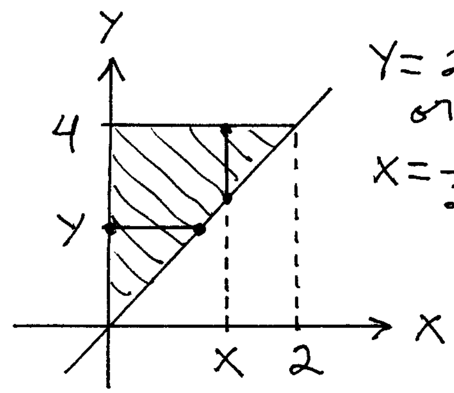
Section 15.2

2.)

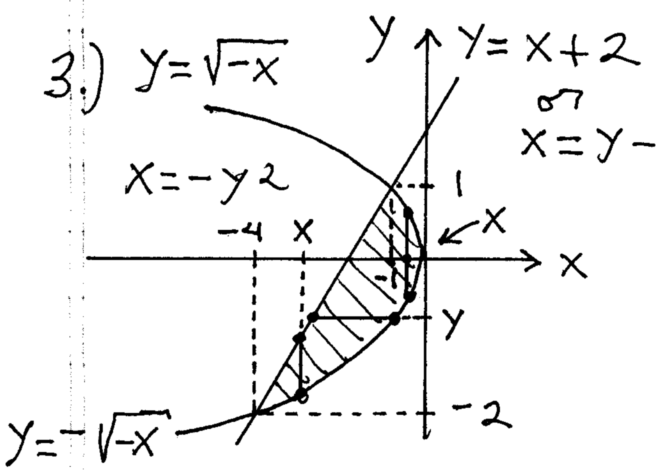



$y = 2x$ or $x = \frac{1}{2}y$

$$\text{Area} = \int_0^2 \int_{2x}^4 1 \, dy \, dx$$

$$= \int_0^4 \int_0^{\frac{1}{2}y} 1 \, dx \, dy$$

3.)




$y = x + 2$ or $x = y - 2$

$$-y^2 = y - 2 \rightarrow$$

$$0 = y^2 + y - 2 \rightarrow$$

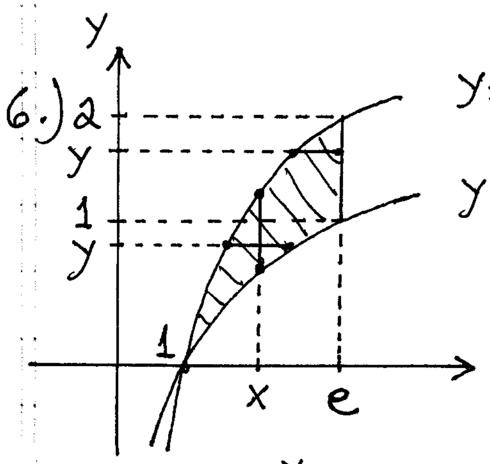
$$0 = (y-1)(y+2)$$

$$y = 1 \qquad y = -2$$

$$\text{Area} = \int_{-2}^1 \int_{y-2}^{-y^2} 1 \, dx \, dy$$

$$= \int_{-4}^{-1} \int_{-\sqrt{-x}}^{x+2} 1 \, dy \, dx + \int_{-1}^{0} \int_{-\sqrt{-x}}^{\sqrt{-x}} 1 \, dy \, dx$$

6.)




$y = 2\ln x$ or $x = e^{y/2}$

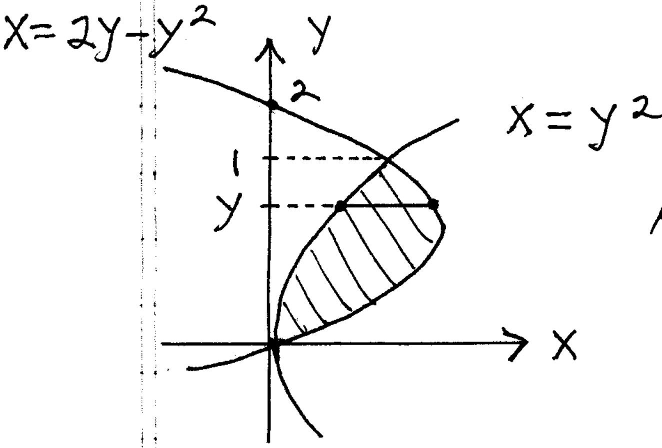
$y = \ln x$ or $x = e^y$

$$\text{Area} = \int_1^e \int_{\ln x}^{2\ln x} 1 \, dy \, dx$$

$$= \int_0^1 \int_{e^{y/2}}^{e^y} 1 \, dx \, dy + \int_1^2 \int_{e^{y/2}}^{e} 1 \, dx \, dy$$

7.) $y^2 = 2y - y^2 \rightarrow 2y^2 - 2y = 0 \rightarrow 2y(y-1) = 0$

$y = 0 \quad y = 1$

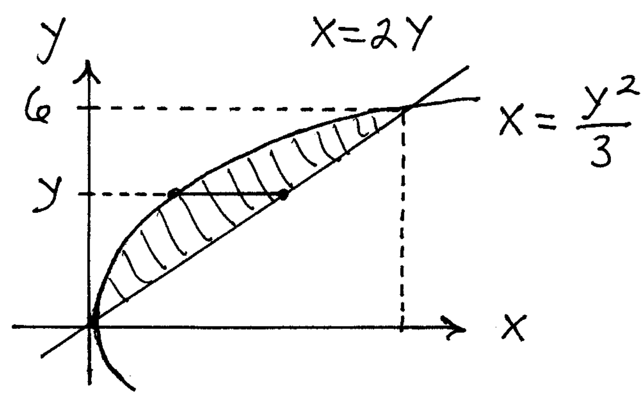



$$\text{Area} = \int_0^1 \int_{y^2}^{2y-y^2} 1 \, dx \, dy$$

9.) $\int_0^6 \int_{y^2/3}^{2y} 1 \, dx \, dy$

$0 \le y \le 6$ ,

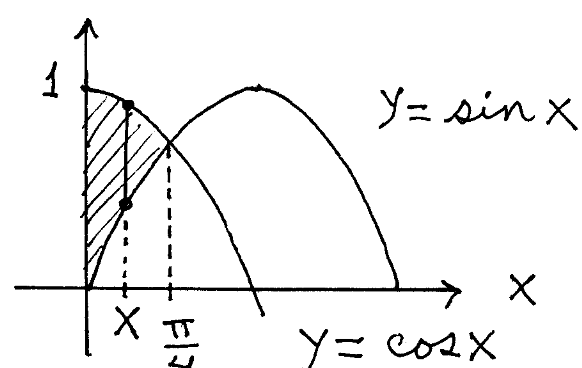
$\frac{y^2}{3} \le x \le 2y$




11.) $\int_0^{\frac{\pi}{4}} \int_{\sin x}^{\cos x} 1 \, dy \, dx$

$0 \le x \le \frac{\pi}{4}$ ,

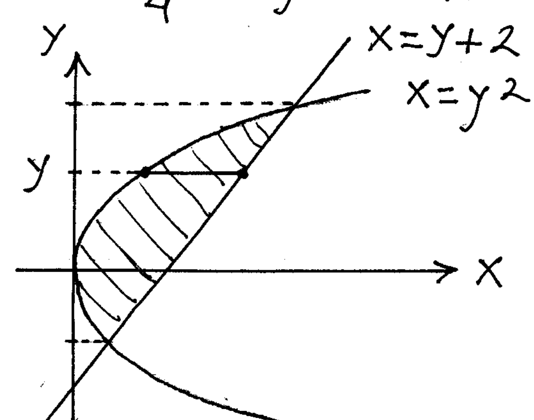
$\sin x \le y \le \cos x$




12.) $\int_{-1}^{2} \int_{y^2}^{y+2} 1 \, dx \, dy$

$-1 \le y \le 2$ ,

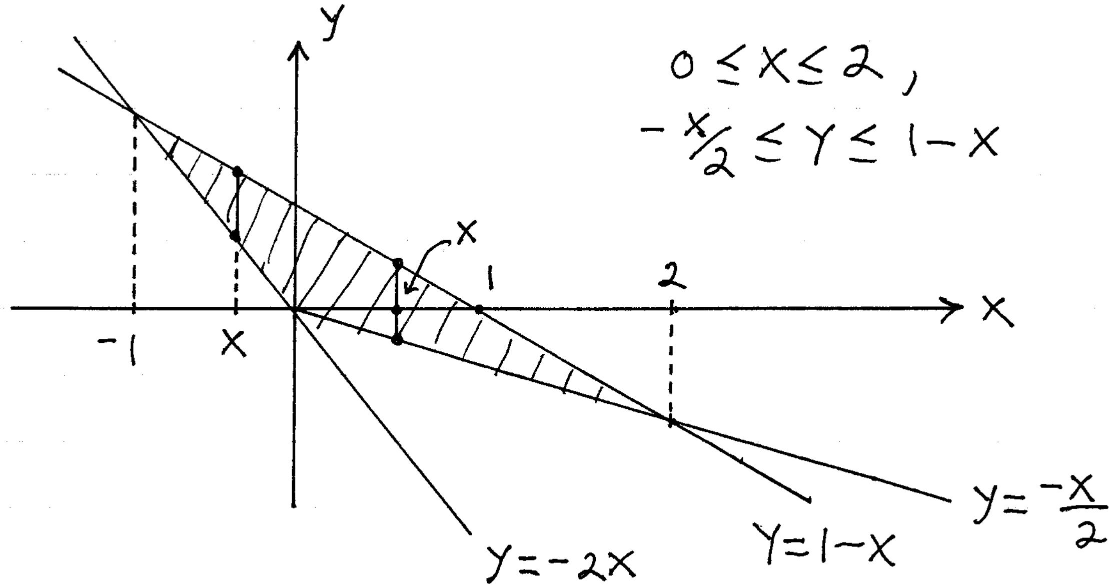
$y^2 \le x \le y + 2$



13.) $\int_{-1}^{0}\int_{-2x}^{1-x} 1\,dy\,dx + \int_{0}^{2}\int_{-x/2}^{1-x} 1\,dy\,dx$

$-1 \le x \le 0,$
$-2x \le y \le 1-x$

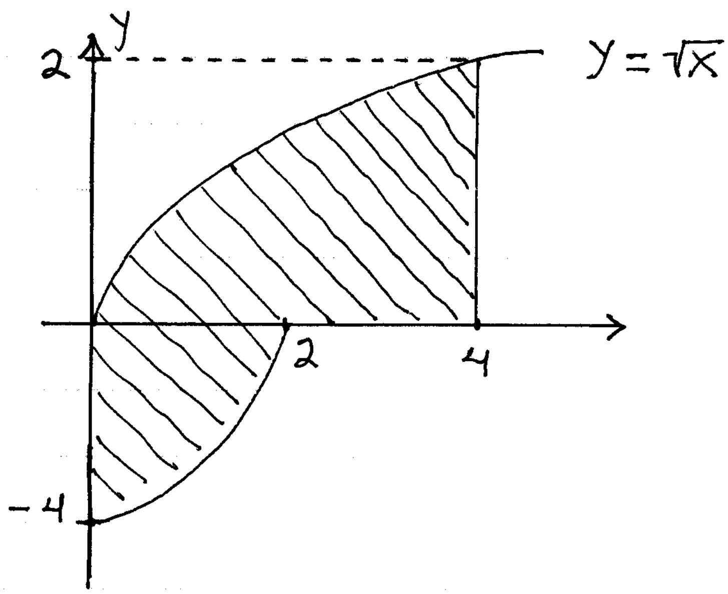
$0 \le x \le 2,$
$-x/2 \le y \le 1-x$



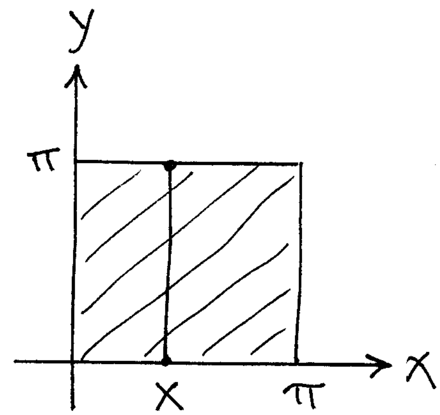

14.) $\int_{0}^{2}\int_{x^2-4}^{0} 1\,dy\,dx + \int_{0}^{4}\int_{0}^{\sqrt{x}} 1\,dy\,dx$

$0 \le x \le 2,$
$x^2 - 4 \le y \le 0$

$0 \le x \le 4,$
$0 \le y \le \sqrt{x}$



15a.) $\text{Ave} = \frac{1}{\text{area}R} \iint\limits_R f(x,y)\, dA$




$= \frac{1}{\pi^2} \int_0^{\pi} \int_0^{\pi} \sin(x+y)\, dy\, dx$

$= \frac{1}{\pi^2} \int_0^{\pi} \left( -\cos(x+y) \Big|_{y=0}^{y=\pi} \right) dx$

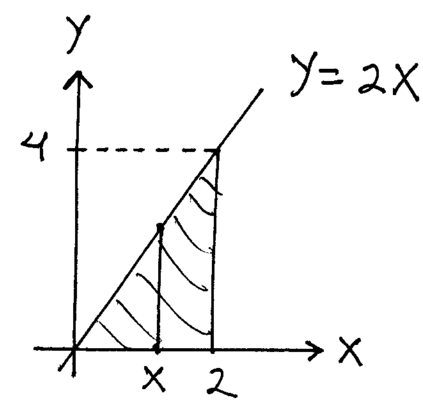
$= \frac{1}{\pi^2} \int_0^{\pi} \left[ -\cos(x+\pi) - -\cos x \right] dx$

$= \frac{1}{\pi^2} \left( -\sin(x+\pi) + \sin x \right) \Big|_0^{\pi}$

$= \frac{1}{\pi^2} \left( -\overset{0}{\cancel{\sin 2\pi}} + \overset{0}{\cancel{\sin \pi}} \right) - \frac{1}{\pi^2} \left( -\overset{0}{\cancel{\sin \pi}} + \overset{0}{\cancel{\sin 0}} \right)$

$= 0$

17.) $\text{Ave} = \frac{1}{\text{area}R} \iint\limits_R f(x,y)\, dA$




$= \frac{1}{\frac{1}{2}(2)(4)} \int_0^{2} \int_0^{2x} (x^2+y^2)\, dy\, dx$

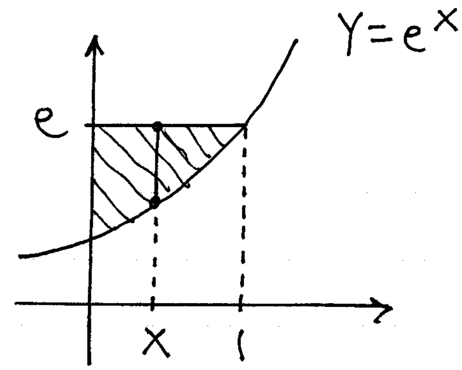
$= \frac{1}{4} \int_0^{2} \left( x^2 y + \frac{1}{3} y^3 \right) \Big|_{y=0}^{y=2x} dx$

$= \frac{1}{4} \int_0^{2} \left[ \left( 2x^3 + \frac{8}{3} x^3 \right) - (0+0) \right] dx$

$= \frac{1}{4} \int_0^{2} \frac{14}{3} x^3\, dx = \frac{7}{6} \cdot \frac{1}{4} x^4 \Big|_0^2$

$= \frac{7}{24} (16) = \frac{14}{3}$

18.) $\text{Ave} = \frac{1}{\text{area}R} \iint_R f(x,y)\,dy\,dx$ ;




$\text{area } R = \int_0^1 \int_{e^x}^{e} 1\,dy\,dx$

$= \int_0^1 \left( y \Big|_{y=e^x}^{y=e} \right) dx$

$= \int_0^1 (e - e^x)\,dx = (ex - e^x)\Big|_0^1$

$= (e-e) - (0-1) = 1$ ;

$\text{Ave} = \frac{1}{1} \int_0^1 \int_{e^x}^{e} \frac{1}{(x+1)y}\,dy\,dx$

$= \int_0^1 \left( \frac{1}{x+1} \cdot \ln y \Big|_{y=e^x}^{y=e} \right) dx$

$= \int_0^1 \left[ \frac{1}{x+1} \cdot \overset{1}{\cancel{\ln e}} - \frac{1}{x+1} \cdot \overset{x}{\cancel{\ln e^x}} \right] dx$

$= \int_0^1 \left[ \frac{1}{x+1} - \frac{x}{x+1} \right] dx = \int_0^1 \frac{1-x}{x+1}\,dx$

(~~Let~~ $u = x+1 \xrightarrow{D} du = dx$ and $x = u-1$, and $x: 0 \to 1$ so $u: 1 \to 2$)

$= \int_1^2 \frac{1-(u-1)}{u}\,du = \int_1^2 \frac{2-u}{u}\,du$

$= \int_1^2 \left( \frac{2}{u} - 1 \right) du = (2\ln|u| - u)\Big|_1^2$

$= (2\ln 2 - 2) - (2\overset{0}{\cancel{\ln 1}} - 1)$

$= 2\ln 2 - 1$