Section 16.1

1.) $\vec{r}(t) = t\vec{i} + (1-t)\vec{j} + 0\cdot\vec{k}$, $0 \le t \le 1 \rightarrow$

$\begin{cases} x = t \\ y = 1-t \\ z = 0 \end{cases}$ (SUB) $\rightarrow$ $y = 1 - x$ c.

2.) $\vec{r}(t) = 1\cdot\vec{i} + 1\cdot\vec{j} + t\cdot\vec{k}$, $-1 \le t \le 1 \rightarrow$

$\begin{cases} x = 1 \\ y = 1 \\ z = t \end{cases}$ e.

3.) $\vec{r}(t) = (2\cos t)\vec{i} + (2\sin t)\vec{j} + 0\cdot\vec{k}$, $0 \le t \le 2\pi$

$\begin{cases} x = 2\cos t \\ y = 2\sin t \\ z = 0 \end{cases} \rightarrow$ $x^2 + y^2 = (2\cos t)^2 + (2\sin t)^2$

$= 4\cos^2 t + 4\sin^2 t$

$= 4(\cos^2 t + \sin^2 t) = 4\cdot(1) = 2^2$

g.

4.) $\vec{r}(t) = t\vec{i} + 0\cdot\vec{j} + 0\cdot\vec{k}$, $-1 \le t \le 1$

$\begin{cases} x = t \\ y = 0 \\ z = 0 \end{cases}$ a.

5.) $\vec{r}(t) = t\vec{i} + t\vec{j} + t\vec{k}$, $0 \le t \le 2$

$\begin{cases} x = t \\ y = t \\ z = t \end{cases}$ d.

6.) $\vec{r}(t) = 0\cdot\vec{i} + t\cdot\vec{j} + (2-2t)\vec{k}\ , \quad 0 \le t \le 1 \rightarrow$

$$\begin{cases} x = 0 \\ y = t \\ z = 2-2t \end{cases} \quad \text{(SUB)} \quad \longrightarrow \quad z = 2-2y$$

b.

7.) $\vec{r}(t) = 0\cdot\vec{i} + (t^2-1)\vec{j} + 2t\cdot\vec{k}\ , \quad -1 \le t \le 1 \rightarrow$

$$\begin{cases} x = 0 \\ y = t^2-1 \\ z = 2t \rightarrow t = \frac{1}{2}z \end{cases} \quad \text{(SUB} \quad \longrightarrow \quad y = \left(\tfrac{1}{2}z\right)^2 - 1$$

f.

8.) $\vec{r}(t) = (2\cos t)\vec{i} + (0)\vec{j} + (2\sin t)\vec{k}\ , \quad 0 \le t \le \pi \rightarrow$

$$\begin{cases} x = 2\cos t \\ y = 0 \\ z = 2\sin t \end{cases} \rightarrow \quad x^2 + z^2 = 4\cos^2 t + 4\sin^2 t$$
$$= 4(\cos^2 t + \sin^2 t) = 4\cdot 1 = 2^2$$

h.

10.) $C: \vec{r}(t) = t\,\vec{i} + (1-t)\vec{j} + 1\cdot\vec{k}\ , \quad 0 \le t \le 1 \rightarrow$

$\vec{v}(t) = 1\cdot\vec{i} + (-1)\cdot\vec{j} + 0\cdot\vec{k} \rightarrow$

$|\vec{v}(t)| = \sqrt{(1)^2 + (-1)^2 + (0)^2} = \sqrt{2}$ then

$$\cancel{\int_C (x - y + z - 2)\, ds = \int_0^1 (t - (1-t) + 1 - 2))\, |\vec{v}(t)|\, dt}$$
$$= \int_0^1 (2t-2)\sqrt{2}\, dt = \sqrt{2}\,(t^2 - 2t)\Big|_0^1$$
$$\cancel{= \sqrt{2}(1-2) - \sqrt{2}\cdot(0) = -\sqrt{2}}$$

11.) $C: \vec{r}(t) = 2t\cdot\vec{i} + t\cdot\vec{j} + (2-2t)\vec{k}\ , \quad 0 \le t \le 1 \rightarrow$

$\vec{v}(t) = 2\cdot\vec{i} + 1\cdot\vec{j} + (-2)\cdot\vec{k} \longrightarrow$

$|\vec{v}(t)| = \sqrt{2^2+1^2+(-2)^2} = \sqrt{9} = 3$ then

$\int_C (xy+y+z)\,ds = \int_0^1 (2t\cdot t + t + (2-2t))\cdot|\vec{v}(t)|\,dt$

$= \int_0^1 (2t^2+t+2-2t)(3)\,dt$

$= 3\int_0^1 (2t^2-t+2)\,dt = 3\left(\frac{2}{3}t^3 - \frac{1}{2}t^2+2t\right)\Big|_0^1$

$= 3\left(\frac{2}{3}-\frac{1}{2}+2\right) = 3\left(\frac{4}{6}-\frac{3}{6}+\frac{12}{6}\right)$

$= 3\cdot\left(\frac{13}{6}\right) = 13/2$

12.) $\vec{r}(t) = (4\cos t)\vec{i} + (4\sin t)\vec{j} + (3t)\vec{k}$
for $-2\pi \le t \le 2\pi \xrightarrow{D}$
$\vec{v}(t) = (-4\sin t)\vec{i} + (4\cos t)\vec{j} + (3)\vec{k}$ and

$|\vec{v}(t)| = \sqrt{(-4\sin t)^2+(4\cos t)^2+(3)^2}$

$= \sqrt{16(\underbrace{\sin^2 t+\cos^2 t}_{1})+9}$

$= \sqrt{25} = 5$; then

$\int_C \sqrt{x^2+y^2}\,ds = \int_{-2\pi}^{2\pi} \sqrt{(4\cos t)^2+(4\sin t)^2}\cdot\frac{ds}{dt}\,dt$

$= \int_{-2\pi}^{2\pi} \sqrt{16(\underbrace{\cos^2 t+\sin^2 t}_{1})}\,(5)\,dt$

$= \int_{-2\pi}^{2\pi} (4)(5)\,dt = 20t\Big|_{-2\pi}^{2\pi}$

$= 20(2\pi) - 20(-2\pi) = 80\pi$

13.) direction vector:

$\vec{w} = (0-1)\vec{i} + (-1-2)\vec{j} + (1-3)\vec{k}$

$= (-1)\vec{i} + (-3)\vec{j} + (-2)\vec{k}$, so

line L passing through points $(1,2,3)$ and $(0,-1,1)$ is

$$L: \begin{cases} x = 1 - t \\ y = 2 - 3t \\ z = 3 - 2t \end{cases} \quad \text{for } 0 \le t \le 1 \quad ;$$

then

$\vec{r}(t) = (1-t)\vec{i} + (2-3t)\vec{j} + (3-2t)\vec{k} \xrightarrow{D}$

$\vec{v}(t) = (-1)\vec{i} + (-3)\vec{j} + (-2)\vec{k}$ and

$|\vec{v}(t)| = \sqrt{(-1)^2 + (-3)^2 + (-2)^2} = \sqrt{14}$; then

$$\int_C (x+y+z)\, ds = \int_C (x+y+z) \frac{ds}{dt}\, dt$$

$$= \int_0^1 [(1-t) + (2-3t) + (3-2t)] \cdot \sqrt{14}\, dt$$

$$= \int_0^1 (6-6t)\sqrt{14}\, dt = \sqrt{14}\,(6t - 3t^2)\Big|_0^1$$

$$= 3\sqrt{14}$$

15.) $C_1 : \vec{r}(t) = t\,\vec{i} + t^2\,\vec{j}$ for $0 \le t \le 1 \xrightarrow{D}$

$\vec{v}(t) = (1)\vec{i} + (2t)\vec{j}$ and

$$|\vec{v}(t)| = \sqrt{(1)^2+(2t)^2} = \sqrt{1+4t^2} \; ;$$

C_2: $\vec{r}(t) = (1)\vec{i} + (1)\vec{j} + t\vec{k}$ for $0 \le t \le 1 \xrightarrow{D}$

$\vec{v}(t) = (0)\vec{i} + (0)\vec{j} + (1)\vec{k}$ and

$|\vec{v}(t)| = \sqrt{0^2+0^2+1^2} = 1$; now

$$\int_C (x+\sqrt{y}-z^2)\,ds$$

$$= \int_{C_1} (x+\sqrt{y}-z^2)\,ds + \int_{C_2} (x+\sqrt{y}-z^2)\,ds$$

$$= \int_0^1 (t+\sqrt{t^2}-0^2)\frac{ds}{dt}\,dt + \int_0^1 (1+\sqrt{1}-t^2)\frac{ds}{dt}\,dt$$

$$= \int_0^1 2t\cdot\sqrt{1+4t^2}\,dt + \int_0^1 (2-t^2)(1)\,dt$$

$$= \frac{1}{4}\cdot\frac{2}{3}(1+4t^2)^{3/2}\Big|_0^1 + (2t-\frac{1}{3}t^3)\Big|_0^1$$

$$= \frac{1}{6}(5)^{3/2} - \frac{1}{6}(1)^{3/2} + (2-1/3)$$

$$= \frac{1}{6}(5)^{3/2} - \frac{1}{6} + \frac{12}{6} - \frac{2}{6} = \frac{9}{6} - \frac{1}{6}(5)^{3/2}$$

$$= \frac{3}{2} - \frac{1}{6}(5)^{3/2}$$

16.) C_1: $\vec{r}(t) = t\vec{k}$ for $0 \le t \le 1 \xrightarrow{D}$

$\vec{v}(t) = (1)\vec{k}$ and $|\vec{v}(t)| = 1$;

C_2: $\vec{r}(t) = t\vec{j} + (1)\vec{k}$ for $0 \le t \le 1 \xrightarrow{D}$

$\vec{v}(t) = (1)\vec{j} + (0)\vec{k}$ and

$|\vec{v}(t)| = \sqrt{1^2+0^2} = 1$;

C_3: $\vec{r}(t) = t\vec{i} + (1)\vec{j} + (1)\vec{k}$ for $0 \le t \le 1 \xrightarrow{D}$

$\vec{v}(t) = (1)\vec{i} + (0)\vec{j} + (0)\vec{k}$ and

$|\vec{v}(t)| = \sqrt{1^2+0^2+0^2} = 1$; then

$$\int_C (x+\sqrt{y}-z^2)\,ds = \int_{C_1} (x+\sqrt{y}-z^2)\frac{ds}{dt}\,dt$$

$$+\int_{C_2}(x+\sqrt{y}-z^2)\frac{ds}{dt}\,dt + \int_{C_3}(x+\sqrt{y}-z^2)\frac{ds}{dt}\,dt$$

$$= \int_0^1 (0+\sqrt{0}-t^2)(1)\,dt + \int_0^1 (0+\sqrt{t}-1^2)(1)\,dt$$

$$+\int_0^1 (t+\sqrt{1}-1^2)(1)\,dt$$

$$= \int_0^1 -t^2\,dt + \int_0^1 (t^{1/2}-1)\,dt + \int_0^1 t\,dt$$

$$= -\frac{1}{3}t^3\Big|_0^1 + \left(\frac{2}{3}t^{3/2}-t\right)\Big|_0^1 + \frac{1}{2}t^2\Big|_0^1$$

$$= \frac{-1}{3} + \frac{2}{3} - 1 + \frac{1}{2} = \frac{-2}{3} + \frac{1}{2} = \frac{-4}{6} + \frac{3}{6} = \frac{-1}{6}$$

17.) $\vec{r}(t) = t\vec{i} + t\vec{j} + t\vec{k}$ for $0 < a \le t \le b \xrightarrow{D}$

$\vec{v}(t) = (1)\vec{i} + (1)\vec{j} + (1)\vec{k}$ and

$|\vec{v}(t)| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$; then

$$\int_C \frac{x+y+z}{x^2+y^2+z^2}\,ds = \int_a^b \frac{t+t+t}{t^2+t^2+t^2}\,\frac{ds}{dt}\,dt$$

$$= \int_a^b \frac{\cancel{3}\cancel{t}}{\cancel{3}t^{\cancel{2}}}\sqrt{3}\,dt = \sqrt{3}\int_a^b \frac{1}{t}\,dt$$

$$= \sqrt{3}\ln|t|\Big|_a^b = \sqrt{3}(\ln b - \ln a) = \sqrt{3}\ln\left(\frac{b}{a}\right).$$

18.) $\vec{r}(t) = (a\cos t)\vec{j} + (a\sin t)\vec{k}$
for $0 \le t \le 2\pi \xrightarrow{D}$
$\vec{v}(t) = (-a\sin t)\vec{j} + (a\cos t)\vec{k}$ and

$$|\vec{v}(t)| = \sqrt{(-a\sin t)^2 + (a\cos t)^2}$$

$$= \sqrt{a^2(\underbrace{\sin^2 t + \cos^2 t}_{1})} = a \quad (a > 0) \; ;$$

$$\int_C -\sqrt{x^2+z^2}\, ds = -\int_0^{2\pi} \sqrt{0^2 + (a\sin t)^2}\, \frac{ds}{dt}\, dt$$

$$= -\int_0^{2\pi} \sqrt{a^2 \sin^2 t}\, (a)\, dt$$

$$= -a^2 \int_0^{2\pi} |\sin t|\, dt$$

$$= -a^2 \cdot 2\int_0^{\pi} \sin t\, dt$$

$$= -2a^2 (-\cos t)\Big|_0^{\pi} = 2a^2(\cos\pi - \cos 0)$$

$$= 2a^2(-1-1) = -4a^2 .$$

19.) $C: y = \frac{1}{2}x^2 \longrightarrow \begin{cases} x = t \\ y = \frac{1}{2}t^2 \end{cases}$ for $0 \le x \le 2$
$\rightarrow 0 \le t \le 2$

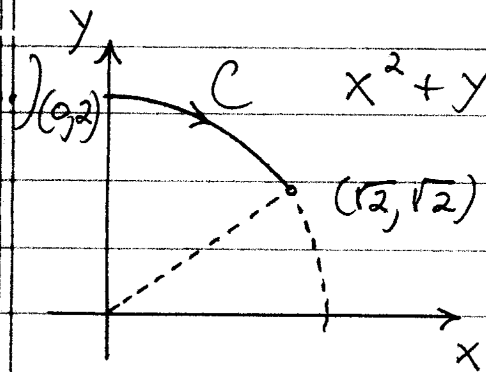
$\longrightarrow \vec{r}(t) = t\vec{i} + \frac{1}{2}t^2\vec{j} \xrightarrow{D}$
$\vec{v}(t) = (1)\vec{i} + t\vec{j}$ and
$|\vec{v}(t)| = \sqrt{1+t^2}$; then

$$\int_C \frac{x^3}{y}\, ds = \int_0^2 \frac{t^3}{\frac{1}{2}t^2} \cdot \frac{ds}{dt}\, dt$$

$$= 2\int_0^2 t\sqrt{1+t^2}\,dt = \frac{2}{3}(1+t^2)^{3/2}\Big|_0^2$$

$$= \frac{2}{3}(5)^{3/2} - \frac{2}{3}(1)^{3/2} = \frac{2}{3}(5)^{3/2} - \frac{2}{3}$$

22.)



$C \quad x^2+y^2=2^2$

$$C: \begin{cases} x = 2\sin t \\ y = 2\cos t \end{cases}$$
for $0 \le t \le \frac{\pi}{4}$;

$$\vec{r}(t) = (2\sin t)\,\vec{i} + (2\cos t)\,\vec{j} \xrightarrow{D}$$

$$\vec{v}(t) = (2\cos t)\,\vec{i} + (-2\sin t)\,\vec{j} \quad \text{and}$$

$$|\vec{v}(t)| = \sqrt{(2\cos t)^2 + (-2\sin t)^2}$$

$$= \sqrt{4\,(\underbrace{\cos^2 t + \sin^2 t}_{1})} = 2 \text{ ; then}$$

$$\int_C (x^2 - y)\,ds = \int_0^{\frac{\pi}{4}} \left((2\sin t)^2 - 2\cos t\right)\frac{ds}{dt}\,dt$$

$$= \int_0^{\frac{\pi}{4}} (4\sin^2 t - 2\cos t)(2)\,dt$$

$$= 4\int_0^{\frac{\pi}{4}} (2\sin^2 t - \cos t)\,dt$$

$$= 4\int_0^{\frac{\pi}{4}} \left[\cancel{2}\cdot\frac{1}{\cancel{2}}(1-\cos 2t) - \cos t\right] dt$$

$$= 4\left(t - \frac{1}{2}\sin 2t - \sin t\right)\Big|_0^{\frac{\pi}{4}}$$

$$= 4\left(\frac{\pi}{4} - \frac{1}{2}\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right)$$

$$- 4\left(0 - \frac{1}{2}\underbrace{\sin 0}_{0} - \underbrace{\sin 0}_{0}\right)$$

$= 4\left(\frac{\pi}{4} - \frac{1}{2}(1) - \frac{\sqrt{2}}{2}\right)$
$= \pi - 2 - 2\sqrt{2}$

23.) $C: \vec{r}(t) = (t^2-1)\vec{j} + (2t)\vec{k}$ for $0 \le t \le 1 \xrightarrow{D}$
$\vec{v}(t) = (2t)\vec{j} + (2)\vec{k}$ and
$|\vec{v}(t)| = \sqrt{(2t)^2+2^2} = \sqrt{4t^2+4} = 2\sqrt{t^2+1}$;
density $\delta(x,y,z) = \frac{3}{2}t$;
$Mass = \int_C \delta(P)\, ds = \int_0^1 \frac{3}{2}t \cdot \frac{ds}{dt}\, dt$
$= \int_0^1 \frac{3}{2}t \cdot 2\sqrt{t^2+1}\, dt = 3\int_0^1 t\sqrt{t^2+1}\, dt$
$= \cancel{3} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{3}}(t^2+1)^{3/2}\Big|_0^1 = 2^{3/2} - 1^{3/2}$
$= 2^{3/2} - 1$

24.) $C: \vec{r}(t) = (t^2-1)\vec{j} + (2t)\vec{k}$ for $-1 \le t \le 1$
$\xrightarrow{D} \vec{v}(t) = (2t)\vec{j} + (2)\vec{k}$ and
$|\vec{v}(t)| = \sqrt{(2t)^2+(2)^2} = \sqrt{4t^2+4} = 2\sqrt{t^2+1}$;
density $\delta(x,y,z) = 15\sqrt{y+2}$, then

$$\bar{y} = \frac{\int_C y \cdot \delta(P)\, ds}{\int_C \delta(P)\, ds} \; ;$$

$$\int_C \delta(P)\,ds = \int_{-1}^{1} 15\sqrt{(t^2-1)+2}\cdot\frac{ds}{dt}\,dt$$

$$= 15\int_{-1}^{1}\sqrt{t^2+1}\cdot 2\sqrt{t^2+1}\,dt$$

$$= 30\int_{-1}^{1}(t^2+1)\,dt = 30\left(\frac{1}{3}t^3+t\right)\Big|_{-1}^{1}$$

$$= 30\left(\frac{1}{3}+1\right) - 30\left(\frac{-1}{3}-1\right) = \cancel{30}^{10}\left(\frac{4}{\cancel{3}}\right) - \cancel{30}^{10}\left(\frac{-4}{\cancel{3}}\right)$$

$$= 80 \quad ;$$

$$\int_C y\cdot\delta(P)\,dt = \int_{-1}^{1}(t^2-1)\sqrt{t^2+1}\cdot \overbrace{2\sqrt{t^2+1}}^{\frac{ds}{dt}}\,dt$$

$$= 2\int_{-1}^{1}(t^2-1)(t^2+1)\,dt = 2\int_{-1}^{1}(t^4-1)\,dt$$

$$= 2\left(\frac{1}{5}t^5-t\right)\Big|_{-1}^{1} = 2\left(\frac{1}{5}-1\right) - 2\left(\frac{-1}{5}+1\right)$$

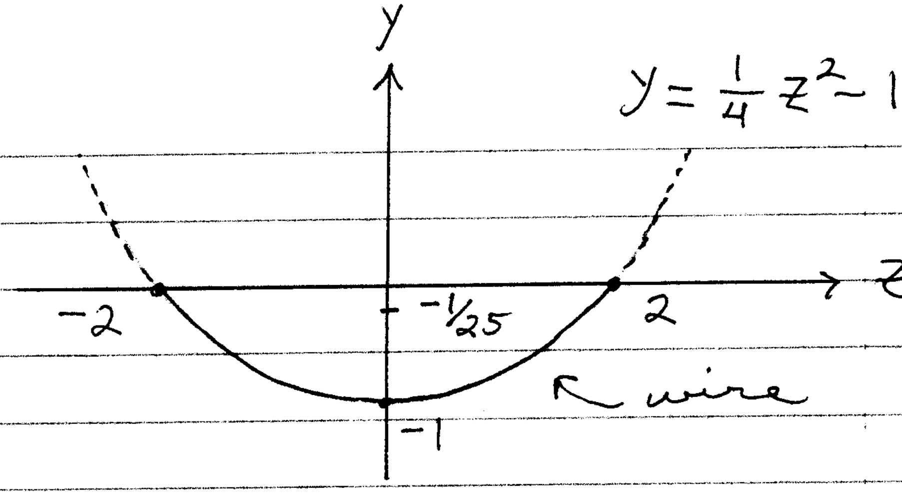
$$= 2\left(-4/5\right) - 2\left(4/5\right) = -16/5 \text{ so}$$

$$\bar{y} = \frac{-16/5}{\frac{80}{1}} = \frac{-\cancel{16}}{5}\cdot\frac{1}{\cancel{80}5} = \frac{-1}{25} \quad ;$$

$$\begin{cases} y = t^2-1 \\ z = 2t \end{cases} \quad \text{for } -1 \le t \le 1 \longrightarrow$$

$$t = \frac{1}{2}z \longrightarrow (\text{SUB}) \longrightarrow y = \left(\frac{1}{2}z\right)^2 - 1 \longrightarrow$$

$$y = \frac{1}{4}z^2 - 1$$

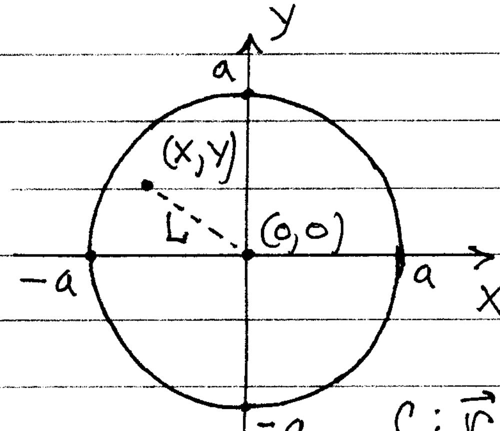


25.) a.) $C: \vec{r}(t) = \sqrt{2}t\,\vec{i} + \sqrt{2}t\,\vec{j} + (4-t^2)\vec{k}$
for $0 \le t \le 1 \xrightarrow{D}$
$\vec{v}(t) = \sqrt{2}\,\vec{i} + \sqrt{2}\,\vec{j} + (-2t)\vec{k}$ then

$|\vec{v}(t)| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2 + (-2t)^2} = \sqrt{4+4t^2}$
$= 2\sqrt{1+t^2}$; density $\delta(x,y,z) = 3t \longrightarrow$

$\text{Mass} = \int_C \delta(P)\,ds = \int_0^1 3t \cdot \frac{ds}{dt}\,dt$
$= \int_0^1 3t \cdot 2\sqrt{1+t^2}\,dt = 2(1+t^2)^{3/2}\Big|_0^1$
$= 2(2)^{3/2} - 2(1)^{3/2} = 4\sqrt{2} - 2$

27.)



density is constant value δ ;
distance
$L = \sqrt{x^2+y^2}$;

$C: \vec{r}(t) = a\cos t \cdot \vec{i} + a\sin t \cdot \vec{j}$
for $0 \le t \le 2\pi \xrightarrow{D} \vec{v}(t) = -a\sin t\,\vec{i} + a\cos t\,\vec{j}$
$\longrightarrow |\vec{v}(t)| = \sqrt{(-a\sin t)^2 + (a\cos t)^2} = \cdots = a$;

$$M.\ of\ I. = \int_C (\text{distance})^2 \cdot \delta(P)\, ds$$

$$= \int_0^{2\pi} (x^2 + y^2) \cdot \delta \, \frac{ds}{dt}\, dt$$

$$= \int_0^{2\pi} \left((a\cos t)^2 + (a \sin t)^2\right) \cdot \delta \cdot (a)\, dt$$

$$= \int_0^{2\pi} \left(a^2 \underbrace{(\cos^2 t + \sin^2 t)}_{1}\right) \delta a \, dt$$

$$= a^3 \delta t \Big|_0^{2\pi} = 2\pi a^3 \delta$$

29.) $C: \vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j} + t\vec{k}$

for $0 \le t \le 2\pi \xrightarrow{D}$

$\vec{v}(t) = (-\sin t)\vec{i} + (\cos t)\vec{j} + (1)\vec{k}$ and

$|\vec{v}(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (1)^2}$

$= \sqrt{1+1} = \sqrt{2}$; assume

density is $\delta(x, y, z) = z$; then

a.) $I_z = \int_C (\text{distance})^2 \, \delta(P)\, ds$

from (x, y, z) to $(0, 0, z)$

$$= \int_0^{2\pi} (x^2 + y^2) \cdot z \, \frac{ds}{dt}\, dt$$

$$= \int_0^{2\pi} \underbrace{(\cos^2 t + \sin^2 t)}_{1} \cdot t \cdot \sqrt{2}\, dt$$

$$= \sqrt{2} \cdot \frac{1}{2} t^2 \Big|_0^{2\pi} = \frac{\sqrt{2}}{2} \cdot 4\pi^2 = 2\sqrt{2}\,\pi^2$$

b.) M. of I. $= \int_C (\text{distance})^2 \cdot \delta(P)\, ds$

from (x,y,z) to $(0,0,0)$

$$= \int_0^{2\pi} (x^2+y^2+z^2) \cdot z \cdot \frac{ds}{dt}\, dt$$

$$= \int_0^{2\pi} (\underbrace{\cos^2 t + \sin^2 t}_{1} + t^2)\, t \cdot \sqrt{2}\, ds$$

$$= \sqrt{2} \int_0^{2\pi} (t + t^3)\, dt = \sqrt{2}\left(\frac{1}{2}t^2 + \frac{1}{4}t^4\right)\Big|_0^{2\pi}$$

$$= \sqrt{2}\left(\frac{1}{2}(4\pi^2) + \frac{1}{4}(16\pi^4)\right)$$

$$= \sqrt{2}\left(2\pi^2 + 4\pi^4\right).$$

c.) $\bar{z} = \dfrac{\int_C z\, \delta(P)\, ds}{\int_C \delta(P)\, ds}$;

$$\int_C \delta(P)\, ds = \int_0^{2\pi} z \cdot \frac{ds}{dt}\, dt = \int_0^{2\pi} t \cdot \sqrt{2}\, dt$$

$$= \sqrt{2} \cdot \frac{1}{2} t^2 \Big|_0^{2\pi} = \sqrt{2} \cdot \frac{1}{2}(4\pi^2) = 2\sqrt{2}\,\pi^2 ;$$

$$\int_C z\, \delta(P)\, ds = \int_0^{2\pi} t \cdot z \cdot \frac{ds}{dt}\, dt$$

$$= \int_0^{2\pi} t \cdot t \cdot \sqrt{2}\, dt = \sqrt{2} \int_0^{2\pi} t^2\, dt$$

$$= \sqrt{2} \cdot \frac{1}{3} t^3 \Big|_0^{2\pi} = \frac{\sqrt{2}}{3} \cdot 8\pi^3 = \frac{8}{3}\sqrt{2}\,\pi^3 \; ;$$

then

$$\bar{z} = \frac{\frac{8}{3}\cancel{\sqrt{2}}\,\pi^{\cancel{3}\,1}}{2\cancel{\sqrt{2}}\,\cancel{\pi^2}} = \frac{4}{3}\pi \; .$$

d.) $$\bar{z} = \frac{\int_C 1 \cdot \delta(P)\, ds}{\int_C 1\, ds} \; ;$$

$$\int_C 1\, ds = \int_0^{2\pi} \frac{ds}{dt} \cdot dt = \int_0^{2\pi} \sqrt{2}\, dt$$

$$= \sqrt{2}\, t \Big|_0^{2\pi} = 2\sqrt{2}\,\pi \; ;$$

$$\int_C \delta(P)\, ds = \cdots = 2\sqrt{2}\,\pi^2$$; *then*

$$\bar{z} = \frac{\cancel{2}\cancel{\sqrt{2}}\,\pi^{\cancel{2}}}{\cancel{2}\cancel{\sqrt{2}}\,\cancel{\pi}} = \pi \; .$$