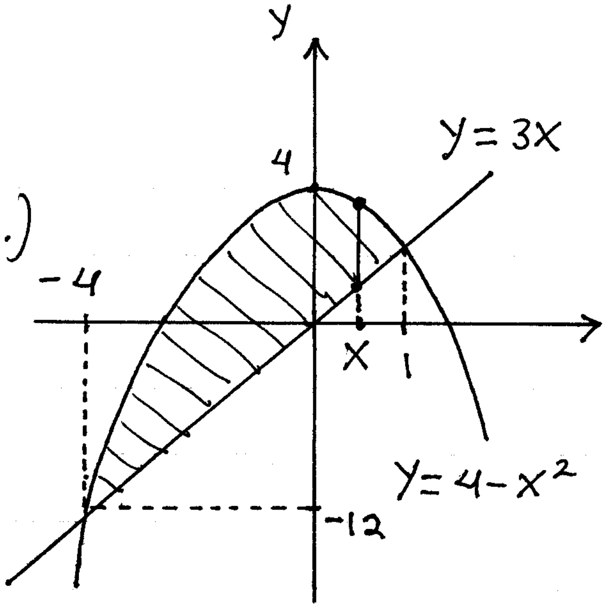
41.)



$$\text{Vol} = \int_0^1 \int_x^{2-x} (x^2+y^2)\, dy\, dx$$



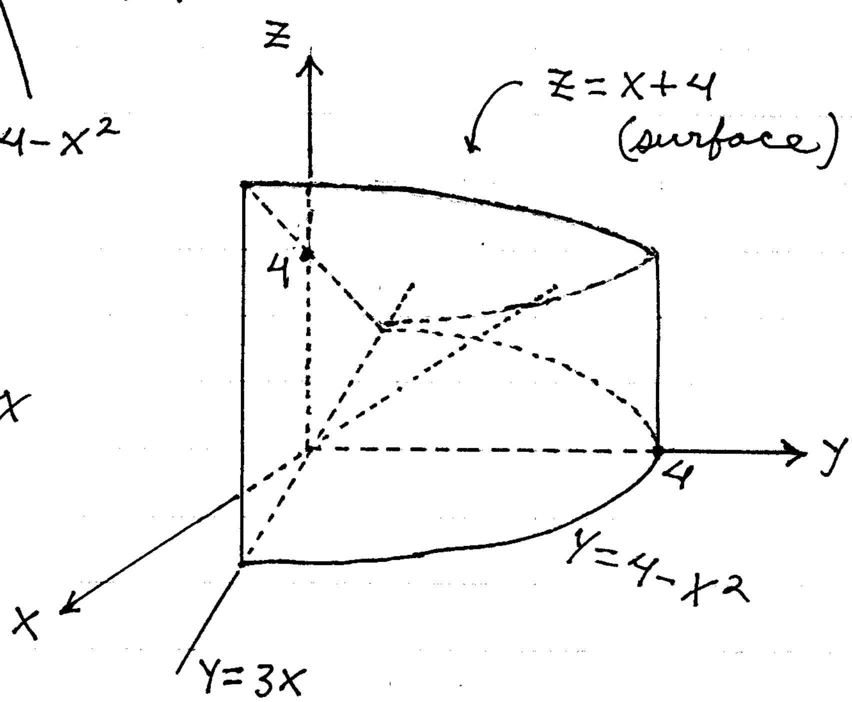
43.)



$$4-x^2 = 3x \rightarrow x^2+3x-4=0$$

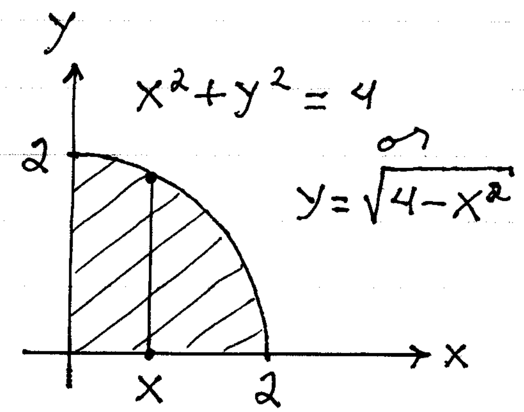
$$\rightarrow (x-1)(x+4)=0$$

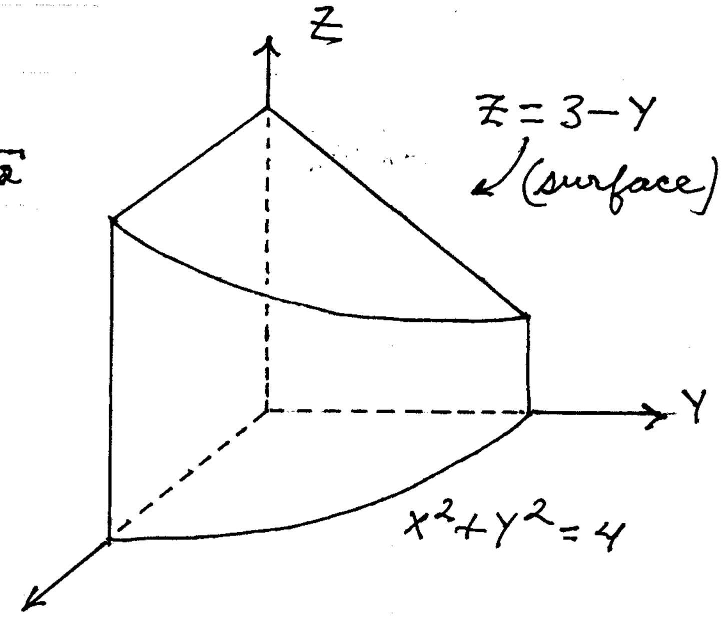
$$x=1 \quad x=-4$$



$$Vol = \int_{-4}^{1}\int_{3x}^{4-x^2} (x+4)\, dy\, dx$$

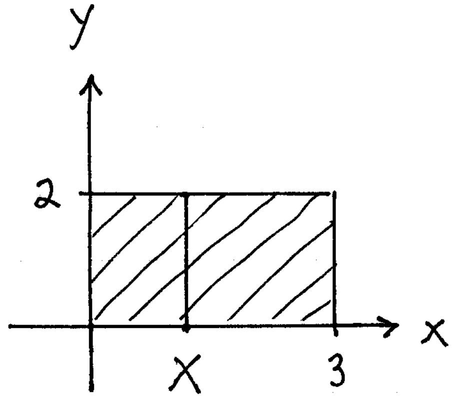
44.)

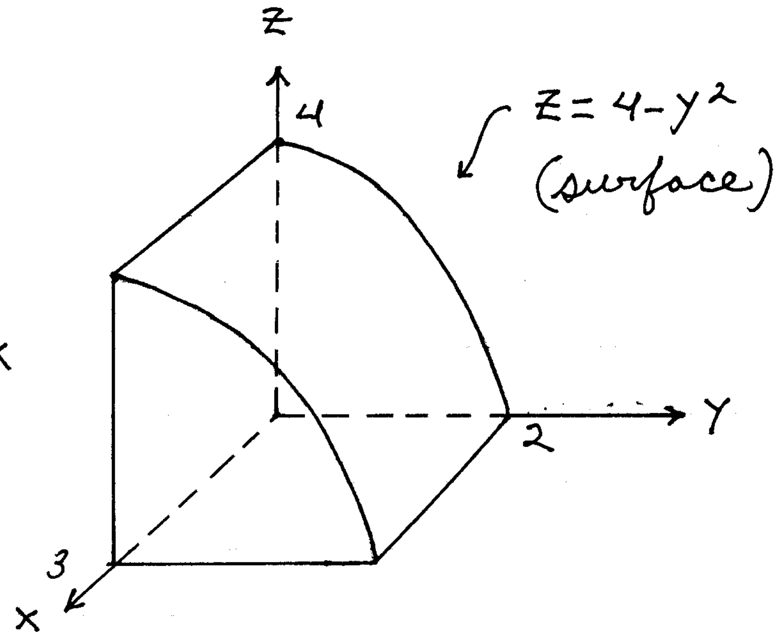




$$Vol = \int_{0}^{2}\int_{0}^{\sqrt{4-x^2}} (3-y)\, dy\, dx$$

45.)



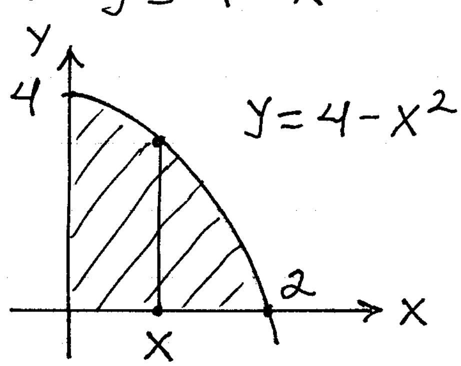


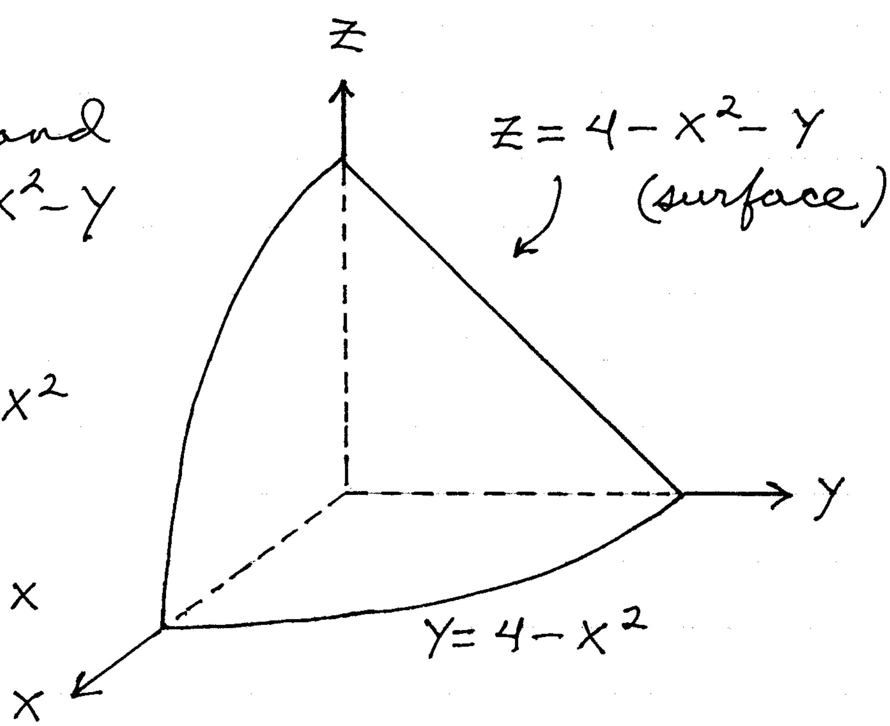
$$\text{Vol} = \int_0^3 \int_0^2 (4 - y^2)\, dy\, dx$$

46.) $z = 4 - x^2 - y$ and

$z = 0 \rightarrow 0 = 4 - x^2 - y$

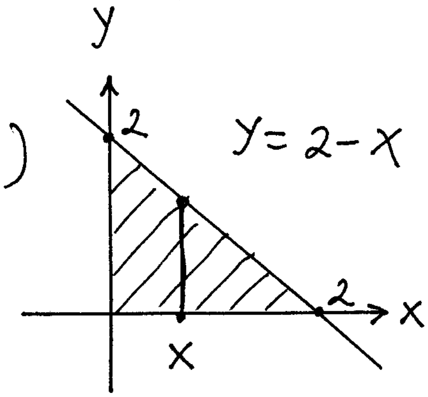
$\rightarrow \quad y = 4 - x^2$

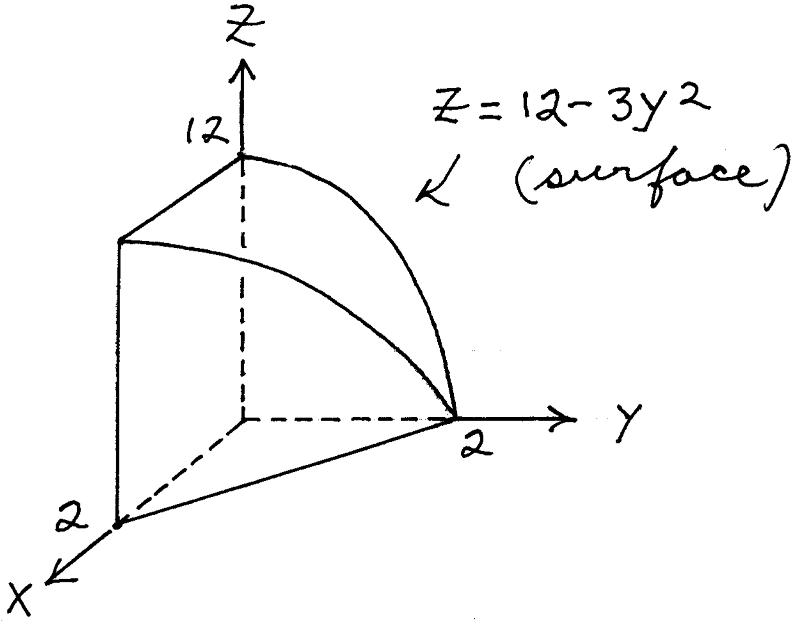




$$\text{Vol} = \int_0^2 \int_0^{4-x^2} (4 - x^2 - y)\, dy\, dx$$

47.)





$$\text{Vol} = \int_0^2 \int_0^{2-x} (12 - 3y^2)\, dy\, dx$$

51.) $$\int_1^\infty \int_{e^{-x}}^1 \frac{1}{x^3 y}\, dy\, dx = \int_1^\infty \left(\frac{1}{x^3} \ln y \Big|_{y=e^{-x}}^{y=1} \right) dx$$

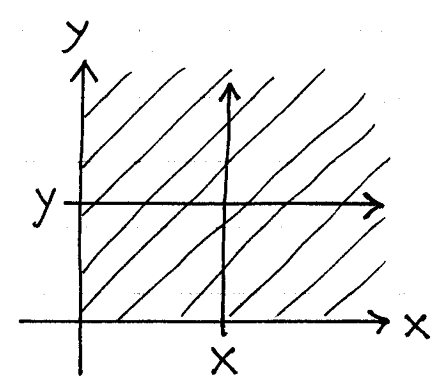
$$= \int_1^\infty \left(\frac{1}{x^3} \overbrace{\cancel{\ln 1}}^{0} - \frac{1}{x^3} \ln e^{-x} \right) dx$$

$$= \int_1^\infty \frac{-1}{x^3} \cdot (-x)\, dx = \int_1^\infty \frac{1}{x^2}\, dx$$

$$= \lim_{A \to \infty} \int_1^A x^{-2}\, dx = \lim_{A \to \infty} -x^{-1} \Big|_1^A$$

$$= \lim_{A \to \infty} \left(\frac{-1}{A} - \frac{-1}{1} \right) = 0 + 1 = 1$$

54.) $$\int_0^\infty \int_0^\infty x e^{-x-2y}\, dx\, dy$$



$$= \int_0^\infty \int_0^\infty x e^{-x-2y}\, dy\, dx$$

$$= \int_0^\infty \left(\lim_{A \to \infty} \int_0^A x e^{-x-2y}\, dy \right) dx$$

$$= \int_0^\infty \left(\lim_{A\to\infty} -\frac{1}{2} x e^{-x-2y} \Big|_{y=0}^{y=A} \right) dx$$

$$= \int_0^\infty \left(\lim_{A\to\infty} \left[-\frac{1}{2} x \cancel{e^{-x-2A}}^{\,0} - -\frac{1}{2} x e^{-x} \right] \right) dx$$

$$= \int_0^\infty \frac{1}{2} x e^{-x}\, dx \qquad \left(\text{Let } u = \frac{1}{2}x,\ dv = e^{-x}dx \ \rightarrow du = \frac{1}{2}dx,\ v = -e^{-x}\right)$$

$$= -\frac{1}{2} x e^{-x} \Big|_0^\infty - -\frac{1}{2} \int_0^\infty e^{-x}\, dx$$

$$= \lim_{A\to\infty} \left[-\frac{1}{2} x e^{-x} \Big|_0^A + \frac{1}{2} \int_0^A e^{-x}\, dx \right]$$

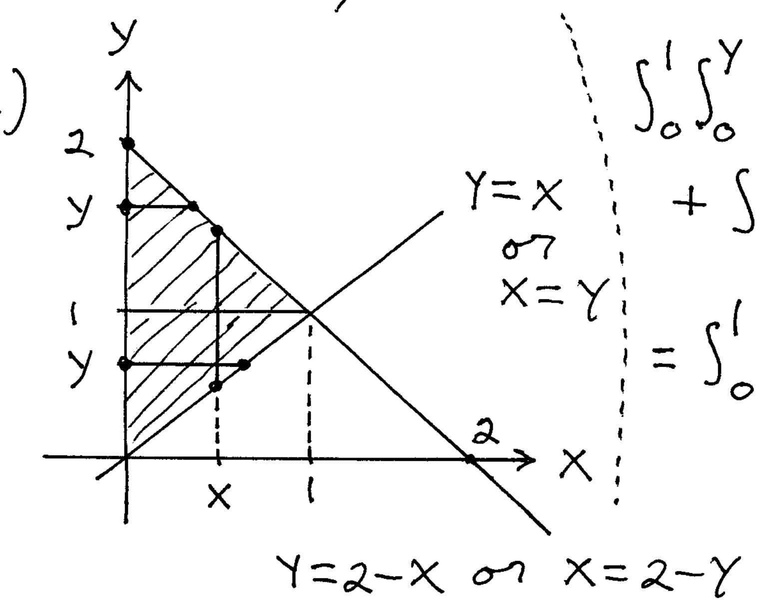
$$= \lim_{A\to\infty} \left[\left(-\frac{1}{2} \cdot \frac{A}{e^A} - 0 \right) + \frac{1}{2} \cdot -e^{-x} \Big|_0^A \right]$$

$$= \lim_{A\to\infty} \left[-\frac{1}{2} \cdot \frac{A}{e^A} + \left(-\frac{1}{2} \cdot \cancel{\frac{1}{e^A}}^{\,0} - -\frac{1}{2}(1) \right) \right]$$

← L'Hopital's Rule, "$\frac{\infty}{\infty}$"

$$= \lim_{A\to\infty} \left[-\frac{1}{2} \cdot \cancel{\frac{1}{e^A}}^{\,0} + \frac{1}{2} \right] = \frac{1}{2}$$

59.)



$$\int_0^1 \int_0^y (x^2+y^2)\, dx\, dy$$

$$+ \int_1^2 \int_0^{2-y} (x^2+y^2)\, dx\, dy$$

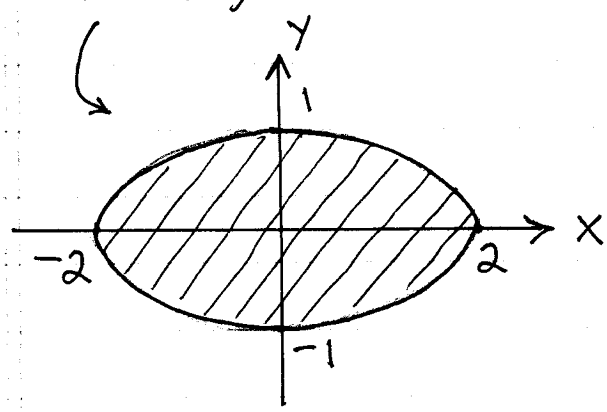
$$= \int_0^1 \int_x^{2-x} (x^2+y^2)\, dy\, dx$$

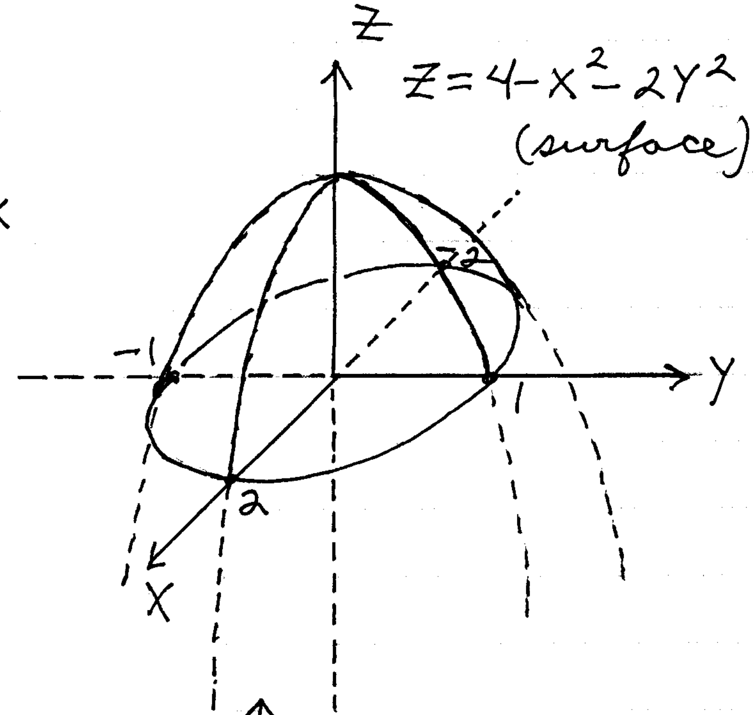
61.) Consider trace of surface

$z = 4 - x^2 - 2y^2$ in xy-plane :

$z = 0 \rightarrow 0 = 4 - x^2 - 2y^2 \rightarrow$

$x^2 + 2y^2 = 4$ (ellipse)





i.) If region R is inside ellipse, then solid will be smaller than →.

ii.) If region R is outside ellipse, then value of z is NEGATIVE and surface is below xy-plane creating "NEGATIVE" volume

iii) Volume is MAXIMUM when

R : pts. (x, y) satisfying

$x^2 + 2y^2 \leq 4$

Math 21D
Kouba
Worksheet 1
Intersections of Surfaces, Projections

1.) Sketch each pair of surfaces and their intersection on the same set of axes. On a separate set of axes plot the projection of this intersection in the xy-plane.

a.) $x + 2y + 3z = 6$ and $z = x$

b.) $z = x^2 + y^2$ and $z = 4$

c.) $z = 2x^2 + 2y^2$ and $z = y + 1$

d.) $z = x^2 + y^2$ and $z = 4 - 3x^2$

e.) $z = \sqrt{x^2 + y^2}$ and $z = 6 - x^2 - y^2$

f.) $x^2 + y^2 + z^2 = 9$ and $y = x$

g.) $x^2 + z^2 = y^2 + 3$ and $z = 2$

h.) $x^2 + y^2 - z^2 = 0$ and $z = y + 1$

2.) Consider the intersection of the surfaces $z = x^2 + y - 1$ and $y = x^2 + 1$. Plot the projection of this intersection in the

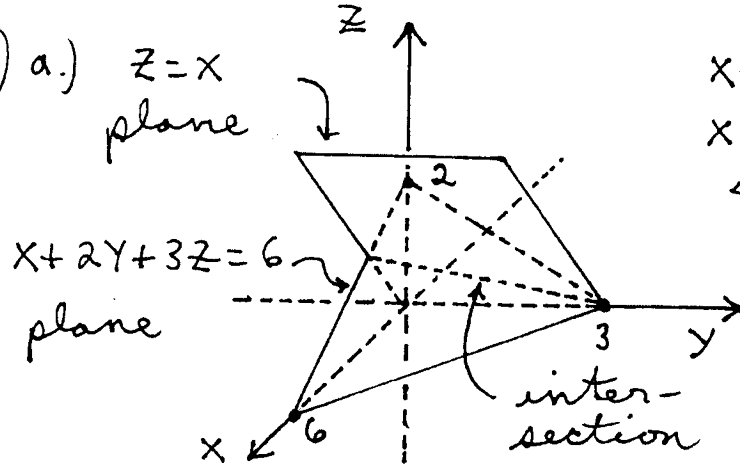
a.) xz-plane.

b.) yz-plane.

c.) xy-plane.

Worksheet 1 Solutions

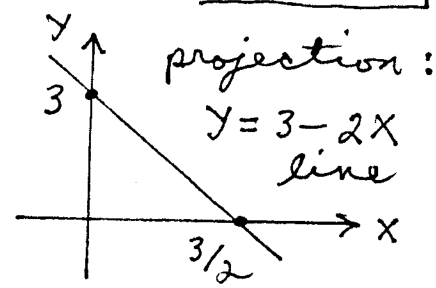
1.) a.)



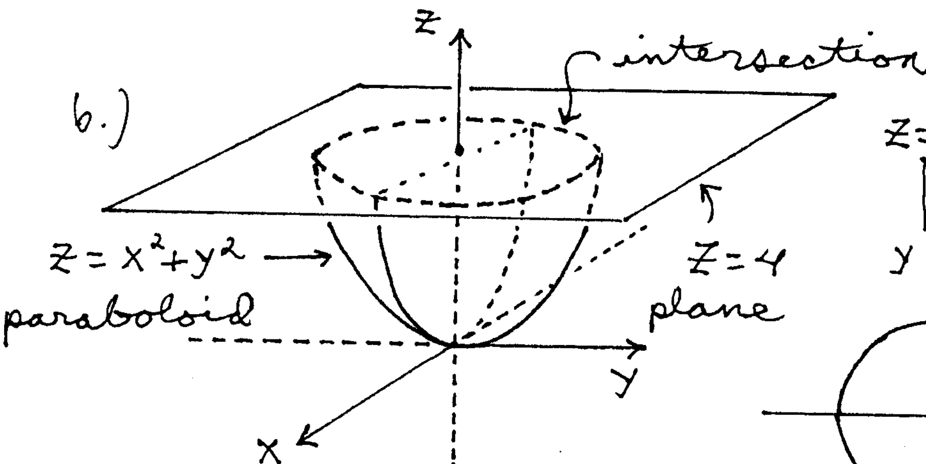
$x + 2y + 3z = 6$ and $z = x \rightarrow$

$x + 2y + 3(x) = 6 \rightarrow$

$4x + 2y = 6 \rightarrow \boxed{y = 3 - 2x}$

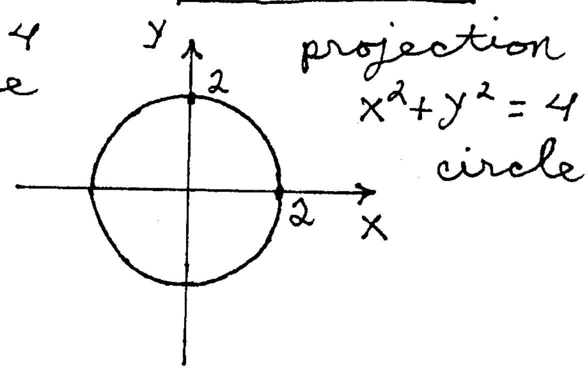


b.)

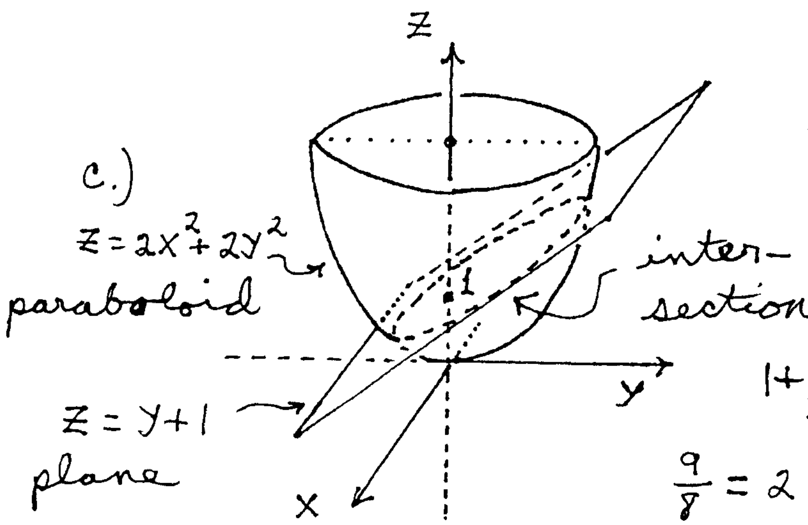


$z = x^2 + y^2$ and $z = 4 \rightarrow$

$\boxed{x^2 + y^2 = 2^2}$



c.)



$z = 2x^2 + 2y^2$ and $z = y + 1 \rightarrow$

$y + 1 = 2x^2 + 2y^2 \rightarrow$

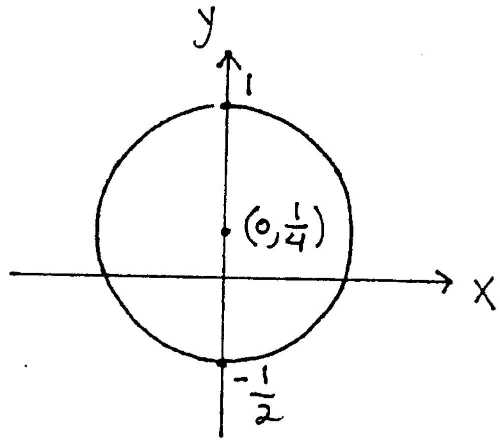
$1 = 2x^2 + 2y^2 - y \rightarrow$

$1 = 2x^2 + 2\left(y^2 - \frac{1}{2}y\right) \rightarrow$

$1 + \frac{1}{8} = 2x^2 + 2\left(y^2 - \frac{1}{2}y + \frac{1}{16}\right) \rightarrow$

$\frac{9}{8} = 2\left[x^2 + \left(y - \frac{1}{4}\right)^2\right] \rightarrow$

$$\boxed{x^2 + \left(y - \frac{1}{4}\right)^2 = \left(\frac{3}{4}\right)^2}$$

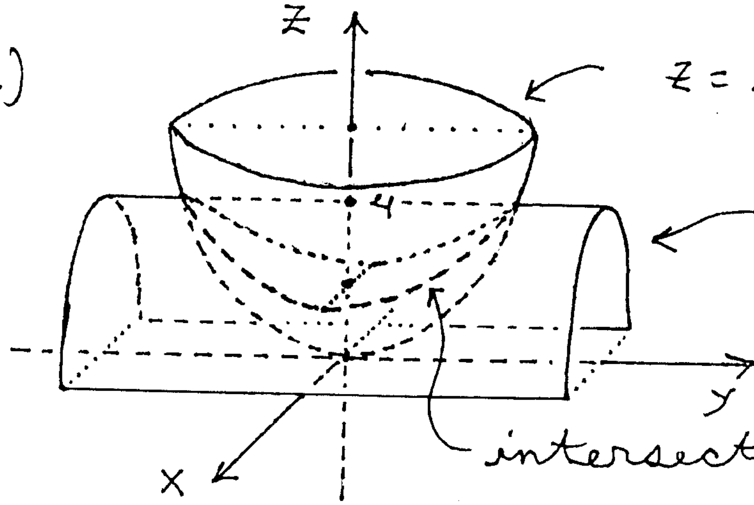


projection:

$x^2 + (y - \frac{1}{4})^2 = \left(\frac{3}{4}\right)^2$

circle

d.)



$z = x^2 + y^2$

paraboloid,

$z = 4 - 3x^2$

parabolic cylinder

intersection

$z = x^2 + y^2$ and $z = 4 - 3x^2 \rightarrow$

$4 - 3x^2 = x^2 + y^2 \rightarrow$

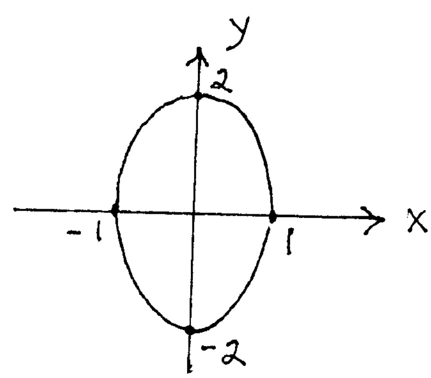
$4 = 4x^2 + y^2 \rightarrow$

$\boxed{1 = x^2 + \frac{y^2}{2^2}}$

projection:

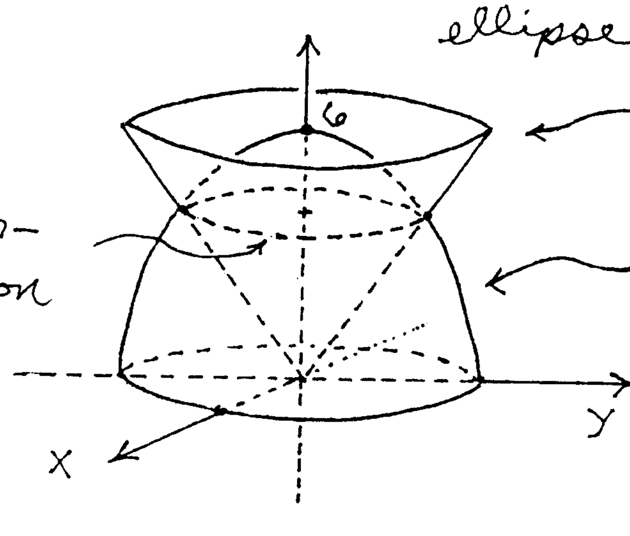
$1 = x^2 + \frac{y^2}{4}$

ellipse



e.)

inter-
section



$z = \sqrt{x^2 + y^2}$

cone,

$z = 6 - x^2 - y^2$

paraboloid

$z = \sqrt{x^2+y^2}$ and $z = 6 - x^2 - y^2 \rightarrow$

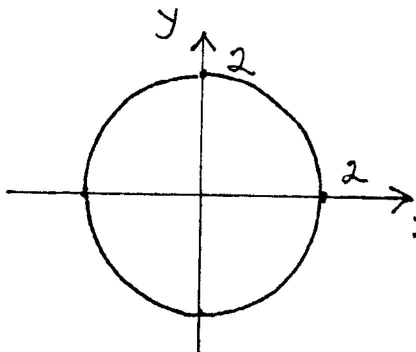
$z^2 = x^2+y^2$ and $z = 6 - (x^2+y^2) \rightarrow$

$z = 6 - z^2 \rightarrow$

$z^2 + z - 6 = 0 \rightarrow (z-2)(z+3) = 0 \rightarrow$

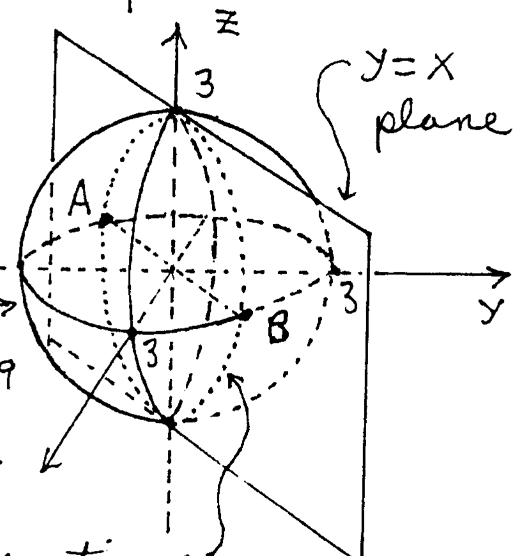
$z = -3$ (NO) or $\underline{z = 2}$; then

$2 = \sqrt{x^2+y^2} \rightarrow \boxed{x^2+y^2 = 2^2}$;

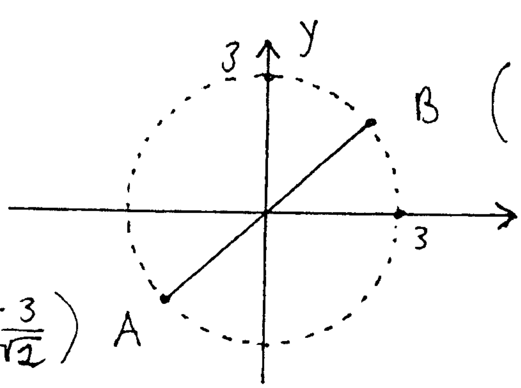


projection : $x^2+y^2 = 4$ circle

f.)



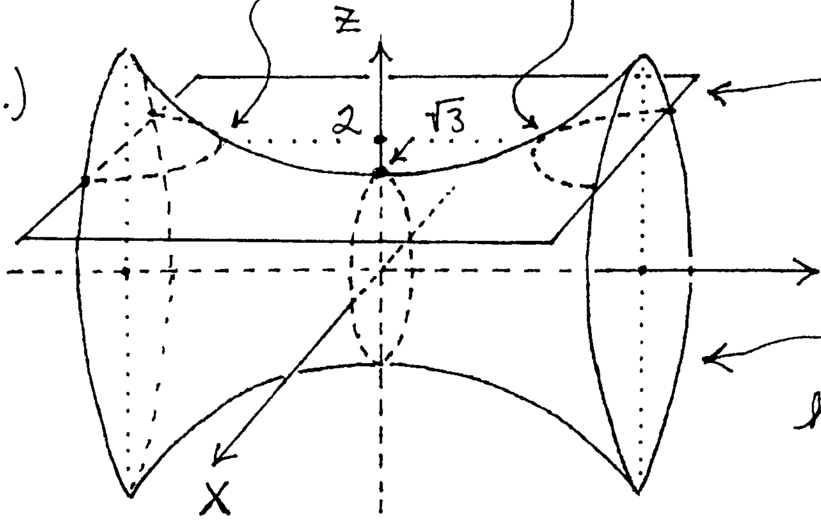
Since the plane $y=x$ is perpendicular to the xy-plane, the projection ~~of the~~ intersection in the xy-plane is the __line segment joining__ __A and B__.



projection : segment AB

g.)

intersection



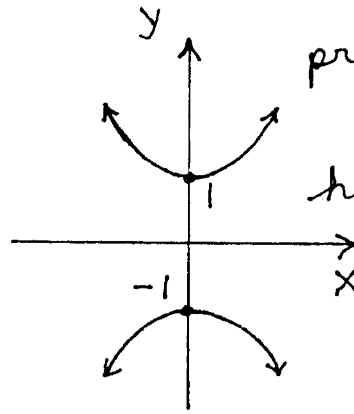
$z = 2$ plane,

$x^2 + z^2 = y^2 + 3$ hyperboloid of one sheet

$x^2 + z^2 = y^2 + 3$ and $z = 2 \rightarrow$

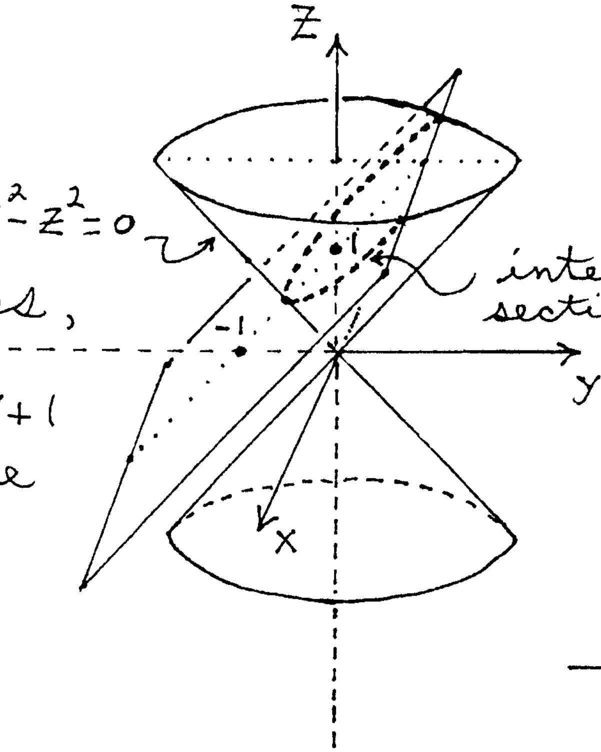
$x^2 + 4 = y^2 + 3 \rightarrow$

$\boxed{y^2 - x^2 = 1}$



projection:
$y^2 - x^2 = 1$
hyperbola

h.)



$x^2 + y^2 - z^2 = 0$ cones,

inter-section

$z = y + 1$ plane

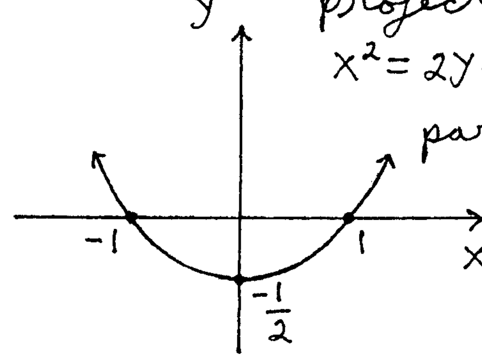
$x^2 + y^2 - z^2 = 0$ and $z = y + 1 \rightarrow$

$x^2 + y^2 - (y+1)^2 = 0 \rightarrow$

$x^2 + \cancel{y^2} - \cancel{y^2} - 2y - 1 = 0 \rightarrow$

$\boxed{x^2 = 2y + 1}$

projection:
$x^2 = 2y + 1$
parabola

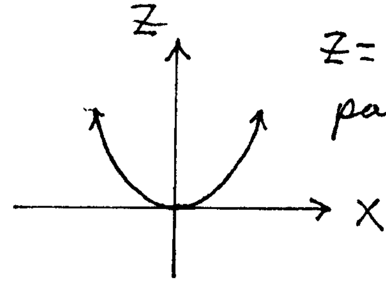


2.) $z = x^2 + y - 1$ and $y = x^2 + 1$

a.) xz-plane projection:

$z = x^2 + (x^2+1) - 1 \rightarrow$

$\boxed{z = 2x^2}$



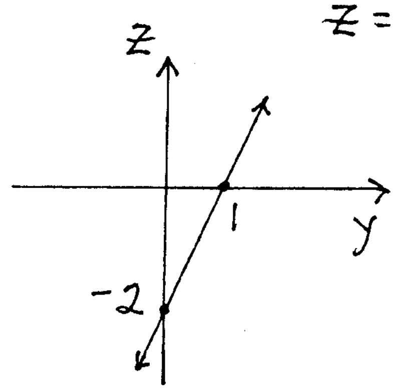
$z = 2x^2$
parabola

b.) yz-plane projection:

$y = x^2 + 1 \rightarrow x^2 = y - 1$ so

$z = (y-1) + y - 1 \rightarrow$

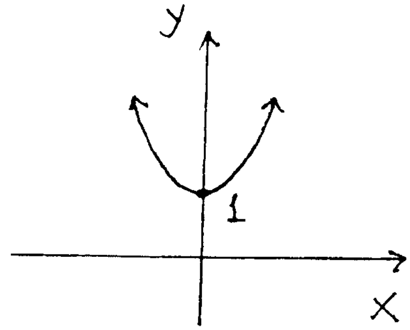
$\boxed{z = 2y - 2}$



$z = 2y - 2$
line

c.) xy-plane projection:

Since $y = x^2 + 1$ is a cylinder (surface) perpendicular to the xy-plane, the projection of the intersection in the xy-plane is $\boxed{y = x^2 + 1}$.



$y = x^2 + 1$
parabola