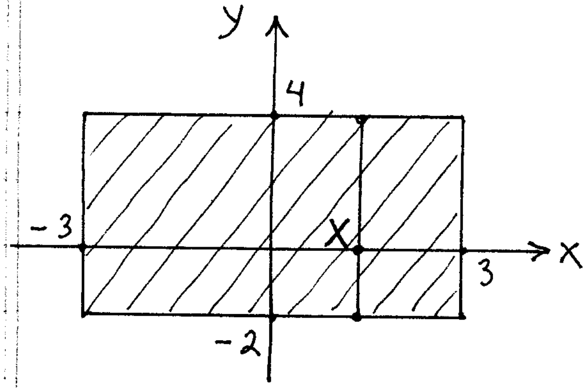
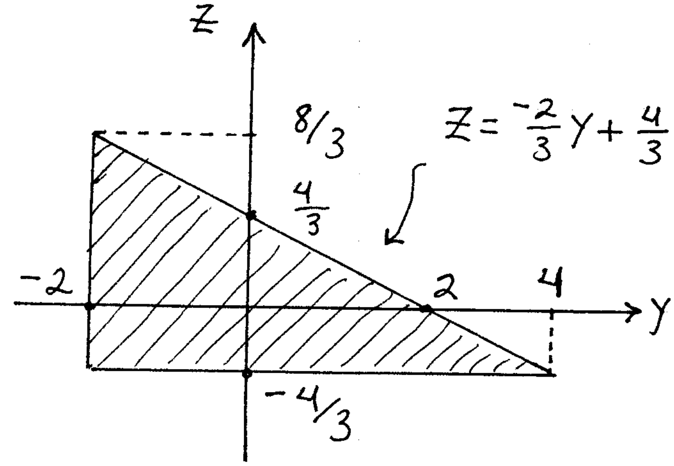
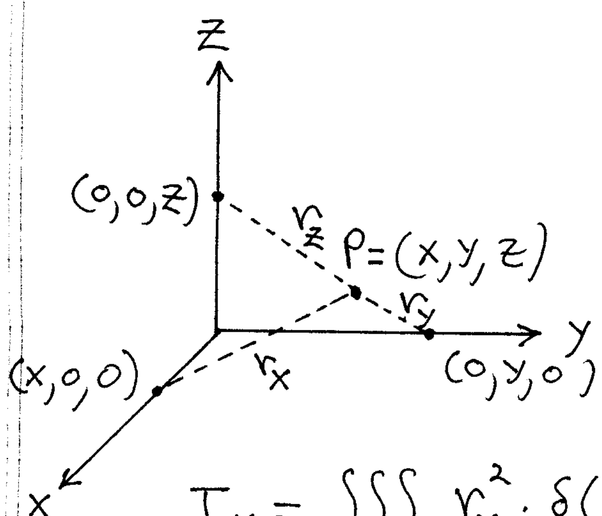
Section 15.5

2.) projection of R onto xy-plane:



projection onto yz-plane:





$\delta = 1$

$$I_x = \iiint_R r_x^2 \cdot \delta(P)\, dV = \int_{-3}^{3}\int_{-2}^{4}\int_{-\frac{4}{3}}^{-\frac{2}{3}y+\frac{4}{3}} (y^2+z^2)(1)\, dz\, dy\, dx,$$

$$I_y = \iiint_R r_y^2 \cdot \delta(P)\, dV = \int_{-3}^{3}\int_{-2}^{4}\int_{-\frac{4}{3}}^{-\frac{2}{3}y+\frac{4}{3}} (x^2+z^2)(1)\, dz\, dy\, dx,$$

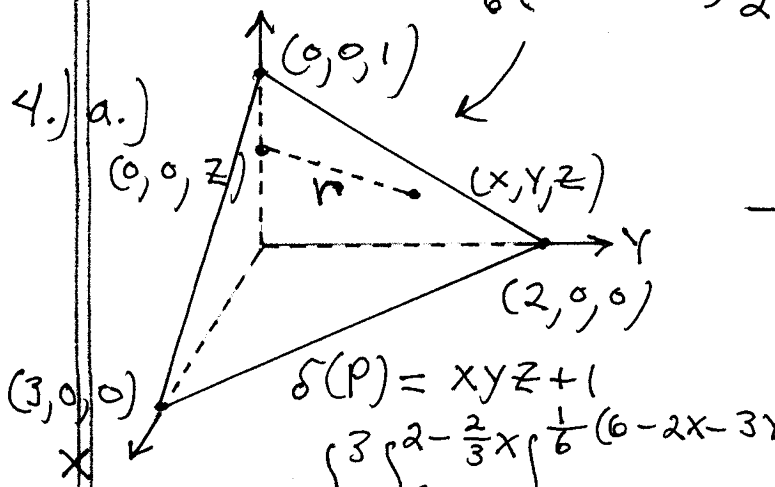
$$I_z = \iiint_R r_z^2 \cdot \delta(P)\, dV = \int_{-3}^{3}\int_{-2}^{4}\int_{-\frac{4}{3}}^{-\frac{2}{3}y+\frac{4}{3}} (x^2+y^2)(1)\, dz\, dy\, dx$$

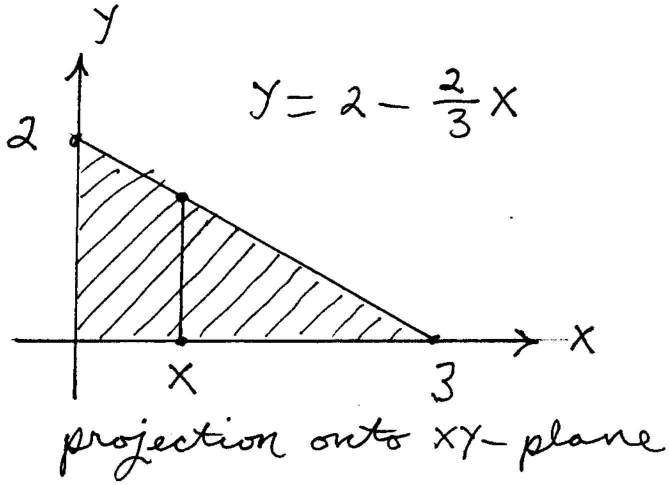
3.) $$I_x = \iiint_R r^2 \delta(P)\, dV = \int_0^a\int_0^b\int_0^c (y^2+z^2)\cdot(1)\, dz\, dy\, dx,$$

$$I_0 = \int_0^a\int_0^b\int_0^c (x^2+y^2+z^2)(1)\, dz\, dy\, dx$$

4.) a.)

(plane) $2X+3Y+6Z=6$ or $Z=\frac{1}{6}(6-2X-3Y)$





$\delta(P) = XYZ+1$

centroid is

$$\bar{X} = \frac{\int_0^3 \int_0^{2-\frac{2}{3}X} \int_0^{\frac{1}{6}(6-2X-3Y)} X \, dZ\, dY\, dX}{\int_0^3 \int_0^{2-\frac{2}{3}X} \int_0^{\frac{1}{6}(6-2X-3Y)} 1 \, dZ\, dY\, dX},$$

$$\bar{Y} = \frac{\int_0^3 \int_0^{2-\frac{2}{3}X} \int_0^{\frac{1}{6}(6-2X-3Y)} Y \, dZ\, dY\, dX}{\int_0^3 \int_0^{2-\frac{2}{3}X} \int_0^{\frac{1}{6}(6-2X-3Y)} 1 \, dZ\, dY\, dX},$$

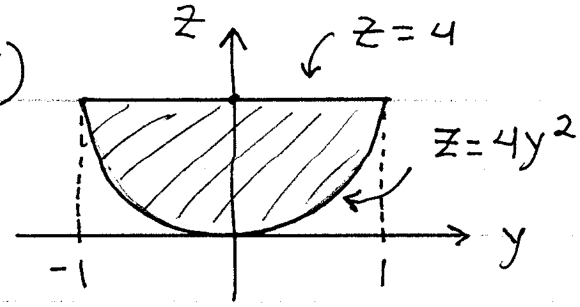
$$\bar{Z} = \frac{\int_0^3 \int_0^{2-\frac{2}{3}X} \int_0^{\frac{1}{6}(6-2X-3Y)} Z \, dZ\, dY\, dX}{\int_0^3 \int_0^{2-\frac{2}{3}X} \int_0^{\frac{1}{6}(6-2X-3Y)} 1 \, dZ\, dY\, dX};$$

moment of inertia around z-axis is

$$I_Z = \int_0^3 \int_0^{2-\frac{2}{3}X} \int_0^{\frac{1}{6}(6-2X-3Y)} (X^2+Y^2)(XYZ+1)\, dZ\, dY\, dX$$

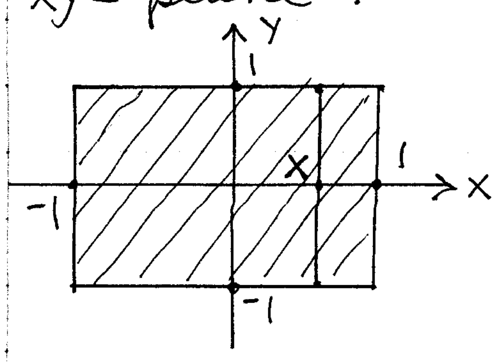
$$r = \sqrt{(X-0)^2+(Y-0)^2+(Z-Z)^2} = \sqrt{X^2+Y^2}$$

5) projection onto yz-plane:



$\delta(P) = x + y + z + 2$

projection onto xy-plane:



moment of inertia about y-axis is

$$I_y = \int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} (x^2+z^2)(x+y+z+2)\, dz\, dy\, dx\, ;$$

center of mass is

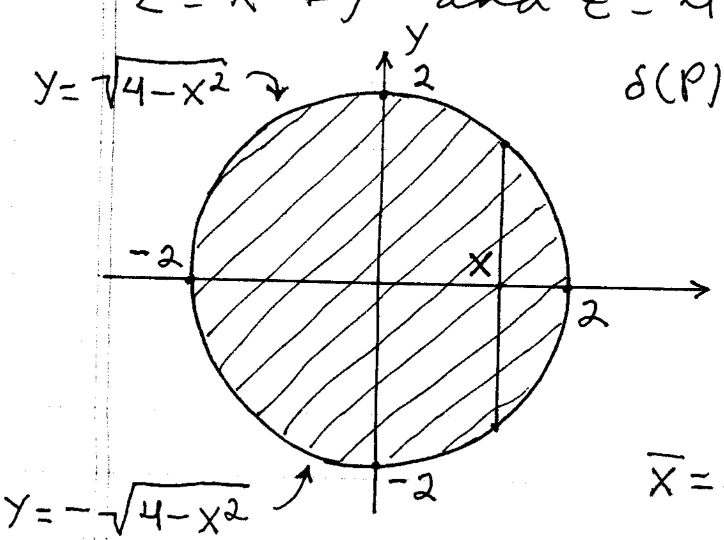
$$\bar{x} = \frac{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} x\cdot(x+y+z+2)\,dz\,dy\,dx}{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} (x+y+z+2)\,dz\,dy\,dx},$$

$$\bar{y} = \frac{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} y\cdot(x+y+z+2)\,dz\,dy\,dx}{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} (x+y+z+2)\,dz\,dy\,dx},$$

$$\bar{z} = \frac{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} z\,(x+y+z+2)\,dz\,dy\,dx}{\int_{-1}^{1}\int_{-1}^{1}\int_{4y^2}^{4} (x+y+z+2)\,dz\,dy\,dx}.$$

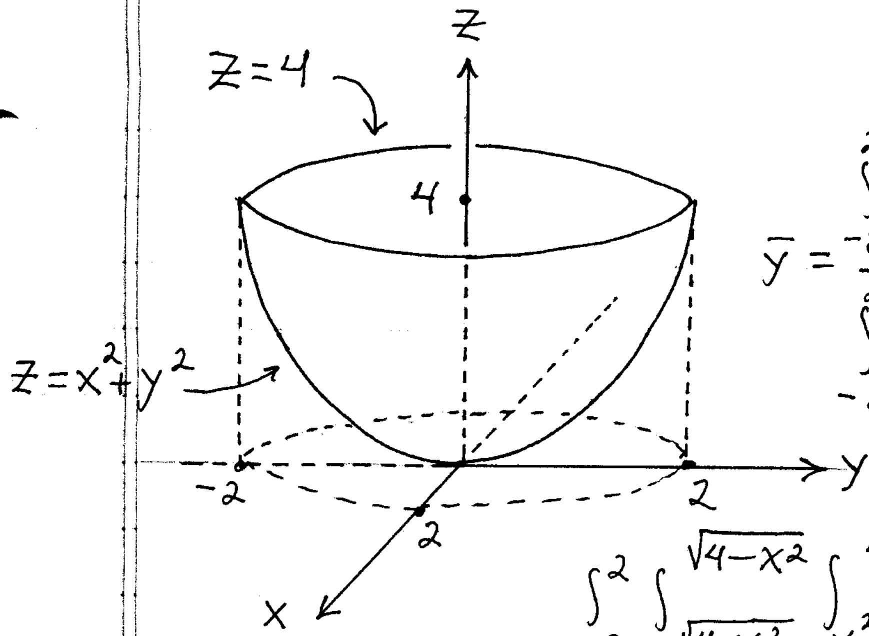
7.) a.) projection onto XY-plane :

$z = x^2+y^2$ and $z = 4 \rightarrow x^2+y^2 = 4$



$\delta(P) = e^{-(x^2+y^2+z^2)}$, center of mass is

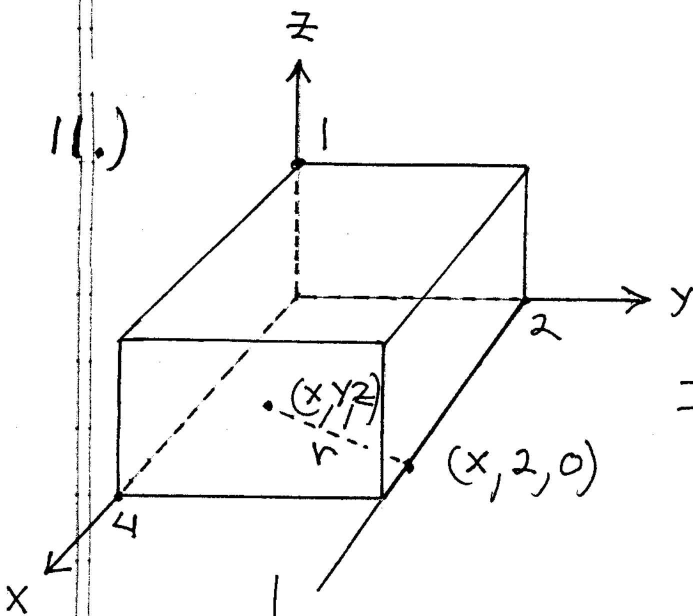
$$\bar{x} = \frac{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} x\cdot e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx}{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx},$$



$$\bar{y} = \frac{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} y e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx}{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx},$$

$$\bar{z} = \frac{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} z e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx}{\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{4} e^{-(x^2+y^2+z^2)}\,dz\,dy\,dx}$$

11.)

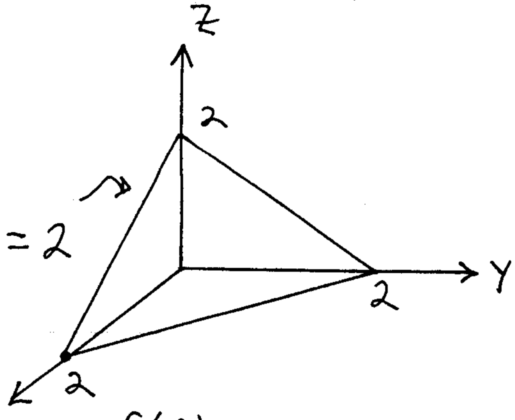


$$r^2 = (y-2)^2 + z^2,$$

$$\delta(P) = \frac{z}{x+y+2},$$

$$I_L = \int_0^4\int_0^2\int_0^1 \left((y-2)^2+z^2\right)\frac{z}{x+y+2}\,dz\,dy\,dx$$

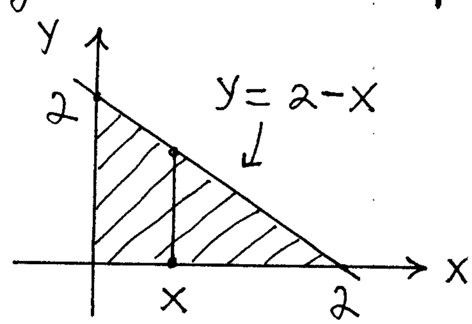
13.)



$x+y+z=2$ or $z=2-x-y$

$\delta(P) = 2x$

projection onto xy-plane:



a.) Mass $= \iiint_R \delta(P)\,dV = \int_0^2 \int_0^{2-x} \int_0^{2-x-y} 2x \, dz\,dy\,dx$

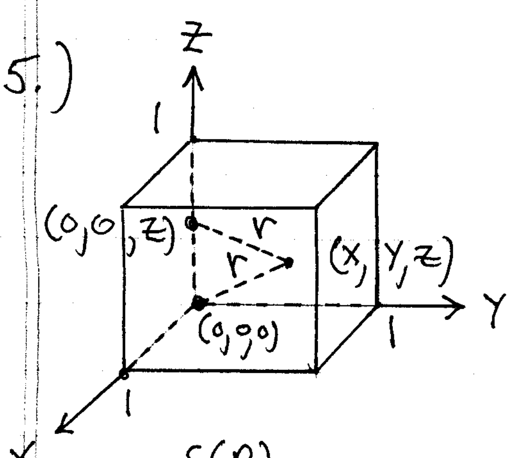
b.) center of mass is

$$\bar{x} = \frac{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} x\cdot(2x)\, dz\,dy\,dx}{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} 2x\, dz\,dy\,dx},$$

$$\bar{y} = \frac{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} y\cdot(2x)\, dz\,dy\,dx}{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} 2x\, dz\,dy\,dx},$$

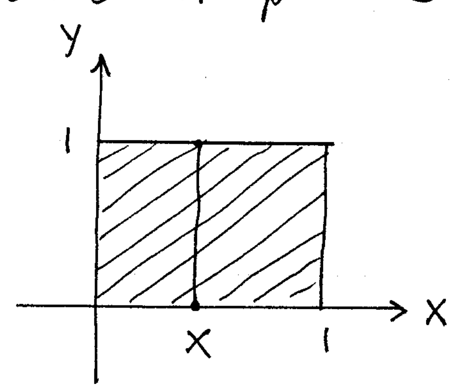
$$\bar{z} = \frac{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} z\cdot(2x)\, dz\,dy\,dx}{\int_0^2 \int_0^{2-x} \int_0^{2-x-y} 2x\, dz\,dy\,dx}$$

15.)



$\delta(P) = x+y+z+1$

projection onto xy-plane:



a.) $\text{Mass} = \int_0^1 \int_0^1 \int_0^1 (x+y+z+1)\, dz\, dy\, dx$

b.) center of mass is

$$\bar{x} = \frac{\int_0^1 \int_0^1 \int_0^1 x(x+y+z+1)\, dz\, dy\, dx}{\int_0^1 \int_0^1 \int_0^1 (x+y+z+1)\, dz\, dy\, dx},$$

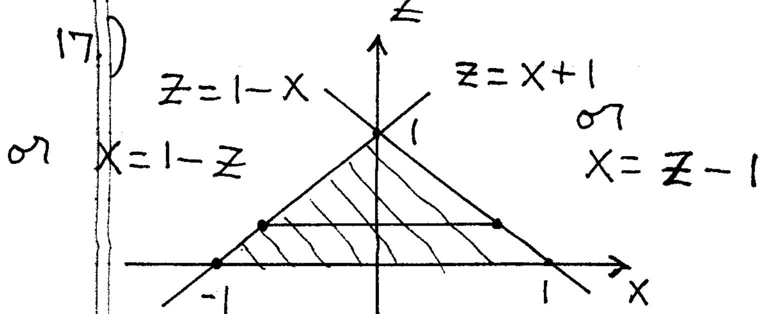
$$\bar{y} = \frac{\int_0^1 \int_0^1 \int_0^1 y(x+y+z+1)\, dz\, dy\, dx}{\int_0^1 \int_0^1 \int_0^1 (x+y+z+1)\, dz\, dy\, dx},$$

$$\bar{z} = \frac{\int_0^1 \int_0^1 \int_0^1 z(x+y+z+1)\, dz\, dy\, dx}{\int_0^1 \int_0^1 \int_0^1 (x+y+z+1)\, dz\, dy\, dx}$$

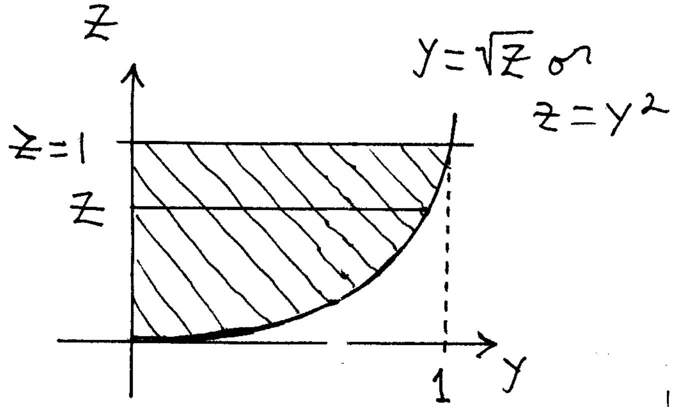
c.) $I_z = \int_0^1 \int_0^1 \int_0^1 (x^2+y^2)(x+y+z+1)\, dz\, dy\, dx$,

$I_0 = \int_0^1 \int_0^1 \int_0^1 (x^2+y^2+z^2)(x+y+z+1)\, dz\, dy\, dx$

17.)

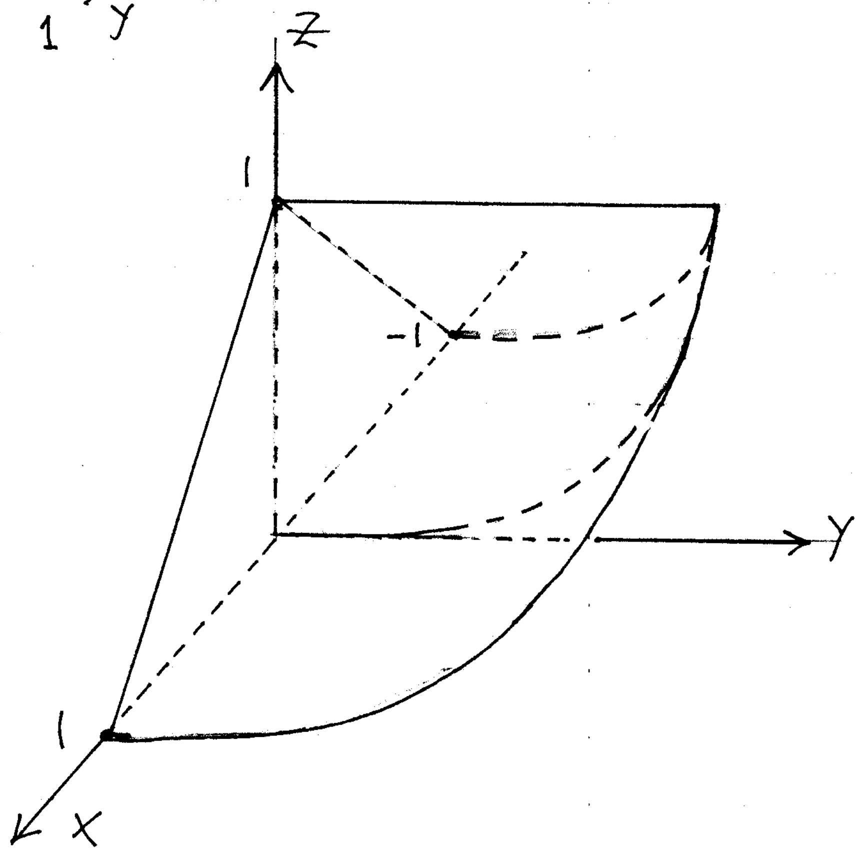


projection onto xz-plane:



projection onto yz-plane :

$\delta(P) = 2y + 5$



$\delta(P) = 2y+5$

$0 \le z \le 1$,
$1-z \le x \le z-1$,
$0 \le y \le \sqrt{z}$

$$\text{Mass} = \int_0^1 \int_{1-z}^{z-1} \int_0^{\sqrt{z}} (2y+5)\, dy\, dx\, dz$$

26.) mass A : $c_1 = \langle 1, \frac{3}{2}, 1 \rangle$, $m_1 = 12$
mass B : $c_2 = \langle \frac{1}{2}, 4, \frac{1}{2} \rangle$, $m_2 = 2$
mass C : $c_3 = \langle 1, \frac{11}{2}, \frac{-1}{2} \rangle$, $m_3 = 12$

a.) $A \cup B$: $\dfrac{m_1 c_1 + m_2 c_2}{m_1 + m_2} = \dfrac{12\langle 1, \frac{3}{2}, 1 \rangle + 2\langle \frac{1}{2}, 4, \frac{1}{2} \rangle}{12+2}$

$$= \frac{\langle 12, 18, 12\rangle + \langle 1, 8, 1\rangle}{14} = \frac{\langle 13, 26, 13\rangle}{14}$$

$$= \left\langle \frac{13}{14}, \frac{26}{14}, \frac{13}{14} \right\rangle = \left\langle \frac{13}{14}, \frac{13}{7}, \frac{13}{14} \right\rangle$$

c.) $B \cup C$: $\dfrac{2\langle \frac{1}{2}, 4, \frac{1}{2}\rangle + 12\langle 1, \frac{11}{2}, \frac{-1}{2}\rangle}{2+12}$

$$= \frac{\langle 1, 8, 1\rangle + \langle 12, 66, -6\rangle}{14} = \frac{\langle 13, 74, -5\rangle}{14}$$

$$= \left\langle \frac{13}{14}, \frac{74}{14}, \frac{-5}{14} \right\rangle = \left\langle \frac{13}{14}, \frac{37}{7}, \frac{-5}{14} \right\rangle$$

d.) $A \cup B \cup C$: $\dfrac{m_1 c_1 + m_2 c_2 + m_3 c_3}{m_1 + m_2 + m_3}$

$$= \frac{12\langle 1, \frac{3}{2}, 1\rangle + 2\langle \frac{1}{2}, 4, \frac{1}{2}\rangle + 12\langle 1, \frac{11}{2}, \frac{-1}{2}\rangle}{12 + 2 + 12}$$

$$= \frac{\langle 12, 18, 12\rangle + \langle 1, 8, 1\rangle + \langle 12, 66, -6\rangle}{26}$$

$$= \frac{\langle 25, 92, 7\rangle}{26} = \left\langle \frac{25}{26}, \frac{92}{26}, \frac{7}{26} \right\rangle$$

$$= \left\langle \frac{25}{26}, \frac{46}{13}, \frac{7}{26} \right\rangle$$