Section 15.6

1) $\int_0^{2\pi}\int_0^1\int_r^{\sqrt{2-r^2}} dz \cdot r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \left(z \Big|_{z=r}^{z=\sqrt{2-r^2}} \right) r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \left(\sqrt{2-r^2} - r\right) r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \left(r\sqrt{2-r^2} - r^2\right) dr\, d\theta$

$= \int_0^{2\pi} \left(-\frac{1}{3}(2-r^2)^{3/2} - \frac{1}{3}r^3\right)\Big|_{r=0}^{r=1} d\theta$

$= \int_0^{2\pi} \left[\left(\frac{-1}{3} - \frac{1}{3}\right) - \left(\frac{-1}{3}\cdot 2^{3/2} - 0\right)\right] d\theta$

$= \int_0^{2\pi} \left(-\frac{2}{3} + \frac{1}{3}2^{3/2}\right) d\theta = \left(\frac{-2}{3} + \frac{1}{3}\cdot 2\cdot\sqrt{2}\right)\theta\Big|_0^{2\pi}$

$= \left(\frac{-2}{3} + \frac{2}{3}\sqrt{2}\right)(2\pi - 0) = \frac{4\pi}{3}\left(\sqrt{2}-1\right)$

6) $\int_0^{2\pi}\int_0^1\int_{\frac{-1}{2}}^{\frac{1}{2}} \left(r^2\sin^2\theta + z^2\right) dz \cdot r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \left(r^2\sin^2\theta \cdot z + \frac{1}{3}z^3\right)\Big|_{z=-\frac{1}{2}}^{z=\frac{1}{2}} \cdot r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \Big[\left(r^2\sin^2\theta\cdot\frac{1}{2} + \frac{1}{3}\left(\frac{1}{8}\right)\right)$
$\qquad - \left(r^2\sin^2\theta\left(\frac{-1}{2}\right) + \frac{1}{3}\left(\frac{-1}{8}\right)\right)\Big] r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \left(\frac{1}{12} + r^2\sin^2\theta\right)\cdot r\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 \left(\frac{1}{12}r + r^3\sin^2\theta\right) dr\, d\theta$

$$= \int_0^{2\pi} \left(\frac{1}{24} r^2 + \frac{1}{4} r^4 \sin^2\theta \right) \Big|_{r=0}^{r=1} d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{24} + \frac{1}{4} \sin^2\theta \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{24} + \frac{1}{4} \cdot \frac{1}{2} (1 - \cos 2\theta) \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{24} + \frac{1}{8} - \frac{1}{8} \cos 2\theta \right) d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{6} - \frac{1}{8} \cos 2\theta \right) d\theta$$

$$= \left(\frac{1}{6}\theta - \frac{1}{8} \cdot \frac{1}{2} \sin 2\theta \right) \Big|_0^{2\pi}$$

$$= \left(\frac{2\pi}{6} - \frac{1}{16} \cancel{\sin 4\pi}^{\,0} \right) - \left(0 - \frac{1}{16} \cancel{\sin 0}^{\,0} \right) = \frac{\pi}{3}$$

8.) $\int_{-1}^{1} \int_0^{2\pi} \int_0^{1+\cos\theta} 4r \, dr \, d\theta \, dz$

$$= \int_{-1}^{1} \int_0^{2\pi} \left(2r^2 \Big|_{r=0}^{r=1+\cos\theta} \right) d\theta \, dz$$

$$= \int_{-1}^{1} \int_0^{2\pi} 2(1+\cos\theta)^2 \, d\theta \, dz$$

$$= \int_{-1}^{1} \int_0^{2\pi} 2(1 + 2\cos\theta + \cos^2\theta) \, d\theta \, dz$$

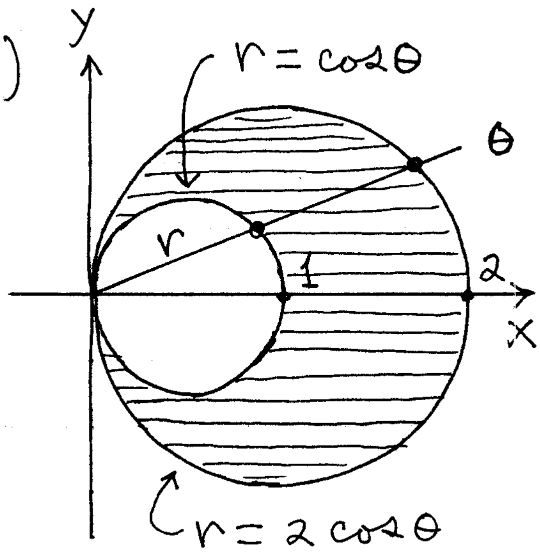
$$= \int_{-1}^{1} \int_0^{2\pi} 2\left(1 + 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta)\right) d\theta \, dz$$

$$= \int_{-1}^{1} \int_0^{2\pi} (3 + 4\cos\theta + \cos 2\theta) \, d\theta \, dz$$

$$= \int_{-1}^{1} \left(3\theta + 4\sin\theta + \frac{1}{2}\sin 2\theta\right) \Big|_{\theta=0}^{\theta=2\pi} dz$$

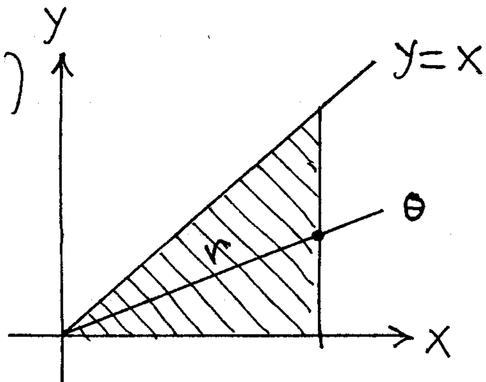
$$= \int_{-1}^{1} \Big[\left(6\pi + 4\cancel{\sin 2\pi}^{\,0} + \frac{1}{2}\cancel{\sin(4\pi)}^{\,0}\right) - \left(0 + 4\cancel{\sin 0}^{\,0} + \frac{1}{2}\cancel{\sin 0}^{\,0}\right) \Big] \, dz$$

18.)



$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\cos\theta}^{2\cos\theta} \int_{0}^{3-r\sin\theta} f(r,\theta,z)\cdot r \, dz \, dr \, d\theta$$

19.)



$$\int_{0}^{\frac{\pi}{4}} \int_{0}^{\sec\theta} \int_{0}^{2-r\sin\theta} f(r,\theta,z)\cdot r \, dz \, dr \, d\theta$$

$x = 1$ or $r = \sec\theta$

43.) $\left.\begin{array}{l} z = 4 - 4(x^2+y^2) \\ z = (x^2+y^2)^2 - 1 \end{array}\right\} \rightarrow 4(x^2+y^2) = 4 - z \rightarrow$

$x^2+y^2 = 1 - \frac{1}{4}z \rightarrow$

(SUB) $z = (1 - \frac{1}{4}z)^2 - 1 \rightarrow z = \cancel{1} - \frac{1}{2}z + \frac{1}{16}z^2 - \cancel{1} \rightarrow$

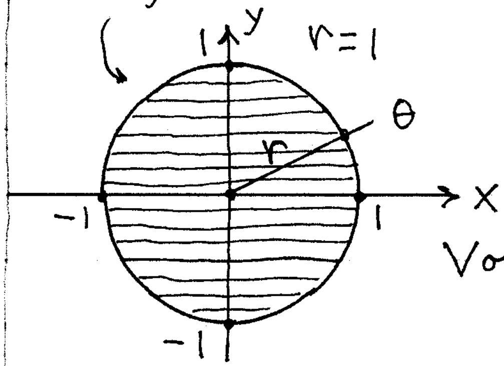
$16z = -8z + z^2 \rightarrow z^2 - 24z = 0 \rightarrow z(z-24) = 0$

$\rightarrow z = 0$ or $\cancel{z = 24}$; $0 = 4 - 4(x^2+y^2) \rightarrow$

$x^2+y^2 = 1$ or $r = 1$

$z = 4 - 4(x^2+y^2)$ or $z = 4 - 4r^2$,

$z = (x^2+y^2)^2 - 1$ or $z = r^4 - 1$:

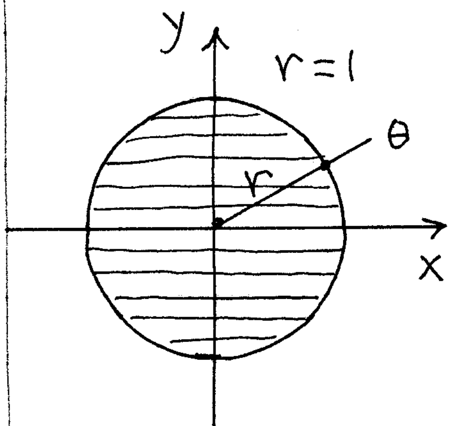


$$\text{Vol} = \int_{0}^{2\pi} \int_{0}^{1} \int_{r^4-1}^{4-4r^2} r \, dz \, dr \, d\theta$$

44.) $\left.\begin{array}{l} z = 1 - r \\ z = -\sqrt{1-r^2} \end{array}\right\} \rightarrow 1 - r = -\sqrt{1-r^2} \rightarrow (1-r)^2 = 1 - r^2 \rightarrow$

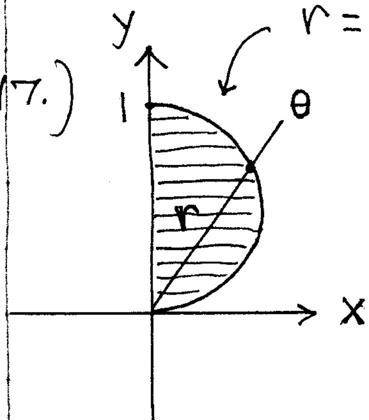
$\cancel{1} - 2r + r^2 = \cancel{1} - r^2 \rightarrow 2r^2 - 2r = 0 \rightarrow$

$2r(r-1) = 0 \rightarrow \cancel{r = 0}$ or $r = 1$ and $z = 0$:



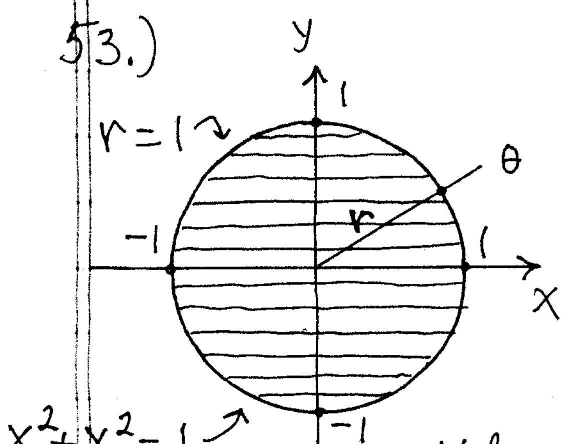
$$Vol = \int_0^{2\pi}\int_0^1\int_{-\sqrt{1-r^2}}^{1-r} r \, dz \, dr \, d\theta$$

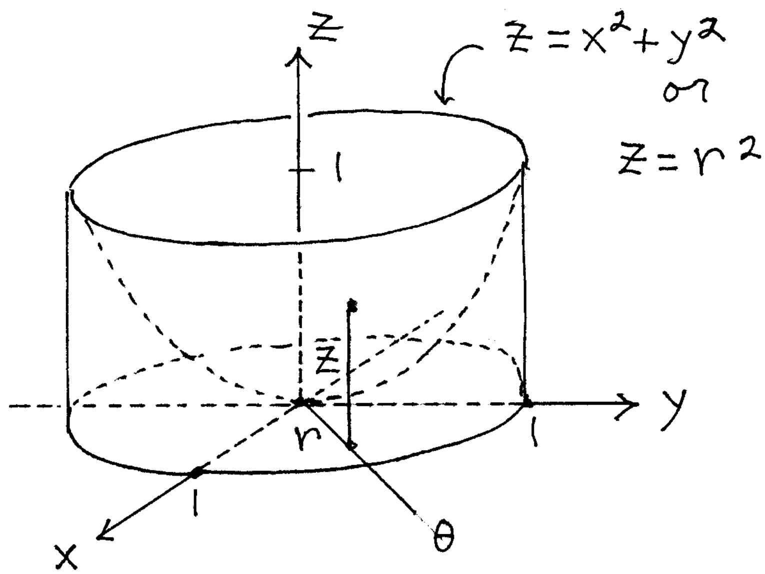
47.)



$$Vol = \int_0^{\frac{\pi}{2}}\int_0^{\sin\theta}\int_0^{\sqrt{1-r^2}} r \, dz \, dr \, d\theta$$

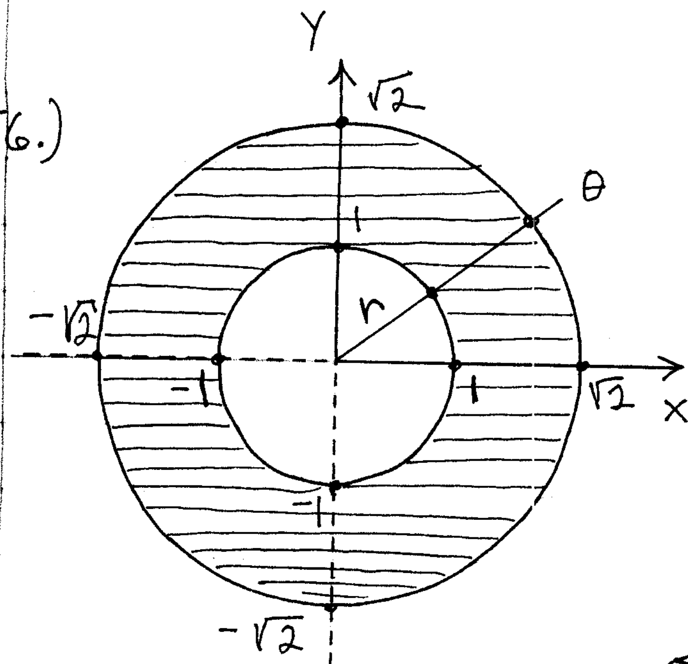
53.)





$$Vol = \int_0^{2\pi}\int_0^1\int_0^{r^2} r \, dz \, dr \, d\theta$$

56.)

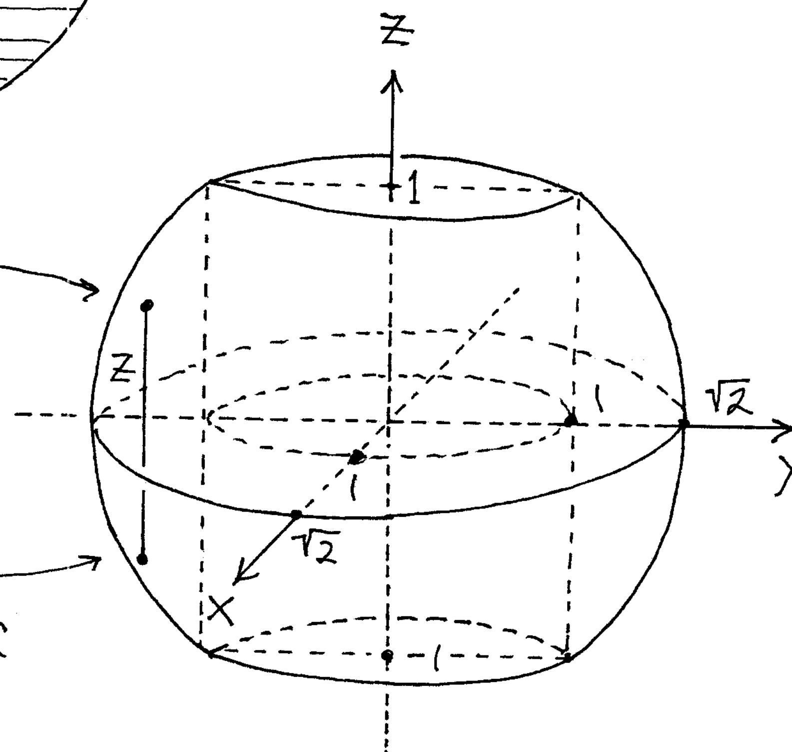


$$\left.\begin{array}{l} x^2+y^2+z^2=2 \\ x^2+y^2=1 \end{array}\right\} \rightarrow$$

$1+z^2=2 \rightarrow z^2=1 \rightarrow$

$z=\pm 1$;

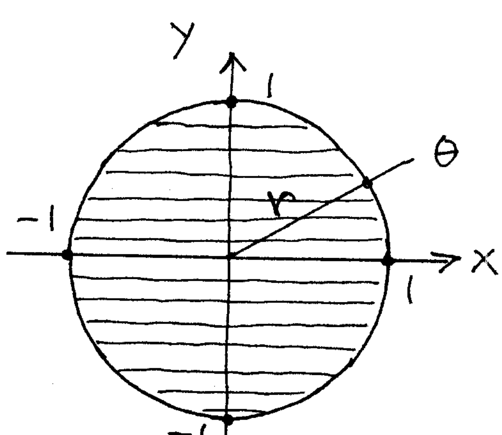
$z=\sqrt{2-x^2-y^2}$
or
$z=\sqrt{2-r^2}$

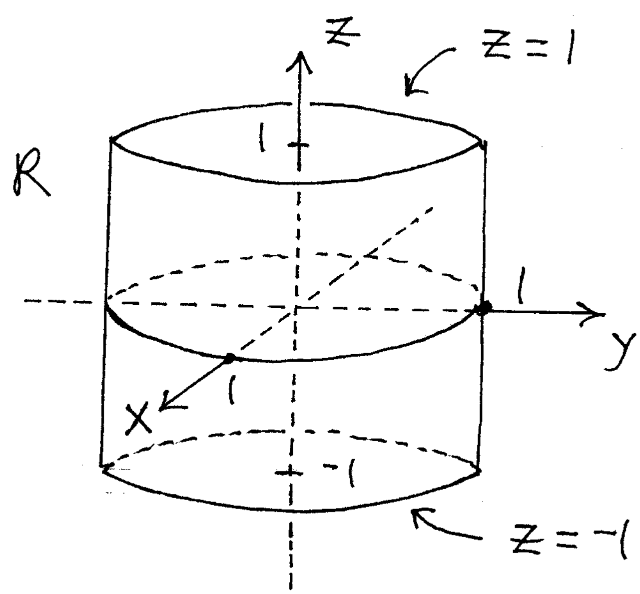


$z=-\sqrt{2-x^2-y^2}$
or $z=-\sqrt{2-r^2}$

$$\text{Vol} = \int_0^{2\pi}\int_1^{\sqrt{2}}\int_{-\sqrt{2-r^2}}^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta$$

63.)

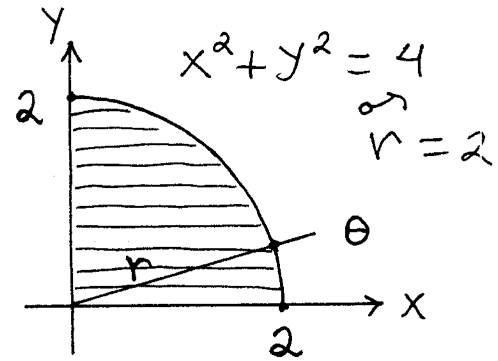


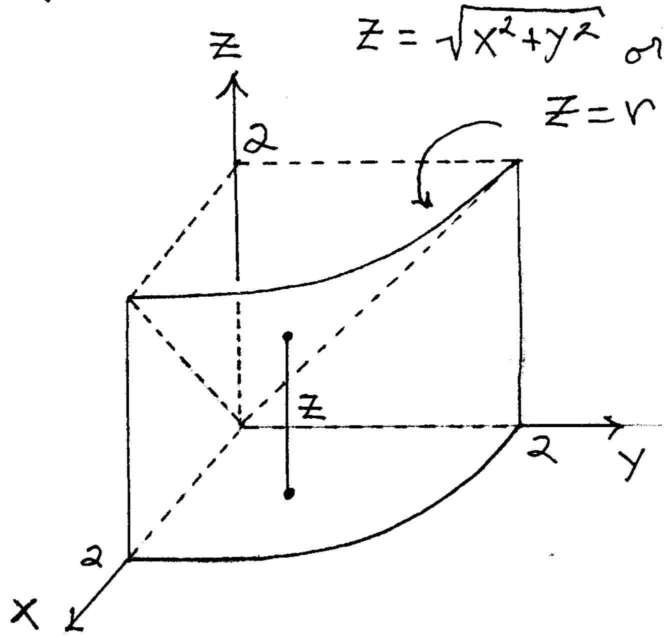


$$AVE = \frac{1}{\text{vol. } R} \cdot \iiint_R f(P)\, dV$$

$$= \frac{1}{\pi(1)^2(2)} \int_0^{2\pi}\int_0^1\int_{-1}^1 r \cdot r\, dz\, dr\, d\theta$$

68.)





$$\left.\begin{array}{l} z=\sqrt{x^2+y^2} \\ x^2+y^2=4 \end{array}\right\} \to$$

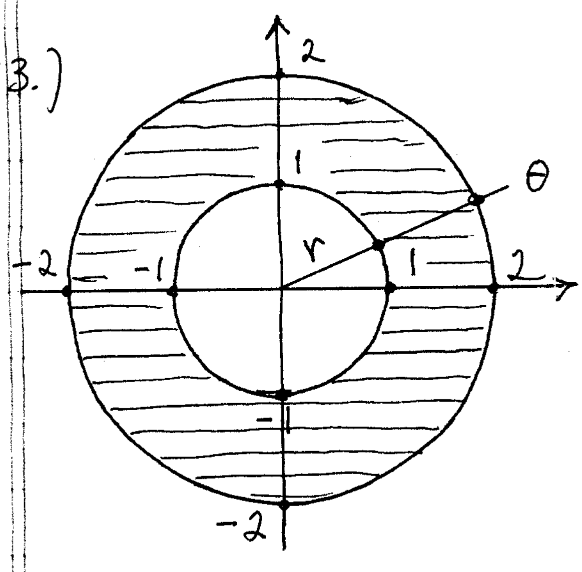
$$z=\sqrt{4}=2$$

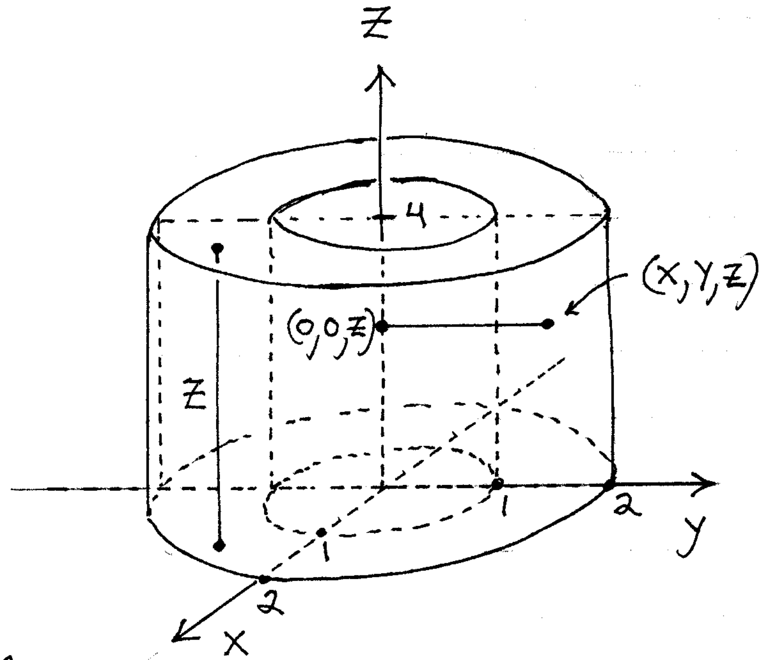
$$\bar{x} = \frac{\int_0^{\frac{\pi}{2}}\int_0^2\int_0^r (r\cos\theta)\cdot r\, dz\, dr\, d\theta}{\int_0^{\frac{\pi}{2}}\int_0^2\int_0^r r\, dz\, dr\, d\theta},$$

$$\bar{y} = \frac{\int_0^{\frac{\pi}{2}}\int_0^2\int_0^r (r\sin\theta)\cdot r\, dz\, dr\, d\theta}{\int_0^{\frac{\pi}{2}}\int_0^2\int_0^r r\, dz\, dr\, d\theta},$$

$$\bar{z} = \frac{\int_0^{\frac{\pi}{2}}\int_0^2\int_0^r z\cdot r\, dz\, dr\, d\theta}{\int_0^{\frac{\pi}{2}}\int_0^2\int_0^r r\, dz\, dr\, d\theta}$$

73.)





distance between $(0,0,z)$ and (x,y,z) is

$$\sqrt{(x-0)^2+(y-0)^2+(z-z)^2} = \sqrt{x^2+y^2} = \sqrt{r^2} = r \ ;$$

$$I_z = \iiint_R (\text{distance})^2 \, \delta(P) \, dV$$

$$= \int_0^{2\pi} \int_1^2 \int_0^4 r^2 \cdot (1) \ r \, dz \, dr \, d\theta$$