Section 16.2

23.) a.) $\vec{F}_1(x,y) = (x)\vec{i} + (y)\vec{j}$ and

$C: \begin{cases} x = \cos t \\ y = \sin t \end{cases}$ for $0 \le t \le 2\pi$

and $\frac{dx}{dt} = -\sin t$, $\frac{dy}{dt} = \cos t$;

$$Circ_1 = \int_C \vec{F}_1 \cdot \vec{T}\, ds = \int_C M\,dx + N\,dy$$

$$= \int_0^{2\pi} \left[x \cdot \frac{dx}{dt} + N \cdot \frac{dy}{dt} \right] dt$$

$$= \int_0^{2\pi} \left[\cos t \cdot {}^{-}\sin t + \sin t \cos t \right] dt$$

$$= \int_0^{2\pi} 0\, dt = 0 \quad ;$$

$$Flux_1 = \int_C \vec{F}_1 \cdot \vec{n}\, ds = \int_C M\,dy - N\,dx$$

$$= \int_0^{2\pi} \left[x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt} \right] dt$$

$$= \int_0^{2\pi} \left[\cos t \cdot \cos t - \sin t \cdot {}^{-}\sin t \right] dt$$

$$= \int_0^{2\pi} (\underbrace{\cos^2 t + \sin^2 t}_{1})\, dt = t \Big|_0^{2\pi} = 2\pi$$

$\vec{F}_2(x,y) = (-y)\vec{i} + (x)\vec{j}$ then

$$Circ_2 = \int_C \vec{F}_2 \cdot \vec{T}\, ds = \int_C M\,dx + N\,dy$$

$$= \int_0^{2\pi} \left[-y \cdot \frac{dx}{dt} + x \frac{dy}{dt} \right] dt$$

$$= \int_0^{2\pi} [-\sin t \cdot {}^-\sin t + \cos t \cdot \cos t]\, dt$$

$$= \int_0^{2\pi} [\underbrace{\sin^2 t + \cos^2 t}_{1}]\, dt = t \Big|_0^{2\pi} = 2\pi \; ;$$

$$\text{Flux}_2 = \int_{C_2} \vec{F} \cdot \vec{n}\, ds = \int_{C_2} M\, dy - N\, dx$$

$$= \int_0^{2\pi} \left[-y \cdot \frac{dy}{dt} - x \cdot \frac{dx}{dt}\right] dt$$

$$= \int_0^{2\pi} [-\sin t \cdot {}^-\cos t - \cos t \cdot {}^-\sin t]\, dt$$

$$= \int_0^{2\pi} 2 \sin t \cos t\, dt = \sin^2 t \Big|_0^{2\pi}$$

$$= \cancel{\sin^2 2\pi}^{\,0} - \cancel{\sin^2 0}^{\,0} = 0 \, .$$

24.) $C: \begin{cases} x = a \cos t \\ y = a \sin t \end{cases}$ for $0 \le t \le 2\pi$,

$\frac{dx}{dt} = -a \sin t$, $\frac{dy}{dt} = a \cos t$; then

$\vec{F}_1(x,y) = (2x)\vec{i} + (-3y)\vec{j}$ so

$$\text{Flux}_1 = \int_C \vec{F}_1 \cdot \vec{n}\, ds = \int_C M\, dy - N\, dx$$

$$= \int_0^{2\pi} \left[2x \cdot \frac{dy}{dt} - (-3y) \frac{dx}{dt}\right] dt$$

$$= \int_0^{2\pi} [2a \cos t \cdot a \cos t + 3a \sin t \cdot {}^-a \sin t]\, dt$$

$$= \int_0^{2\pi} [2a^2 \cos^2 t - 3a^2 \sin^2 t]\, dt$$

$$= \int_0^{2\pi} \left[\cancel{2}a^2 \cdot \frac{1}{\cancel{2}}(1 + \cos 2t) - 3a^2 \cdot \frac{1}{2}(1 - \cos 2t)\right] dt$$

$$= \left[a^2\left(t+\tfrac{1}{2}\sin 2t\right) - \tfrac{3}{2}a^2\left(t-\tfrac{1}{2}\sin 2t\right)\right]\Big|_0^{2\pi}$$

$$= \left[a^2\left(2\pi+\tfrac{1}{2}\overset{0}{\cancel{\sin 4\pi}}\right) - \tfrac{3}{2}a^2\left(2\pi-\tfrac{1}{2}\overset{0}{\cancel{\sin 4\pi}}\right)\right]$$

$$- \left[a^2\left(0+\tfrac{1}{2}\overset{0}{\cancel{\sin 0}}\right) - \tfrac{3}{2}a^2\left(0-\tfrac{1}{2}\overset{0}{\cancel{\sin 0}}\right)\right]$$

$$= 2a^2\pi - 3a^2\pi = -a^2\pi \ ;$$

$$\vec{F}_2(x,y) = (2x)\vec{i} + (x-y)\vec{j} \quad \text{then}$$

$$\text{Flux}_2 = \int_C \vec{F}_2 \cdot \vec{n}\, ds = \int_C M\,dy - N\,dx$$

$$= \int_0^{2\pi} \left[2x \cdot \frac{dy}{dt} - (x-y)\cdot \frac{dx}{dt}\right] dt$$

$$= \int_0^{2\pi} \left[2 \cdot a\cos t \cdot a\cos t - (a\cos t - a\sin t)\cdot {}^{-}a\sin t\right] dt$$

$$= \int_0^{2\pi} \left[2\cdot a^2\cos^2 t + a^2\sin t\cos t - a^2\sin^2 t\right] dt$$

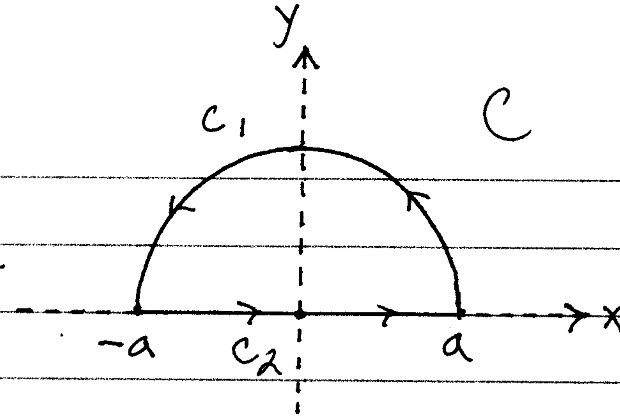
$$= \int_0^{2\pi} \left[\cancel{2}a^2\cdot\tfrac{1}{\cancel{2}}(1+\cos 2t) + a^2\sin t\cos t - a^2\cdot\tfrac{1}{2}(1-\cos 2t)\right] dt$$

$$= \left[a^2\left(t+\tfrac{1}{2}\sin 2t\right) + a^2\cdot\tfrac{1}{2}\sin^2 t - \tfrac{a^2}{2}\left(t-\tfrac{1}{2}\sin 2t\right)\right]\Big|_0^{2\pi}$$

$$= \left[a^2\left(2\pi+\tfrac{1}{2}\underset{0}{\cancel{\sin 4\pi}}\right) + \tfrac{a^2}{2}\cancel{\sin^2 2\pi} - \tfrac{a^2}{2}\left(2\pi - \tfrac{1}{2}\underset{0}{\cancel{\sin 4\pi}}\right)\right]$$

$$- \left[a^2\left(0+\tfrac{1}{2}\overset{0}{\cancel{\sin(0)}}\right) + \tfrac{a^2}{2}\underset{0}{\overset{0}{\cancel{\sin^2 0}}} - \tfrac{a^2}{2}\left(0-\tfrac{1}{2}\underset{0}{\overset{0}{\cancel{\sin(0)}}}\right)\right]$$

$$= 2a^2\pi - a^2\pi = a^2\pi \ .$$



26.) $C_1: \begin{cases} x = a\cos t & \text{for} \\ y = a\sin t & 0 \le t \le \pi \end{cases}$

$C_2: \begin{cases} x = t - a & \text{for} \\ y = 0 & 0 \le t \le 2a \end{cases}$;

$\vec{F}(x,y) = (x^2)\vec{i} + (y^2)\vec{j}$ then

$$\text{Circ} = \int_C \vec{F}\cdot\vec{T}\,ds = \int_C M\,dx + N\,dy$$

$$= \int_{C_1} M\,dx + N\,dy + \int_{C_2} M\,dx + N\,dy$$

$$= \int_0^{\pi}\left[x^2\frac{dx}{dt} + y^2\frac{dy}{dt}\right]dt + \int_0^{2a}\left[x^2\frac{dx}{dt} + y^2\frac{dy}{dt}\right]dt$$

$$= \int_0^{\pi}\left[a^2\cos^2 t\cdot -a\sin t + a^2\sin^2 t\cdot a\cos t\right]dt$$

$$+ \int_0^{2a}\left[(t-a)^2(1) + (0)^2(0)\right]dt$$

$$= \int_0^{\pi}\left[-a^3\cos^2 t\sin t + a^3\sin^2 t\cos t\right]dt$$

$$+ \frac{1}{3}(t-a)^3\Big|_0^{2a}$$

$$= \left(-a^3\cdot -\frac{1}{3}\cos^3 t + a^3\cdot\frac{1}{3}\sin^3 t\right)\Big|_0^{\pi} + \frac{1}{3}a^3 - \frac{-1}{3}a^3$$

$$= \left(\frac{1}{3}a^3\cos^3\pi + \frac{1}{3}a^3\cancelto{0}{\sin^3\pi}\right) - \left(\frac{1}{3}a^3\cos^3 0 + \frac{1}{3}a^3\cancelto{0}{\sin^3 0}\right)$$

$$= \frac{1}{3}a^3(-1)^3 - \frac{1}{3}a^3(1)^3 + \frac{2}{3}a^3 = \frac{-2}{3}a^3 + \frac{2}{3}a^3 = 0;$$

$$\text{Flux} = \int_C \vec{F}\cdot\vec{n}\,ds = \int_C M\,dy - N\,dx$$

$$= \int_{C_1} M\,dy - N\,dx + \int_{C_2} M\,dy - N\,dx$$

$$= \int_0^{\pi} \left[x^2 \cdot \frac{dy}{dt} - y^2 \cdot \frac{dx}{dt}\right] dt + \int_0^{2a} \left[x^2 \cdot \cancel{\frac{dy}{dt}}^{\,0} - 0 \cdot \cancel{\frac{dx}{dt}}^{\,0}\right] dt$$

$$= \int_0^{\pi} a^2\cos^2 t \cdot a\cos t - a^2\sin^2 t \cdot {}^{-}a\sin t \,]\, dt$$

$$= \int_0^{\pi} (a^3\cos^3 t + a^3\sin^3 t)\, dt$$

$$= a^3 \int_0^{\pi} [\cos^2 t \cdot \cos t + \sin^2 t \cdot \sin t]\, dt$$

$$= a^3 \int_0^{\pi} [(1-\sin^2 t)\cos t + (1-\cos^2 t)\sin t]\, dt$$

$$= a^3 \cdot \int_0^{\pi} [\cos t - \sin^2 t\cos t + \sin t - \cos^2 t\sin t]\, dt$$

$$= a^3 \cdot \left[\sin t - \tfrac{1}{3}\sin^3 t - \cos t + \tfrac{1}{3}\cos^3 t\right]\Big|_0^{\pi}$$

$$= a^3 (\cancel{\sin \pi}^{\,0} - \tfrac{1}{3}\cancel{\sin^3 \pi}^{\,0} - \cos\pi + \tfrac{1}{3}\cos^3\pi)$$
$$- a^3 (\cancel{\sin 0}^{\,0} - \tfrac{1}{3}\cancel{\sin^3 0}^{\,0} - \cos 0 + \tfrac{1}{3}\cos^3 0)$$

$$= a^3(-(-1) + \tfrac{1}{3}(-1)^3) - a^3(-1 + \tfrac{1}{3}(1)^3)$$

$$= a^3(2/3) - a^3(-2/3) = \frac{4}{3}a^3.$$

27) $\vec{F}(x,y) = (-y)\,\vec{i} + (x)\,\vec{j}$ then

$$\text{Circ} = \int_C \vec{F} \cdot \vec{T}\, ds = \int_C M\,dx + N\,dy$$

$$= \int_{C_1} M\,dx + N\,dy + \int_{C_2} M\,dx + N\,dy$$

$$= \int_0^{\pi} \left[(-y) \cdot \frac{dx}{dt} + x \cdot \frac{dy}{dt}\right] dt + \int_0^{2a} \left[(-y) \cdot \frac{dx}{dt} + x \cdot \frac{dy}{dt}\right] dt$$

$$= \int_0^{\pi} [-a\sin t \cdot {}^{-}a\sin t + a\cos t \cdot a\cos t]\, dt$$
$$+ \int_0^{2a} [(0)(1) + (t-a)(0)]\, dt$$

$$= \int_0^{\pi} \left[a^2 \sin^2 t + a^2 \cdot \cos^2 t \right] dt$$

$$= a^2 \int_0^{\pi} (\underbrace{\sin^2 t + \cos^2 t}_{1}) \, dt$$

$$= a^2 \cdot t \Big|_0^{\pi} = a^2 \pi \quad ;$$

$$\text{Flux} = \int_C \vec{F} \cdot \vec{n} \, ds = \int_C M\,dy - N\,dx$$

$$= \int_{C_1} M\,dy - N\,dx + \int_{C_2} M\,dy - N\,dx$$

$$= \int_0^{\pi} \left[(-y) \cdot \frac{dy}{dt} - x \cdot \frac{dx}{dt} \right] dt + \int_0^{2a} \left[(-y) \cdot \frac{dy}{dt} - x \cdot \frac{dx}{dt} \right] dt$$

$$= \int_0^{\pi} \left[-a \sin t \cdot a \cos t - a \cos t \cdot {}^{-}a \sin t \right] dt$$

$$+ \int_0^{2a} \left[(0)(0) - (t-a)(1) \right] dt$$

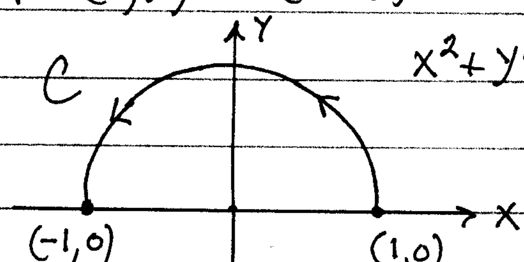
$$= \int_0^{\pi} (\cancel{- a^2 \sin t \cos t + a^2 \sin t \cos t})^{0} \, dt$$

$$+ \int_0^{2a} (a - t) \, dt$$

$$= \left(at - \frac{1}{2} t^2 \right) \Big|_0^{2a} = 2a^2 - 2a^2 = 0$$

29) $\vec{F}(x,y) = (x+y)\,\vec{i} - (x^2+y^2)\,\vec{j}$

a.)



$x^2 + y^2 = 1$

$$C : \begin{cases} x = \cos t & \text{for} \\ y = \sin t & 0 \le t \le \pi \end{cases}$$

$$\frac{dx}{dt} = -\sin t, \quad \frac{dy}{dt} = \cos t$$

$$\text{Flow} = \int_C \vec{F}\cdot\vec{T}\,ds = \int_C M\,dx + N\,dy$$

$$= \int_0^{\pi} \left[(x+y)\frac{dx}{dt} + (-x^2-y^2)\frac{dy}{dt}\right] dt$$

$$= \int_0^{\pi} \left[(\cos t + \sin t)\cdot -\sin t + (\underbrace{-\cos^2 t - \sin^2 t}_{-1})\cdot \cos t\right] dt$$

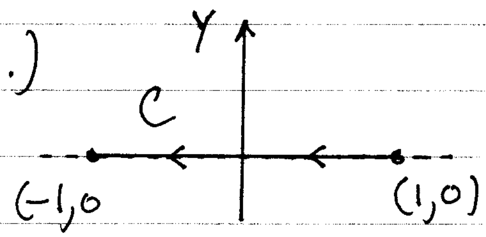
$$= \int_0^{\pi} \left[-\sin t \cos t - \sin^2 t - \cos t\right] dt$$

$$= \int_0^{\pi} \left[-\sin t\cdot\cos t - \tfrac{1}{2}(1-\cos 2t) - \cos t\right] dt$$

$$= \left(-\tfrac{1}{2}\sin^2 t - \tfrac{1}{2}\left(t - \tfrac{1}{2}\sin 2t\right) - \sin t\right)\Big|_0^{\pi}$$

$$= \left(-\tfrac{1}{2}\overset{0}{\cancel{\sin^2\pi}} - \tfrac{1}{2}\pi + \tfrac{1}{4}\overset{0}{\cancel{\sin 2\pi}} - \overset{0}{\cancel{\sin\pi}}\right)$$
$$- \left(-\tfrac{1}{2}\overset{0}{\cancel{\sin^2 0}} - \tfrac{1}{2}(0) + \tfrac{1}{4}\overset{0}{\cancel{\sin 0}} - \overset{0}{\cancel{\sin 0}}\right) = -\tfrac{1}{2}\pi.$$

b.)



$$C: \begin{cases} x = 1-t & \text{for} \\ y = 0 & 0 \le t \le 2 \end{cases}$$

$$\frac{dx}{dt} = -1, \quad \frac{dy}{dt} = 0 \;;$$

$$\text{Flow} = \int_C \vec{F}\cdot\vec{T}\,ds = \int_C M\,dx + N\,dy$$

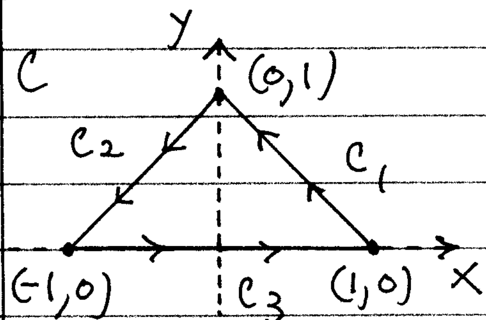
$$= \int_0^{2} \left[(x+y)\frac{dx}{dt} + (-x^2-y^2)\cdot\frac{dy}{dt}\right] dt$$

$$= \int_0^{2} \left[(1-t)\cdot(-1) + (-(1-t)^2 - 0^2)(0)\right] dt$$

$$= \int_0^{2} (t-1)\,dt = \left(\tfrac{1}{2}t^2 - t\right)\Big|_0^{2}$$

$$= 2 - 2 = 0$$

36.) $\vec{F}(x,y) = (x+y)\,\vec{i} - (x^2+y^2)\,\vec{j}$



$C_1: \begin{cases} x = t & \text{for } t=1 \text{ to} \\ y = 1-t & t=0 \end{cases}$,

$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = -1$;

$C_2: \begin{cases} x = t & \text{for } t=0 \text{ to} \\ y = t+1 & t=-1 \end{cases}$,

$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = 1$;

$C_3: \begin{cases} x = t & \text{for} \\ y = 0 & -1 \le x \le 1 \end{cases}$, $\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = 0$;

$$\text{Flux} = \int_C \vec{F}\cdot\vec{n}\,ds$$

$$= \int_{C_1} \vec{F}\cdot\vec{n}\,ds + \int_{C_2} \vec{F}\cdot\vec{n}\,ds + \int_{C_3} \vec{F}\cdot\vec{n}\,ds \ ;$$

$$\int_{C_1} \vec{F}\cdot\vec{n}\,ds = \int_{C_1} M\,dy - N\,dx = \int_1^0 \left[(x+y)\frac{dy}{dt} + (x^2+y^2)\cdot\frac{dx}{dt}\right]dt$$

$$= \int_1^0 \left[(1)(-1) + (t^2 + (1-t)^2)(1)\right] dt$$

$$= \int_1^0 \left[\cancel{-1} + t^2 + \cancel{1} - 2t + t^2\right] dt = \int_1^0 \left[2t^2 - 2t\right] dt$$

$$= \left(\frac{2}{3}t^3 - t^2\right)\Big|_1^0 = (0-0) - \left(\frac{2}{3} - 1\right) = \frac{1}{3} \ ;$$

$$\int_{C_2} \vec{F}\cdot\vec{n}\,ds = \int_{C_2} M\,dy - N\,dx = \int_0^{-1} \left[(x+y)\frac{dy}{dt} + (x^2+y^2)\frac{dx}{dt}\right]dt$$

$$= \int_0^{-1} \left[(2t+1)(1) + (t^2 + (t+1)^2)(1)\right] dt$$

$= \int_0^{-1} [2t+1+t^2+t^2+2t+1]\, dt$

$= \int_0^{-1} (2t^2+4t+2)\, dt = \left(\frac{2}{3}t^3+2t^2+2t\right)\Big|_0^{-1}$

$= \left(-\frac{2}{3}+2-2\right)-(0+0+0) = -2/3$;

$\int_{C_3} \vec{F}\cdot\vec{n}\, ds = \int_{C_3} M\,dy - N\,dx = \int_{-1}^{1} \left[(x+y)\frac{dy}{dt}+(x^2+y^2)\frac{dx}{dt}\right] dt$

$= \int_{-1}^{1} [(t)\cdot(0)+(t^2)(1)]\, dt = \frac{1}{3}t^3\Big|_{-1}^{1} = \frac{1}{3}-\frac{-1}{3} = 2/3$;

so Flux $= \frac{1}{3}+\frac{-2}{3}+\frac{2}{3} = \frac{1}{3}$.

37.) $\vec{F}(x,y,z) = (-4xy)\vec{i}+(8y)\vec{j}+(2)\vec{k}$,

$C: \begin{cases} x = t \\ y = t^2 \\ z = 1 \end{cases}$ for $0 \le t \le 2$; $\frac{dx}{dt} = 1$, $\frac{dy}{dt} = 2t$, $\frac{dz}{dt} = 0$;

Flow $= \int_C \vec{F}\cdot\vec{T}\, ds = \int_C M\,dx+N\,dy+P\,dz$

$= \int_0^2 \left[-4xy\cdot\frac{dx}{dt}+8y\cdot\frac{dy}{dt}+2\cdot\cancelto{0}{\frac{dz}{dt}}\right] dt$

$= \int_0^2 [-4(t)(t^2)(1)+8(t^2)(2t)]\, dt$

$= \int_0^2 12t^3\, dt = 3t^4\Big|_0^2 = 48$

40.) $\vec{F}(x,y,z) = (-y)\vec{i}+(x)\vec{j}+(2)\vec{k}$

$C: \begin{cases} x = -2\cos t \\ y = 2\sin t \\ z = 2t \end{cases}$ for $0 \le t \le 2\pi$

$$\frac{dx}{dt} = 2\sin t, \quad \frac{dY}{dt} = 2\cos t, \quad \frac{dz}{dt} = 2;$$

$$\text{Flow} = \int_C \vec{F}\cdot\vec{T}\, ds = \int_C M\,dx + N\,dy + P\,dz$$

$$= \int_0^{2\pi} \left[(-y)\frac{dx}{dt} + (x)\frac{dY}{dt} + (2)\frac{dz}{dt}\right] dt$$

$$= \int_0^{2\pi} \left[(-2\sin t)(2\sin t) + (-2\cos t)(2\cos t) + (2)(2)\right] dt$$

$$= \int_0^{2\pi} \left[-4\sin^2 t - 4\cos^2 t + 4\right] dt$$

$$= \int_0^{2\pi} \left[-4(\underbrace{\sin^2 t + \cos^2 t}_{1}) + 4\right] dt$$

$$= \int_0^{2\pi} 0\, dt = 0.$$

4.) $\vec{F}(x,Y,z) = (2x)\vec{i} + (2z)\vec{j} + (2Y)\vec{k}$,

$$C_1: \begin{cases} x = \cos t \\ y = \sin t \\ z = t \end{cases} \quad \text{for } 0 \le t \le \frac{\pi}{2},$$

$$\frac{dx}{dt} = -\sin t, \quad \frac{dY}{dt} = \cos t, \quad \frac{dz}{dt} = 1;$$

$$C_2: \begin{cases} x = 0 \\ y = 1 \\ z = \frac{\pi}{2}(1-t) \end{cases} \quad \text{for } 0 \le t \le 1,$$

$$\frac{dx}{dt} = 0, \quad \frac{dY}{dt} = 0, \quad \frac{dz}{dt} = -\frac{\pi}{2};$$

$$C_3: \begin{cases} x = t \\ y = 1-t & \text{for } 0 \le t \le 1 \\ z = 0 \end{cases},$$

$$\frac{dx}{dt} = 1, \quad \frac{dy}{dt} = -1, \quad \frac{dz}{dt} = 0 ;$$

$$\text{Circ} = \int_C \vec{F} \cdot \vec{T}\, ds$$

$$= \int_{C_1} \vec{F} \cdot \vec{T}\, ds + \int_{C_2} \vec{F} \cdot \vec{T}\, ds + \int_{C_3} \vec{F} \cdot \vec{T}\, ds ;$$

$$\int_{C_1} \vec{F} \cdot \vec{T}\, ds = \int_{C_1} M\,dx + N\,dy + P\,dz$$

$$= \int_0^{\frac{\pi}{2}} \left[(2x)\frac{dx}{dt} + (2z)\frac{dy}{dt} + (2y)\frac{dz}{dt} \right] dt$$

$$= \int_0^{\frac{\pi}{2}} \left[(2\cos t)(\sin t) + (2t)\cos t + (2\sin t)(1) \right] dt$$

$$= \left(-\sin^2 t + 2(t\sin t + \cos t) - 2\cos t \right) \Big|_0^{\pi/2}$$

$$= \left(-\sin^2 t + 2t\sin t \right) \Big|_0^{\pi/2}$$

$$= \left(-\sin^2 \frac{\pi}{2} + 2\left(\frac{\pi}{2}\right) \sin \frac{\pi}{2} \right)$$

$$- \left(-\overset{0}{\sin^2} 0 + 0 \right) = -(1)^2 + \pi(1) = \pi - 1 ;$$

$$\int_{C_2} \vec{F} \cdot \vec{T}\, ds = \int_{C_2} M\,dx + N\,dy + P\,dz$$

$$= \int_0^1 \left[(2x)\frac{dx}{dt} + (2z)\frac{dy}{dt} + (2y)\frac{dz}{dt} \right] dt$$

$$= \int_0^1 \left[(0)(0) + \pi(1-t)(0) + (2)\left(-\frac{\pi}{2}\right) \right] dt$$

$$= -\pi t \Big|_0^1 = -\pi ;$$

$$\int_{C_3} \vec{F}\cdot\vec{T}\, ds = \int_{C_3} M\,dx + N\,dy + P\,dz$$

$$= \int_0^1 \left[(2x)\frac{dx}{dt} + (2z)\frac{dy}{dt} + (2y)\frac{dz}{dt}\right] dt$$

$$= \int_0^1 \left[(2t)(1) + (0)(-1) + (2-2t)(0)\right] dt$$

$$= \int_0^1 2t\, dt = t^2 \Big|_0^1 = 1 \; ; \text{ so}$$

$$\text{Flow} = (\pi - 1) + (-\pi) + (1) = 0 .$$

43.) $\vec{F}(x,y,z) = (xy)\vec{i} + (y)\vec{j} + (-yz)\vec{k}$,

$$C: \begin{cases} x = t \\ y = t^2 \\ z = t \end{cases} \text{ for } 0 \le t \le 1 ; \quad \frac{dx}{dt} = 1 , \frac{dy}{dt} = 2t , \frac{dz}{dt} = 1 ;$$

$$\text{Flow} = \int_C \vec{F}\cdot\vec{T}\, ds = \int_C M\,dx + N\,dy + P\,dz$$

$$= \int_0^1 \left[(xy)\frac{dx}{dt} + (y)\frac{dy}{dt} + (-yz)\frac{dz}{dt}\right] dt$$

$$= \int_0^1 \left[\cancel{(t^3)(1)} + (t^2)(2t) + \cancel{(-t^3)(1)}\right] dt$$

$$= \int_0^1 2t^3\, dt = \frac{1}{2} t^4 \Big|_0^1 = \frac{1}{2} .$$