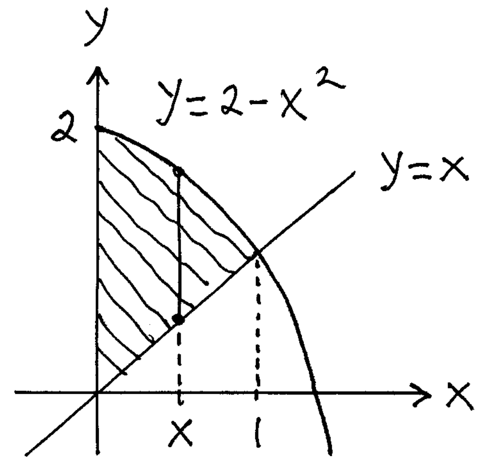
Section 15.2

19.) $M = \iint\limits_R \delta(x,y)\, dA$

$= \int_0^1 \int_x^{2-x^2} 3 \; dy\, dx$,



$\delta(x,y) = 3$

$M_y = \iint\limits_R x\, \delta(x,y)\, dA$

$= \int_0^1 \int_x^{2-x^2} 3x \; dy\, dx$,

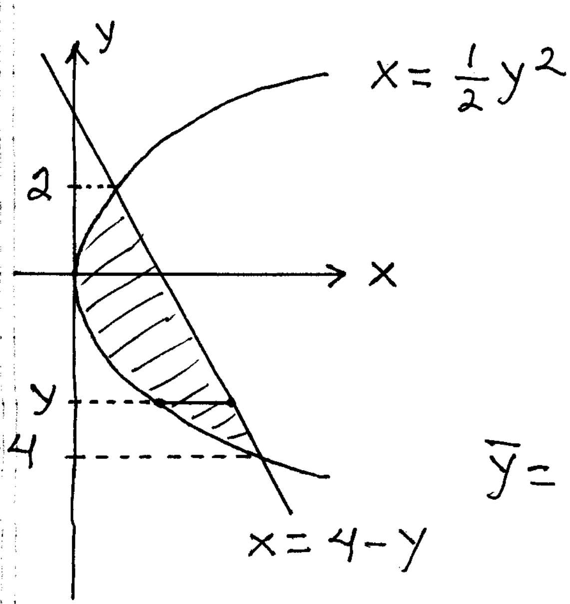
$M_x = \iint\limits_R y\, \delta(x,y)\, dA = \int_0^1 \int_x^{2-x^2} 3y \; dy\, dx \rightarrow$

$\bar{x} = \dfrac{M_y}{M}$ and $\bar{y} = \dfrac{M_x}{M}$

21.) $y^2 = 2x \rightarrow x = \frac{1}{2}y^2$ and $x + y = 4 \rightarrow$

$\frac{1}{2}y^2 + y = 4 \rightarrow y^2 + 2y - 8 = 0 \rightarrow$

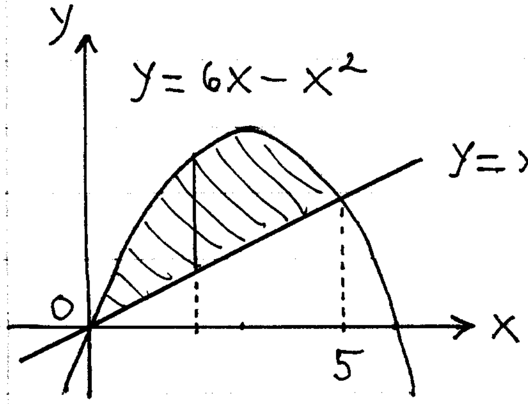
$(y-2)(y+4) = 0 \rightarrow y = 2,\; y = -4$



$$\bar{x} = \frac{\int_{-4}^{2} \int_{\frac{1}{2}y^2}^{4-y} x \; dx\, dy}{\int_{-4}^{2} \int_{\frac{1}{2}y^2}^{4-y} 1 \; dx\, dy} ,$$

$$\bar{y} = \frac{\int_{-4}^{2} \int_{\frac{1}{2}y^2}^{4-y} y \; dx\, dy}{\int_{-4}^{2} \int_{\frac{1}{2}y^2}^{4-y} 1 \; dx\, dy}$$

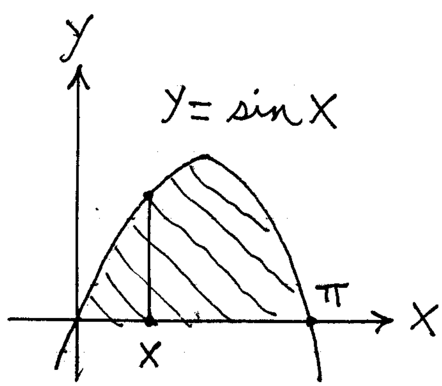
24.) $y = 6x - x^2$ and $y = x \rightarrow x = 6x - x^2 \rightarrow$
$x^2 - 5x = 0 \rightarrow x(x-5) = 0 \rightarrow x=0,\ x=5$



$$\bar{x} = \frac{\int_0^5 \int_x^{6x-x^2} x \, dy \, dx}{\int_0^5 \int_x^{6x-x^2} 1 \, dy \, dx},$$

$$\bar{y} = \frac{\int_0^5 \int_x^{6x-x^2} y \, dy \, dx}{\int_0^5 \int_x^{6x-x^2} 1 \, dy \, dx}$$

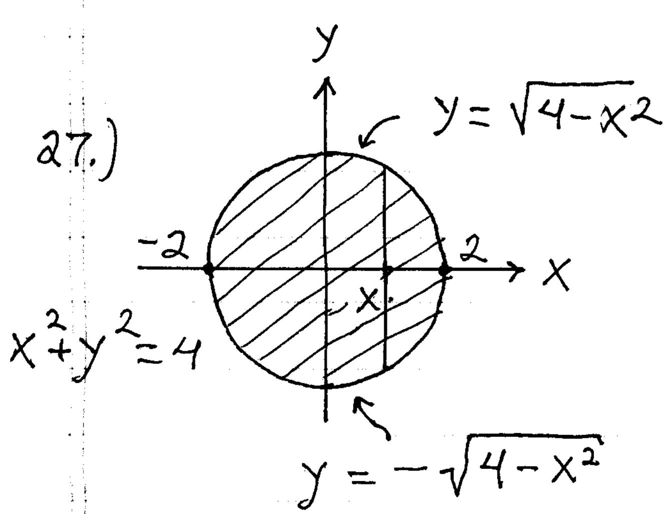
26.)



$$\bar{x} = \frac{\int_0^\pi \int_0^{\sin x} x \, dy \, dx}{\int_0^\pi \int_0^{\sin x} 1 \, dy \, dx},$$

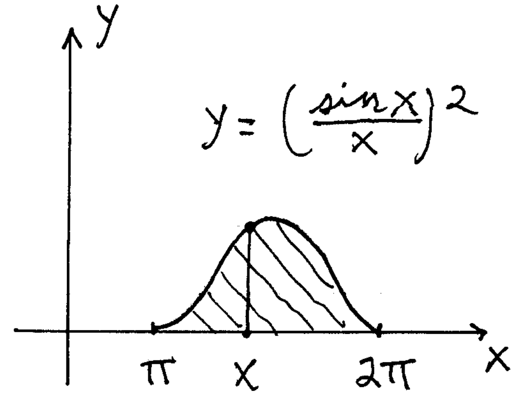
$$\bar{y} = \frac{\int_0^\pi \int_0^{\sin x} y \, dy \, dx}{\int_0^\pi \int_0^{\sin x} 1 \, dy \, dx}$$

27.)



$$I_x = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} y^2 \cdot (1) \, dy \, dx$$

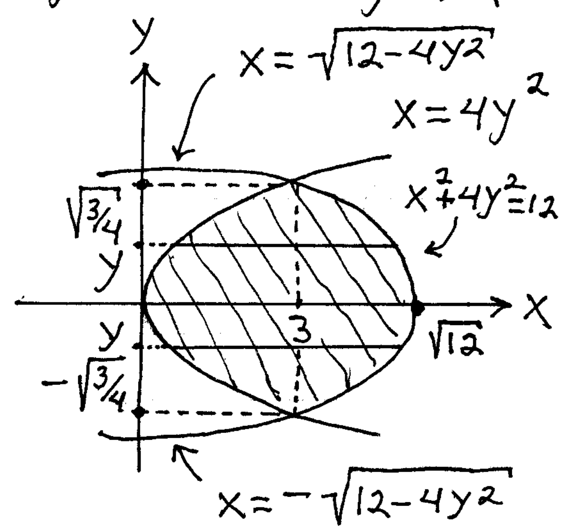
28.) $I_y = \int_{\pi}^{2\pi} \int_0^{\left(\frac{\sin x}{x}\right)^2} x^2(1)\, dy\, dx$



32.) $x^2+4y^2=12$ and $x=4y^2 \rightarrow$
$x^2+x-12=0 \rightarrow (x-3)(x+4)=0 \rightarrow x=3,\ \cancel{x=-4}$
$\rightarrow 3^2+4y^2=12 \rightarrow y^2=3/4 \rightarrow$
$y=\pm\sqrt{3/4}$, $\delta(x,y)=5x$,

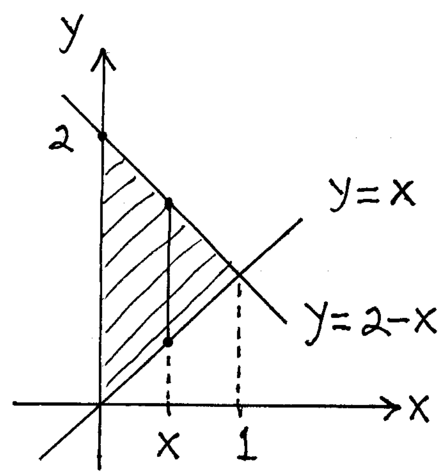
$$\text{Mass} = \int_0^{\sqrt{3/4}} \int_{4y^2}^{\sqrt{12-4y^2}} 5x\, dx\, dy$$

$$+ \int_{-\sqrt{3/4}}^{0} \int_{4y^2}^{-\sqrt{12-4y^2}} 5x\, dx\, dy$$



33.) $\delta(x,y)=6x+3y+3$

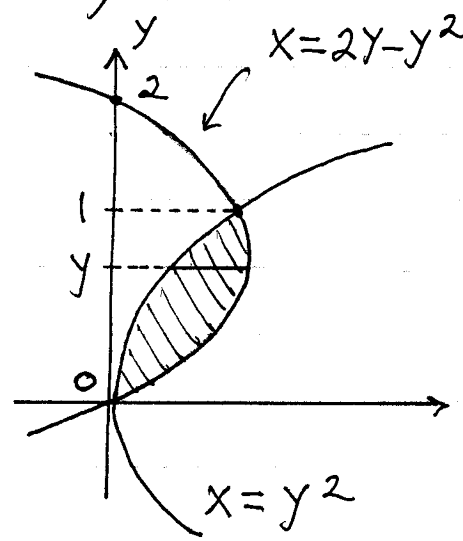
$$\bar{x} = \frac{\int_0^1 \int_x^{2-x} x\cdot(6x+3y+3)\, dy\, dx}{\int_0^1 \int_x^{2-x} (6x+3y+3)\, dy\, dx},$$



$$\bar{y} = \frac{\int_0^1 \int_x^{2-x} y\,(6x+3y+3)\, dy\, dx}{\int_0^1 \int_x^{2-x} (6x+3y+3)\, dy\, dx}$$

34.) $x=y^2$ and $x=2y-y^2 \rightarrow y^2=2y-y^2 \rightarrow$
$2y^2-2y=2y(y-1)=0 \rightarrow y=0,\ y=1$

$\delta(x,y)=y+1$

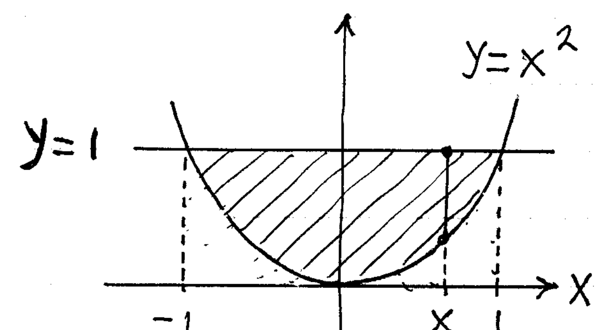


$$\bar{x}=\frac{\int_0^1\int_{y^2}^{2y-y^2} x(y+1)\,dx\,dy}{\int_0^1\int_{y^2}^{2y-y^2}(y+1)\,dx\,dy},$$

$$\bar{y}=\frac{\int_0^1\int_{y^2}^{2y-y^2} y(y+1)\,dx\,dy}{\int_0^1\int_{y^2}^{2y-y^2}(y+1)\,dx\,dy};$$

$$I_x=\int_0^1\int_{y^2}^{2y-y^2} y^2(y+1)\,dx\,dy$$

36.) $\delta(x,y)=y+1$

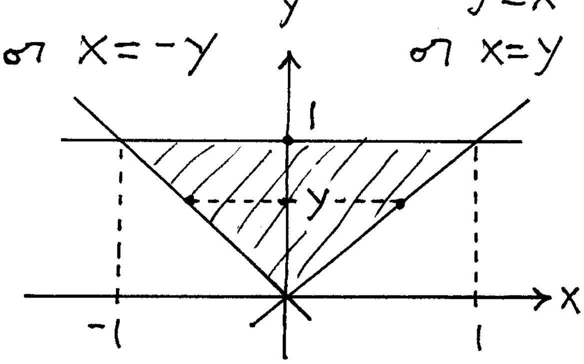


$$\bar{x}=\frac{\int_{-1}^1\int_{x^2}^{1} x(y+1)\,dy\,dx}{\int_{-1}^1\int_{x^2}^{1}(y+1)\,dy\,dx},$$

$$\bar{y}=\frac{\int_{-1}^1\int_{x^2}^{1} y(y+1)\,dy\,dx}{\int_{-1}^1\int_{x^2}^{1}(y+1)\,dy\,dx};$$

$$I_y = \int_{-1}^{1}\int_{x^2}^{1} x^2\,(y+1)\,dy\,dx$$

39.) $\delta(x,y) = y+1$



$$\overline{x} = \frac{\int_0^1\int_{-y}^{y} x\,(y+1)\,dx\,dy}{\int_0^1\int_{-y}^{y} (y+1)\,dx\,dy},$$

$$\overline{y} = \frac{\int_0^1\int_{-y}^{y} y\,(y+1)\,dx\,dy}{\int_0^1\int_{-y}^{y} (y+1)\,dx\,dy};$$

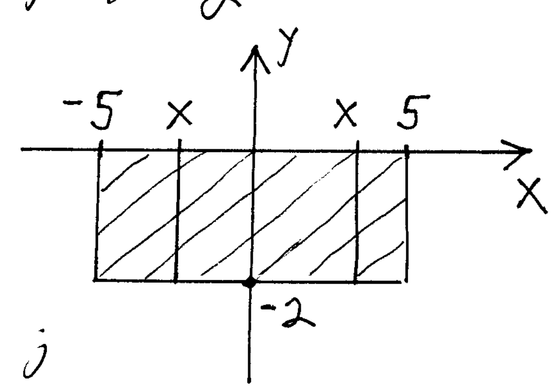
$$I_x = \int_0^1\int_{-y}^{y} y^2\,(y+1)\,dx\,dy,$$

$$I_y = \int_0^1\int_{-y}^{y} x^2\,(y+1)\,dx\,dy,$$

$$I_0 = \int_0^1\int_{-y}^{y} (x^2+y^2)\,(y+1)\,dx\,dy$$

41.) $f(x,y) = \dfrac{10{,}000\,e^{y}}{1+\frac{1}{2}|x|}$

bacteria/area units;



$$\text{TOTAL} = \iint_R f(x,y)\,dA$$

$$= \int_{-5}^{5} \int_{-2}^{0} \frac{10,000}{1+\frac{1}{2}|x|} \cdot e^y \, dy \, dx$$

$$= \int_{-5}^{5} \left(\frac{10,000}{1+\frac{1}{2}|x|} \cdot e^y \Big|_{y=-2}^{y=0} \right) dx$$

$$= \int_{-5}^{5} \left[\frac{10,000}{1+\frac{1}{2}|x|} - \frac{10,000\, e^{-2}}{1+\frac{1}{2}|x|} \right] dx$$

$$= 10,000 \left(1-e^{-2}\right) \int_{-5}^{5} \frac{1}{1+\frac{1}{2}|x|} \cdot \frac{2}{2} \, dx$$

$$= 20,000 \left(1-e^{-2}\right) \int_{-5}^{5} \frac{1}{2+|x|} \, dx \qquad |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$= 20,000 \left(1-e^{-2}\right) \left[\int_{-5}^{0} \frac{1}{2-x} \, dx + \int_{0}^{5} \frac{1}{2+x} \, dx \right]$$

$$= 20,000 \left(1-e^{-2}\right) \left[-\ln|2-x| \Big|_{-5}^{0} + \ln|2+x| \Big|_{0}^{5} \right]$$

$$= 20,000 \left(1-e^{-2}\right) \left[(-\ln 2 - {}^{-}\ln 7) + (\ln 7 - \ln 2) \right]$$

$$= 20,000 \left(1-e^{-2}\right) \cdot \left[2\ln 7 - 2\ln 2 \right]$$

$\approx$ 43,329 bacteria

44.) Minimize $F(a) = \int_0^4 \int_0^2 (y-a)^2 \, dy \, dx$
for $0 \leq a \leq 2$ $\longrightarrow$

$$F(a) = \int_0^4 \left(\frac{1}{3}(y-a)^3 \Big|_{y=0}^{y=2} \right) dx$$

$$= \int_0^4 \left[\frac{1}{3}(2-a)^3 - \frac{1}{3}(-a)^3 \right] dx$$

$$= \left[\frac{1}{3}(2-a)^3 + \frac{1}{3}a^3 \right] \cdot x \Big|_0^4$$

$$= \left[\frac{1}{3}(2-a)^3 + \frac{1}{3}a^3 \right] \cdot (4-0) \rightarrow$$

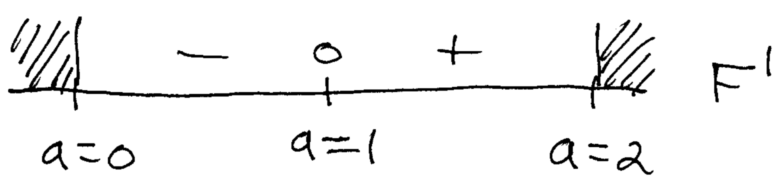
$$F(a) = \frac{4}{3}(2-a)^3 + \frac{4}{3}a^3 \xrightarrow{D}$$

$$F'(a) = 4(2-a)^2 \cdot (-1) + 4a^2$$

$$= -4(\cancel{a^2} - 4a + 4) + \cancel{4a^2}$$

$$= 16a - 16$$

$$= 16(a-1) = 0 \rightarrow a = 1$$



Min. $F(1) = 8/3$

53.) A: $c_1 = \langle 1, 3 \rangle$, $m_1 = 8\delta$

B: $c_2 = \langle 3, 3.5 \rangle$, $m_2 = 2\delta$

C: $c_3 = \langle 5, 2 \rangle$, $m_3 = 6\delta$

a.) $A \cup B$: $\langle \bar{X}, \bar{Y} \rangle = \dfrac{8\delta\langle 1,3 \rangle + 2\delta\langle 3, 3.5 \rangle}{8\delta + 2\delta}$

$$= \frac{\langle 8\delta, 24\delta\rangle + \langle 6\delta, 7\delta\rangle}{10\delta}$$

$$= \frac{\langle 14\delta, 31\delta\rangle}{10\delta} = \left\langle \frac{14\delta}{10\delta}, \frac{31\delta}{10\delta}\right\rangle$$

$$= \left\langle \frac{7}{5}, \frac{31}{10}\right\rangle$$

b.) $A \cup C = \dfrac{8\delta\langle 1,3\rangle + 6\delta\langle 5,2\rangle}{8\delta + 6\delta}$

$$= \frac{\langle 8\delta + 30\delta, 24\delta + 12\delta\rangle}{14\delta}$$

$$= \left\langle \frac{38\delta}{14\delta}, \frac{36\delta}{14\delta}\right\rangle = \left\langle \frac{19}{7}, \frac{18}{7}\right\rangle$$

c.) $B \cup C = \dfrac{2\delta\langle 3, 3.5\rangle + 6\delta\langle 5,2\rangle}{2\delta + 6\delta}$

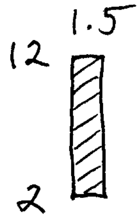
$$= \frac{\langle 6\delta + 30\delta, 7\delta + 12\delta\rangle}{8\delta}$$

$$= \left\langle \frac{36\delta}{8\delta}, \frac{19\delta}{8\delta}\right\rangle = \left\langle \frac{9}{2}, \frac{19}{8}\right\rangle$$

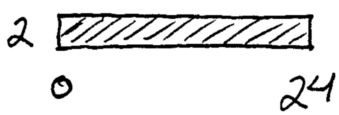
d.) $A \cup B \cup C = \dfrac{8\delta\langle 1,3\rangle + 2\delta\langle 3, 3.5\rangle + 6\delta\langle 5,2\rangle}{8\delta + 2\delta + 6\delta}$

$$= \frac{\langle 8\delta + 6\delta + 30\delta, 24\delta + 7\delta + 12\delta\rangle}{16\delta}$$

$$= \left\langle \frac{44\delta}{16\delta}, \frac{43\delta}{16\delta}\right\rangle = \left\langle \frac{11}{4}, \frac{43}{16}\right\rangle$$

54.) 

: $c_1 = \langle 0.75, 7 \rangle$, $m_1 = 15\delta$



: $c_2 = \langle 12, 1 \rangle$, $m_2 = 48\delta$

$$\langle \bar{x}, \bar{y} \rangle = \frac{15\delta \langle 0.75, 7 \rangle + 48\delta \langle 12, 1 \rangle}{15\delta + 48\delta}$$

$$= \frac{\langle 11.25\delta + 576\delta,\ 105\delta + 48\delta \rangle}{63\delta}$$

$$= \left\langle \frac{577.25\delta}{63\delta},\ \frac{153\delta}{63\delta} \right\rangle$$

$$\approx \langle \underset{\text{in.}}{9.16},\ \underset{\text{in.}}{2.43} \rangle$$