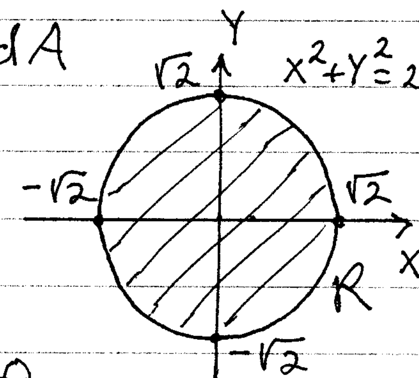
Section 16.5

1.) $\mathcal{S}: x^2+y^2-z=0$ and $z=2 \rightarrow$
$R: x^2+y^2=2$, then
$\vec{\nabla} f=(2x)\vec{i}+(2y)\vec{j}+(-1)\vec{k}$ so that

$$\sec\gamma=\frac{\sqrt{(2x)^2+(2y)^2+(-1)^2}}{|-1|}=\sqrt{4x^2+4y^2+1};$$

$$\text{Area}=\iint_{\mathcal{S}} 1\,dS=\iint_R \sec\gamma\cdot dA$$



$$=\iint_R \sqrt{4x^2+4y^2+1}\,dA$$

$$=\int_0^{2\pi}\int_0^{\sqrt{2}}\sqrt{4r^2+1}\cdot r\,dr\,d\theta$$

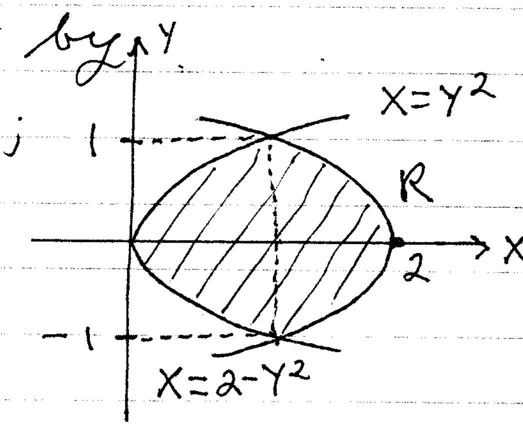
$$=\int_0^{2\pi}\frac{2}{3}\cdot\frac{1}{8}\left(4r^2+1\right)^{3/2}\Big|_{r=0}^{r=\sqrt{2}}$$

$$=\int_0^{2\pi}\left(\frac{1}{12}(9)^{3/2}-\frac{1}{12}(1)^{3/2}\right)d\theta$$

$$=\int_0^{2\pi}\left(\frac{27}{12}-\frac{1}{12}\right)d\theta=\int_0^{2\pi}\frac{26}{12}\,d\theta$$

$$=\int_0^{2\pi}\frac{13}{6}\,d\theta=\frac{13}{6}\theta\Big|_0^{2\pi}=\frac{13}{6}\cdot 2\pi=\frac{13}{3}\pi$$

3.) $\mathcal{S}: x+2y+2z=5$ cut by
$x=y^2$ and $x=2-y^2$;



then
$\vec{\nabla} f=(1)\vec{i}+(2)\vec{j}+(2)\vec{k}$

so that

$$\sec \gamma = \frac{\sqrt{(1)^2+(2)^2+(2)^2}}{|2|} = \frac{3}{2} \ ;$$

$$\text{Area} = \iint_{S} 1 \, dS = \iint_{R} \sec \gamma \cdot dA$$

$$= \iint_{R} \frac{3}{2} \, dA = \int_{-1}^{1}\int_{y^2}^{2-y^2} \frac{3}{2} \, dx \, dy$$

$$= \int_{-1}^{1} \left(\frac{3}{2} x \Big|_{x=y^2}^{x=2-y^2} \right) dy$$

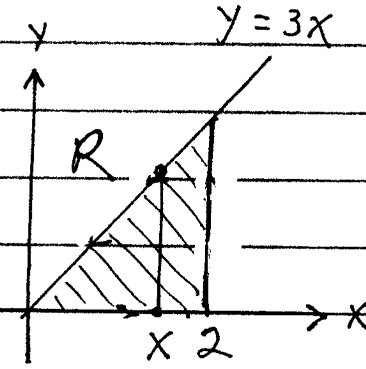
$$= \int_{-1}^{1} \left(\frac{3}{2}(2-y^2) - \frac{3}{2} y^2 \right) dy$$

$$= \int_{-1}^{1} \left(3 - \frac{3}{2} y^2 - \frac{3}{2} y^2 \right) dy$$

$$= \int_{-1}^{1} (3 - 3y^2) \, dy = (3y - y^3) \Big|_{-1}^{1}$$

$$= (3-1) - (-3+1) = 2 - (-2) = 4$$

5.) $S: \ x^2 - 2y - 2z = 0$ and



$$\vec{\nabla} f = (2x)\,\vec{i} + (-2)\,\vec{j} + (-2)\,\vec{k},$$

so that

$$\sec \gamma = \frac{\sqrt{(2x)^2+(-2)^2+(-2)^2}}{|-2|}$$

$$= \frac{1}{2}\sqrt{4(x^2+2)} = \sqrt{x^2+2} \ ;$$

$$\text{Area} = \iint_{S} 1 \, dS = \iint_{R} \sec \gamma \, dA$$

$$= \int_0^2 \int_0^{3x} \sqrt{x^2+2} \, dy \, dx = \int_0^2 \left(\sqrt{x^2+2} \cdot y \Big|_{y=0}^{y=3x} \right) dx$$

$$= \int_0^2 3x\sqrt{x^2+2}\,dx = \cancel{3}\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}(x^2+2)^{3/2}\Big|_0^2$$

$$= 6^{3/2} - 2^{3/2}$$

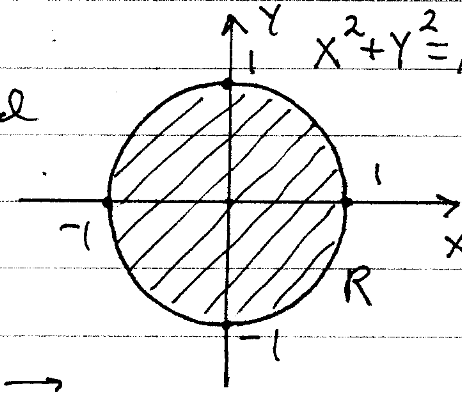
6.) $\mathcal{S}$: $x^2+y^2+z^2=2$ cut by $z=\sqrt{x^2+y^2}$ $\rightarrow$

$x^2+y^2 = 2-z^2$ so $z=\sqrt{2-z^2}$ $\rightarrow$

$z^2 = 2-z^2 \rightarrow 2z^2 = 2 \rightarrow z=1 \rightarrow$

intersection of curves is

$x^2+y^2=1$:



$\vec{\nabla} f = (2x)\vec{i}+(2y)\vec{j}+(2z)\vec{k}$ and

$$\sec\gamma = \frac{\sqrt{(2x)^2+(2y)^2+(2z)^2}}{|2z|}$$

$$= \frac{\sqrt{4(x^2+y^2+z^2)}}{2z} = \frac{\sqrt{4(2)}}{2z} \rightarrow$$

$\sec\gamma = \dfrac{\sqrt{2}}{z}$; then $z=\sqrt{2-(x^2+y^2)}$ and

$$\text{Area} = \iint_{\mathcal{S}} 1\,dS = \iint_R \sec\gamma\,dA$$

$$= \int_0^{2\pi}\int_0^1 \frac{\sqrt{2}}{\sqrt{2-r^2}}\cdot r\,dr\,d\theta$$

$$= \int_0^{2\pi} \sqrt{2}\cdot(-1)(2-r^2)^{1/2}\Big|_{r=0}^{r=1}\,d\theta$$

$$= \int_0^{2\pi}\left[-\sqrt{2}\,(1)^{1/2} - \sqrt{2}\,(2)^{1/2}\right]d\theta$$

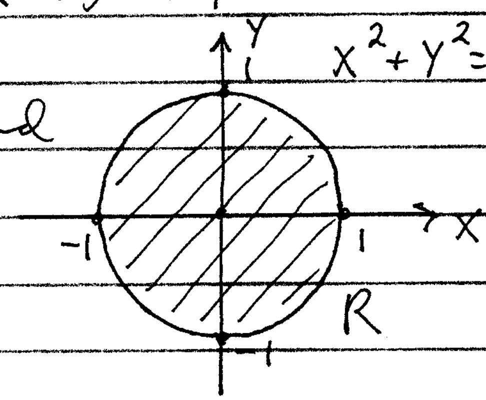
$$= \int_0^{2\pi}(2-\sqrt{2})\,d\theta = (2-\sqrt{2})\,\theta\Big|_0^{2\pi} = (4-2\sqrt{2})\pi.$$

7.) $\mathcal{S}$: $z = 3x$ cut by $x^2 + y^2 = 1$

$\rightarrow 3x - z = 0$ so

$\vec{\nabla} f = (3)\vec{i} + (0)\vec{j} + (-1)\vec{k}$ and

$$\sec\gamma = \frac{\sqrt{(3)^2 + (-1)^2}}{|-1|} = \sqrt{10},$$



then

$$\text{Area} = \iint_{\mathcal{S}} 1\, dS = \iint_{R} \sec\gamma \cdot dA$$

$$= \sqrt{10} \cdot \iint_{R} 1\, dA = \sqrt{10} \cdot \pi (1)^2 = \sqrt{10}\,\pi$$

12.) $\mathcal{S}$: $2x^{3/2} + 2y^{3/2} - 3z = 0$

R: square $0 \le x \le 1$, $0 \le y \le 1$, then

$\vec{\nabla} f = (3\sqrt{x})\vec{i} + (3\sqrt{y})\vec{j} + (-3)\vec{k}$, so that

$$\sec\gamma = \frac{\sqrt{(3\sqrt{x})^2 + (3\sqrt{y})^2 + (-3)^2}}{|-3|}$$

$$= \frac{1}{3}\sqrt{9(x+y+1)} = \sqrt{x+y+1} \text{ ; then}$$

$$\text{Area} = \iint_{\mathcal{S}} 1\, dS = \iint_{R} \sec\gamma \cdot dA$$

$$= \int_0^1 \int_0^1 \sqrt{x+y+1}\, dy\, dx$$

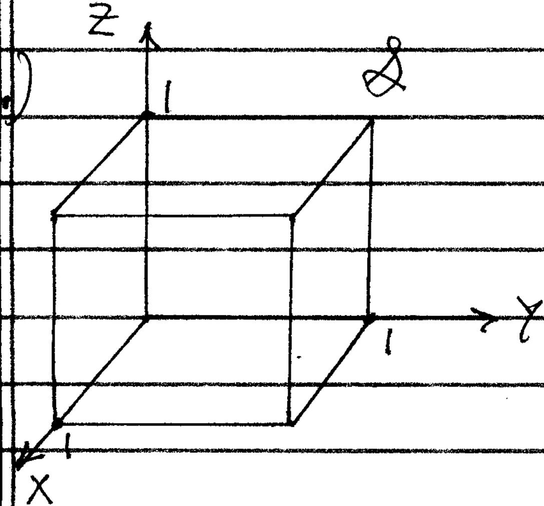
$$= \int_0^1 \frac{2}{3}(x+y+1)^{3/2} \Big|_{y=0}^{y=1} dx$$

$$= \int_0^1 \left[\frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{3/2} \right] dx$$

$$= \left(\frac{2}{3} \cdot \frac{2}{5} (x+2)^{5/2} - \frac{2}{3} \cdot \frac{2}{5} (x+1)^{5/2} \right) \Big|_0^1$$

$$= \frac{4}{15} (3)^{5/2} - \frac{4}{15} (1)^{5/2} = \frac{4}{15} \left(3^{5/2} - 1 \right).$$

13.)



$$\iint_{\mathcal{S}} g(P)\, dS$$

$$= \iint_{\text{bottom}} (x+y+z)\, dS$$

$$= \iint_{\text{top}} (x+y+z)\, dS$$

$$= \iint_{\text{left}} (x+y+z)\, dS + \iint_{\text{right}} (x+y+z)\, dS$$

$$+ \iint_{\text{front}} (x+y+z)\, dS + \iint_{\text{back}} (x+y+z)\, dS$$

$$= \int_0^1 \int_0^1 (x+y+0)\, dy\, dx + \int_0^1 \int_0^1 (x+y+1)\, dy\, dx$$

$$+ \int_0^1 \int_0^1 (x+0+z)\, dz\, dx + \int_0^1 \int_0^1 (x+1+z)\, dz\, dx$$

$$+ \int_0^1 \int_0^1 (1+y+z)\, dz\, dy + \int_0^1 \int_0^1 (0+y+z)\, dz\, dy$$

$$= 3 \int_0^1 \int_0^1 (x+y)\, dy\, dx + 3 \int_0^1 \int_0^1 (x+y+1)\, dy\, dx$$

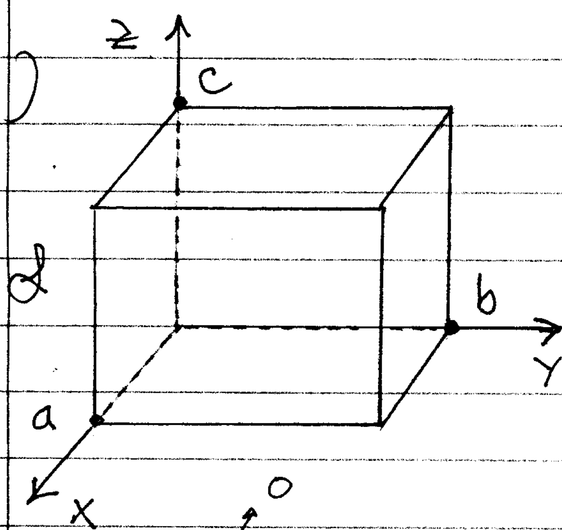
$$= 3 \int_0^1 \left(xy + \frac{1}{2} y^2 \right) \Big|_{y=0}^{y=1} dx + 3 \int_0^1 \left(xy + \frac{1}{2} y^2 + y \right) \Big|_{y=0}^{y=1} dx$$

$$= 3\int_0^1 \left(x+\tfrac{1}{2}\right)dx + 3\int_0^1 \left(x+\tfrac{1}{2}+1\right)dx$$

$$= 3\left(\tfrac{1}{2}x^2+\tfrac{1}{2}x\right)\Big|_0^1 + 3\left(\tfrac{1}{2}x^2+\tfrac{3}{2}x\right)\Big|_0^1$$

$$= 3\left(\tfrac{1}{2}+\tfrac{1}{2}\right) + 3\left(\tfrac{1}{2}+\tfrac{3}{2}\right) = 3(1)+3(2) = 9$$

15.)



$$\iint_{\mathcal{S}} g(P)\,dS$$

$$= \iint_{bottom} xy\cancel{z}^{\,0}\,dS + \iint_{top} xyz\,dS$$

$$+ \iint_{left} x\cancel{y}^{\,0}z\,dS + \iint_{right} xyz\,dS$$

$$+ \iint_{front} xyz\,dS + \iint_{back} \cancel{x}^{\,0}yz\,dS$$

$$= \int_0^a\!\int_0^b xy(c)\,dy\,dx + \int_0^a\!\int_0^c x(b)z\,dz\,dx$$

$$+ \int_0^b\!\int_0^c (a)yz\,dz\,dy$$

$$= \int_0^a \left(cx\cdot\tfrac{1}{2}y^2\Big|_{y=0}^{y=b}\right)dx$$

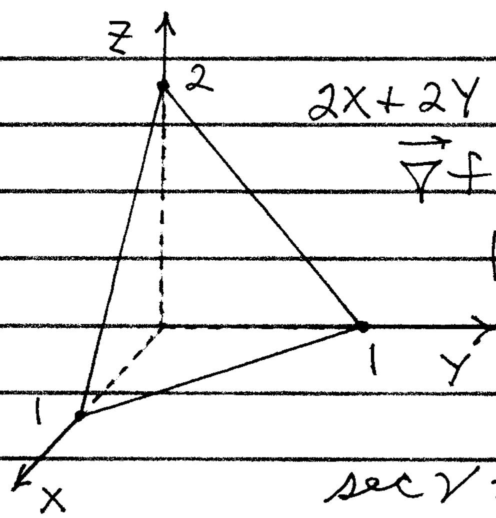
$$+ \int_0^a \left(bx\cdot\tfrac{1}{2}z^2\Big|_{z=0}^{z=c}\right)dx$$

$$+ \int_0^b \left(ay\cdot\tfrac{1}{2}z^2\Big|_{z=0}^{z=c}\right)dy$$

$$= \int_0^a \frac{1}{2} b^2 c x \, dx + \int_0^a \frac{1}{2} b c^2 x \, dx + \int_0^b \frac{1}{2} a c^2 y \, dy$$

$$= \frac{1}{2} b^2 c \cdot \frac{1}{2} x^2 \Big|_0^a + \frac{1}{2} b c^2 \cdot \frac{1}{2} x^2 \Big|_0^a + \frac{1}{2} a c^2 \cdot \frac{1}{2} y^2 \Big|_0^b$$

$$= \frac{1}{4} a^2 b^2 c + \frac{1}{4} a^2 b c^2 + \frac{1}{4} a b^2 c^2$$

17.) 

$2X + 2Y + Z = 2$,

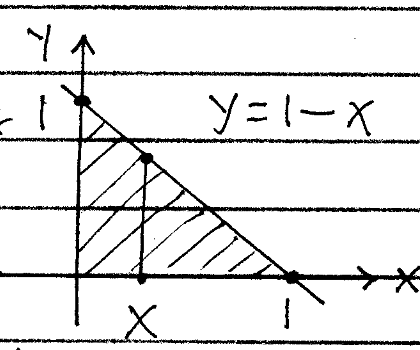
$\vec{\nabla} f = (2)\vec{i} + (2)\vec{j} + (1)\vec{k}$ and

$|\vec{\nabla} f| = \sqrt{(2)^2 + (2)^2 + (1)^2} = \sqrt{9} = 3$,

$\sec \gamma = \dfrac{|\vec{\nabla} f|}{|f_z|} = \dfrac{3}{1} = 3$; then

$$\iint_S g(P)\, dS = \iint_R (X+Y+Z) \cdot \sec\gamma \cdot dA$$

$$= \int_0^1 \int_0^{1-X} (X + Y + (2 - 2X - 2Y)) \cdot 3 \, dy \, dx$$



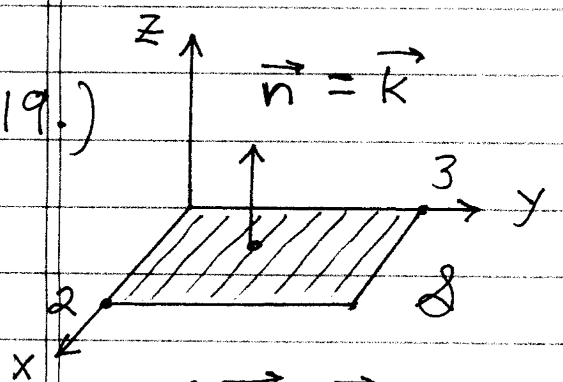
$$= 3 \int_0^1 \int_0^{1-X} (2 - X - Y) \, dy \, dx$$

$$= 3 \int_0^1 \left(2Y - XY - \frac{1}{2} Y^2\right) \Big|_{Y=0}^{Y=1-X} dx$$

$$= 3 \int_0^1 \left(2(1-X) - X(1-X) - \frac{1}{2}(1-X)^2\right) dx$$

$$= 3\int_0^1 \left(2-2x-x+x^2-\frac{1}{2}(x^2-2x+1)\right) dx$$
$$= 3\int_0^1 \left(2-3x+x^2-\frac{1}{2}x^2+x-\frac{1}{2}\right) dx$$
$$= 3\int_0^1 \left(\frac{1}{2}x^2-2x+3/2\right) dx$$
$$= 3\left(\frac{1}{6}x^3-x^2+\frac{3}{2}x\right)\Big|_0^1$$
$$= 3\left(\frac{1}{6}-1+3/2\right) = 3\left(\frac{1}{6}-\frac{6}{6}+\frac{9}{6}\right)$$
$$= 3\left(\frac{4}{6}\right) = 2$$

19.)

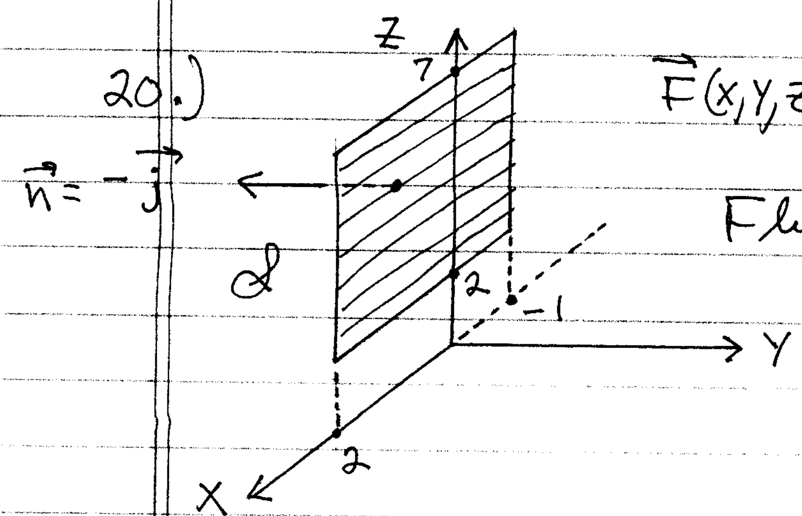


$\vec{F}(x,y,z) = (-1)\vec{i}+(2)\vec{j}+(3)\vec{k}$,

$$\text{Flux} = \iint_{\mathcal{S}} \vec{F}\cdot\vec{n}\, dS$$

$$= \iint_{\mathcal{S}} \vec{F}\cdot\vec{k}\, dS = \int_0^2\int_0^3 3\, dy\, dx$$
$$= 3\left(\int_0^2\int_0^3 1\, dy\, dx\right) = 3\ (\text{area } \mathcal{S})$$
$$= 3(6) = 18$$

20.)



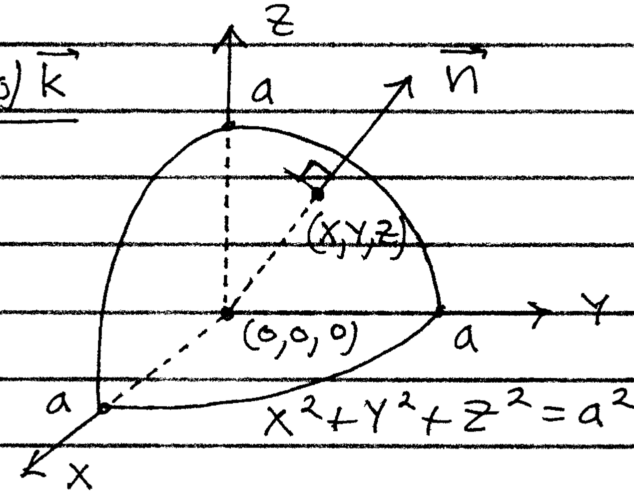
$\vec{F}(x,y,z) = (x^2y)\vec{i}+(-2)\vec{j}+(xz)\vec{k}$,

$$\text{Flux} = \iint_{\mathcal{S}} \vec{F}\cdot\vec{n}\, dS$$
$$= \iint_{\mathcal{S}} \vec{F}\cdot(-\vec{j})\, dS$$

$$= \int_{-1}^{2}\int_{2}^{7} 2 \, dz \, dx = 2\left(\int_{-1}^{2}\int_{2}^{7} 1 \, dz \, dx\right)$$

$$= 2\,(\text{area } \mathcal{S}) = 2\,(15) = 30 .$$

21.)



$$\vec{n} = \frac{(x-0)\vec{i} + (y-0)\vec{j} + (z-0)\vec{k}}{\sqrt{x^2+y^2+z^2}}$$

$$= \frac{x\vec{i} + y\vec{j} + z\vec{k}}{\sqrt{a^2}}$$

$$= \left(\frac{x}{a}\right)\vec{i} + \left(\frac{y}{a}\right)\vec{j} + \left(\frac{z}{a}\right)\vec{k} \, ;$$

$\vec{\nabla} f = (2x)\,\vec{i} + (2y)\,\vec{j} + (2z)\,\vec{k}$ and

$$|\vec{\nabla} f| = \sqrt{(2x)^2 + (2y)^2 + (2z)^2}$$

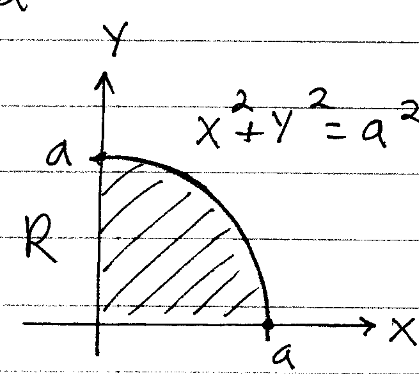
$$= \sqrt{4(x^2+y^2+z^2)} = \sqrt{4(a^2)} = 2a \text{, so}$$

$$\sec\gamma = \frac{|\vec{\nabla} f|}{|f_z|} = \frac{2a}{2z} = \frac{a}{z} \; ; \; \cancel{\text{then}}$$

$\vec{F}(x,y,z) = z\,\vec{k}$ and

$$\text{Flux} = \iint_{\mathcal{S}} \vec{F} \cdot \vec{n} \, dS = \iint_{R} \frac{z^2}{a} \cdot \cancel{\sec\gamma} \, dA$$

$$= \iint_{R} \frac{\cancel{z^2}}{\cancel{a}} \cdot \frac{\cancel{a}}{\cancel{z}} \, dA$$



$$= \iint_{R} z \, dA$$

$$= \iint_R \sqrt{a^2-(x^2+y^2)}\; dA$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{a} \sqrt{a^2-r^2}\cdot r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{2}{3}\cdot\frac{-1}{2}\left(a^2-r^2\right)^{3/2}\Big|_{r=0}^{r=a}\, d\theta$$

$$= \int_0^{\pi/2} \left(-\frac{1}{3}(0)^{3/2} - \frac{-1}{3}(a^2)^{3/2}\right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{3}a^3\, d\theta = \frac{1}{3}a^3\,\theta\Big|_0^{\frac{\pi}{2}} = \frac{1}{6}a^3\pi$$

24.) $\vec{F}(x,y,z) = (xz)\vec{i} + (yz)\vec{j} + (z^2)\vec{k}$,

$\vec{n} = \left(\frac{x}{a}\right)\vec{i} + \left(\frac{y}{a}\right)\vec{j} + \left(\frac{z}{a}\right)\vec{k}$, $\sec\gamma = a/z$;

then Flux $= \iint \vec{F}\cdot\vec{n}\, dS$

$$= \iint_R \left(\frac{x^2z}{a} + \frac{y^2z}{a} + \frac{z^3}{a}\right)\cdot \sec\gamma\, dA$$

$$= \iint_R \cancel{\frac{z}{a}}\underbrace{(x^2+y^2+z^2)}_{a^2}\cdot \cancel{\frac{a}{z}}\, dA$$

$$= a^2 \iint_R 1\, dA = a^2\,(\text{area } R) = a^2\cdot\frac{1}{4}\pi a^2$$

$$= \frac{1}{4}a^4\pi$$

25.) $\vec{F}(x,y,z) = (x)\vec{i} + (y)\vec{j} + (z)\vec{k}$,

$\vec{n} = \left(\frac{x}{a}\right)\vec{i} + \left(\frac{y}{a}\right)\vec{j} + \left(\frac{z}{a}\right)\vec{k}$, $\sec\gamma = a/z$;

then Flux $= \iint_{\mathcal{S}} \vec{F} \cdot \vec{n} \, dS$

$$= \iint_R \left(\frac{x^2}{a} + \frac{y^2}{a} + \frac{z^2}{a} \right) \cdot \sec\gamma \, dA$$

$$= \iint_R \frac{1}{\cancel{a}} (x^2 + y^2 + z^2) \cdot \frac{\cancel{a}}{z} \, dA$$

$$= \iint_R \frac{a^2}{z} \, dA = \int_0^{\frac{\pi}{2}} \int_0^a \frac{a^2}{\sqrt{a^2 - (x^2+y^2)}} \, r \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^a a^2 \cdot \frac{r}{\sqrt{a^2 - r^2}} \, dr \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} -a^2 \cdot (a^2 - r^2)^{1/2} \Big|_{r=0}^{r=a} \, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(-a^2 (0)^{1/2} - -a^2 (a^2)^{1/2} \right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} a^3 \, d\theta = a^3 \theta \Big|_0^{\frac{\pi}{2}} = \frac{1}{2} a^3 \pi$$

26.) $\vec{F}(x,y,z) = \dfrac{(x)\vec{i} + (y)\vec{j} + (z)\vec{k}}{\sqrt{x^2+y^2+z^2}}$,

$\vec{n} = \left(\frac{x}{a}\right)\vec{i} + \left(\frac{y}{a}\right)\vec{j} + \left(\frac{z}{a}\right)\vec{k}$, $\sec\gamma = \frac{a}{z}$;

then

Flux $= \iint_{\mathcal{S}} \vec{F} \cdot \vec{n} \, dS$

$$= \iint_{\mathcal{S}} \frac{1}{a} \cdot \frac{x^2+y^2+z^2}{\sqrt{x^2+y^2+z^2}} \, dS$$

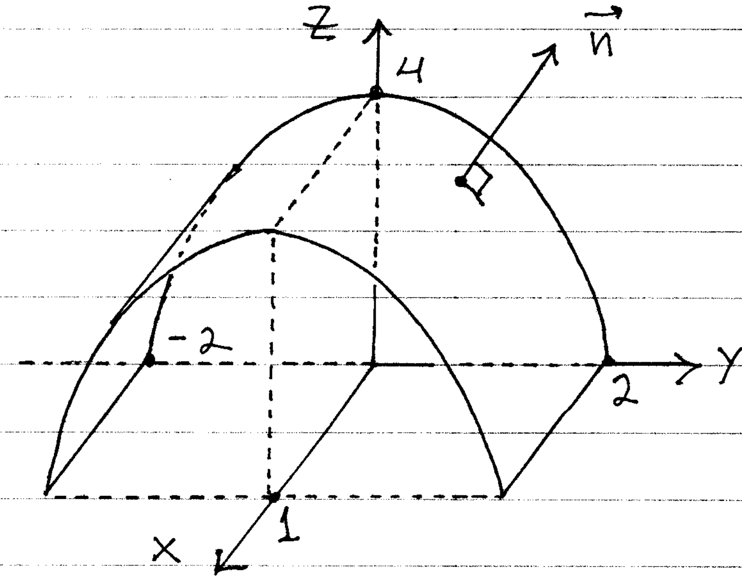
$$= \iint_{\mathcal{S}} \frac{1}{a} \cdot \frac{a^2}{\sqrt{a^2}} \, dS = \iint_R 1 \cdot \sec\gamma \, dA$$

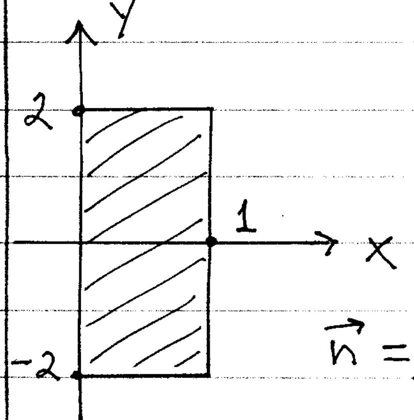
$$= \iint_R \frac{a}{z}\, dA = \frac{1}{a} \iint_R \frac{a^2}{z}\, dA$$

$$= \frac{1}{a}\left(\frac{1}{2} a^3 \pi\right) \qquad \text{(SEE solution 25.)}$$

$$= \frac{1}{2} a^2 \pi$$

27.) S: $z = 4 - y^2$ or $y^2 + z = 4$,





$$\vec{n} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|} = \frac{(2y)\vec{j} + (1)\vec{k}}{\sqrt{(2y)^2 + (1)^2}} \rightarrow$$

$$\vec{n} = \frac{2y}{\sqrt{4y^2+1}}\,\vec{j} + \frac{1}{\sqrt{4y^2+1}}\,\vec{k} \quad \text{and}$$

$$\sec \gamma = \frac{|\vec{\nabla} f|}{|f_z|} = \frac{\sqrt{4y^2+1}}{1} = \sqrt{4y^2+1}\text{; then}$$

$$\vec{F}(x,y,z) = (z^2)\vec{i} + (x)\vec{j} + (-3z)\vec{k} \quad \text{and}$$

$$\text{Flux} = \iint_S \vec{F} \cdot \vec{n}\, dS = \iint_S \left(\frac{2xy}{\sqrt{4y^2+1}} + \frac{-3z}{\sqrt{4y^2+1}}\right) dS$$

$$= \iint_R \frac{2xy - 3z}{\sqrt{4y^2+1}} \cdot \sec \gamma\, dA$$

$$= \int_0^1 \int_{-2}^{2} \frac{2xy - 3z}{\cancel{\sqrt{4y^2+1}}} \cdot \cancel{\sqrt{4y^2+1}}\, dy\, dx$$

$$= \int_0^1 \int_{-2}^{2} \left(2xy - 3(4-y^2)\right) dy\, dx$$

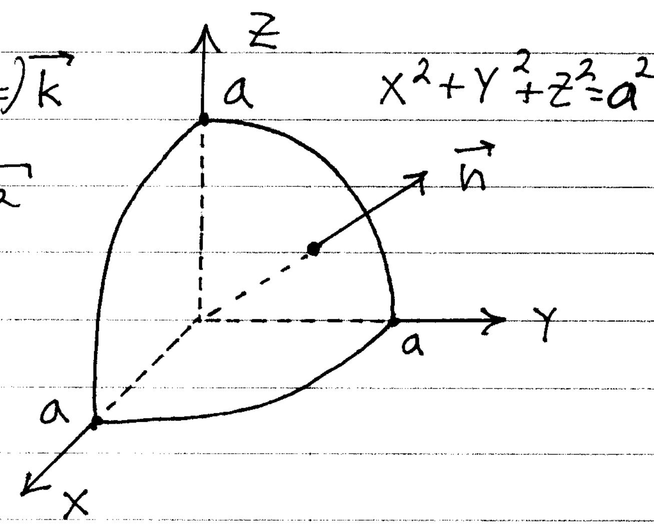
$$= \int_0^1 \int_{-2}^{2} \left(2xy - 12 + 3y^2\right) dy\, dx$$

$$= \int_0^1 \left(xy^2 - 12y + y^3\right)\Big|_{y=-2}^{y=2} dx$$

$$= \int_0^1 \left[(\cancel{4x} - 24 + 8) - (\cancel{4x} + 24 - 8)\right] dx$$

$$= \int_0^1 -32\, dx = -32x\Big|_0^1 = -32$$

33.) $\vec{\nabla} f = (2x)\vec{i} + (2y)\vec{j} + (2z)\vec{k}$



and

$$|\vec{\nabla} f| = \sqrt{(2x)^2 + (2y)^2 + (2z)^2}$$

$$= \sqrt{4(x^2+y^2+z^2)}$$

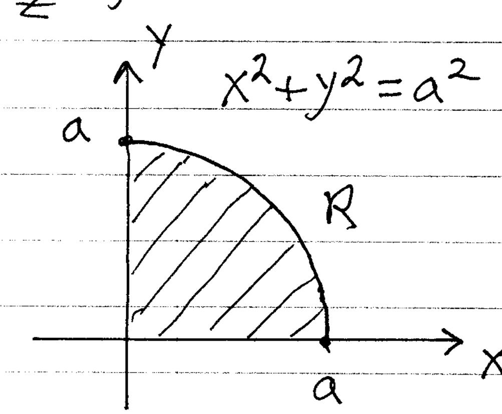
$$= \sqrt{4(a^2)} = 2a,$$

~~so~~

$$\vec{n} = \frac{\vec{\nabla} f}{|\vec{\nabla} f|} = \left(\frac{x}{a}\right)\vec{i} + \left(\frac{y}{a}\right)\vec{j} + \left(\frac{z}{a}\right)\vec{k},$$

$$\sec\gamma = \frac{|\vec{\nabla} f|}{|f_z|} = \frac{2a}{2z} = \frac{a}{z}\text{; then}$$

$\bar{x}$ for centroid is



$$\bar{x} = \frac{\iint_S x\, dS}{\iint_S 1\, dS};$$

$$\iint_S 1\, dS = \iint_R \sec\gamma\, dA$$

$$= \iint_R \frac{a}{z}\, dA = \iint_R \frac{a}{\sqrt{a^2-(x^2+y^2)}}\, dA$$

$$= \int_0^{\frac{\pi}{2}} \int_0^a \frac{a}{\sqrt{a^2-r^2}}\, r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(-a \cdot (a^2-r^2)^{1/2} \Big|_{r=0}^{r=a} \right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(-a(0)^{1/2} - -a(a^2)^{1/2} \right) d\theta$$

$$= \int_0^{\frac{\pi}{2}} a^2\, d\theta = a^2 \theta \Big|_0^{\pi/2} = \frac{1}{2} a^2 \pi \quad ;$$

$$\iint_S x\, dS = \iint_R x \cdot \sec \gamma\, dA$$

$$= \iint_R x \cdot \frac{a}{z}\, dA = \iint_R \frac{ax}{\sqrt{a^2-(x^2+y^2)}}\, dA$$

$$= \int_0^{\frac{\pi}{2}} \int_0^a \frac{a r \cos\theta}{\sqrt{a^2-r^2}}\, r\, dr\, d\theta \quad \text{(TOO DIFFICULT!)}$$

By Symmetry we know that $\bar{x} = \bar{y} = \bar{z}$ and $\bar{z}$ is EASY to compute! Then

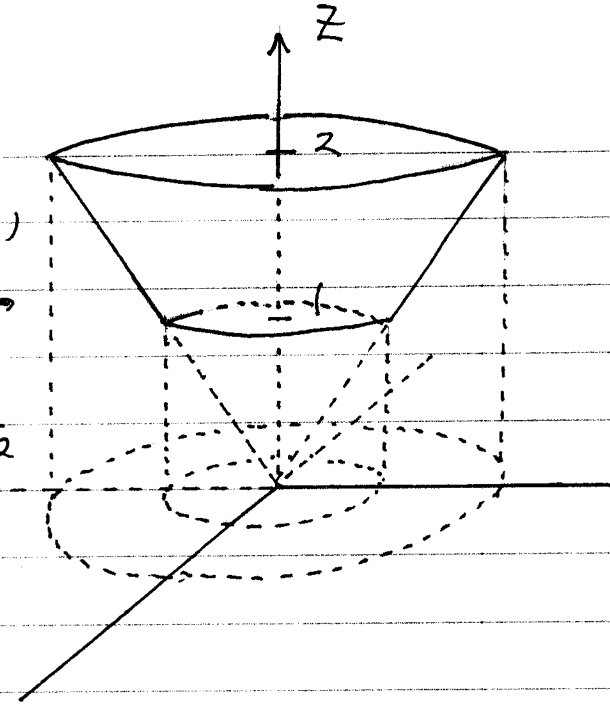
$$\iint_S z\, dS = \iint_R z \cdot \sec \gamma\, dA$$

$$= \iint_R \cancel{z} \cdot \frac{a}{\cancel{z}}\, dA = a \iint_R 1\, dA = a\ (\text{area } R)$$

$$= a \cdot \frac{1}{4} \pi a^2 = \frac{1}{4} a^3 \pi \text{, so}$$

$$\bar{x} = \bar{z} = \frac{\frac{1}{4} a^3 \pi}{\frac{1}{2} a^2 \pi} = \frac{a}{2} .$$

35.) $\mathcal{S}: x^2+y^2-z^2=0$,



$\vec{\nabla} f = (2x)\vec{i} + (2y)\vec{j} + (-2z)\vec{k}$

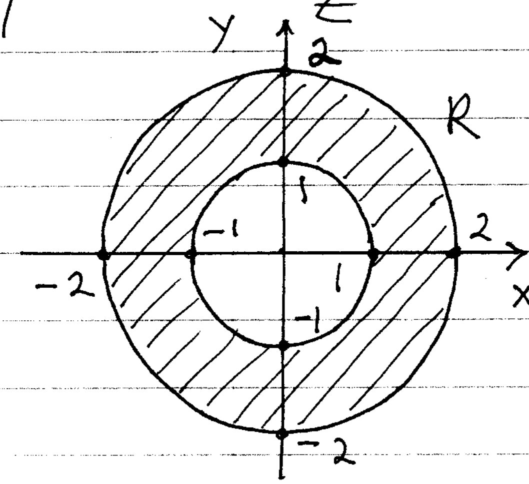
and

$$|\vec{\nabla} f| = \sqrt{(2x)^2+(2y)^2+(-2z)^2}$$
$$= \sqrt{4(x^2+y^2+z^2)}$$
$$= 2\sqrt{x^2+y^2+z^2} ;$$

$$\sec \gamma = \frac{|\vec{\nabla} f|}{|f_z|} = \frac{2\sqrt{x^2+y^2+z^2}}{|-2z|} = \frac{\sqrt{x^2+y^2+z^2}}{z} ;$$

then

$$\bar{z} = \frac{\iint_{\mathcal{S}} z \cdot \cancel{\delta}\, dS}{\iint_{\mathcal{S}} \cancel{\delta}\, dS} ;$$



$$\iint_{\mathcal{S}} 1\, dS = \iint_R \sec \gamma\, dA$$

$$= \iint_R \frac{\sqrt{x^2+y^2+z^2}}{z}\, dA = \iint_R \frac{\sqrt{x^2+y^2+(x^2+y^2)}}{\sqrt{x^2+y^2}}\, dA$$

$$= \iint_R \frac{\sqrt{2(x^2+y^2)}}{\sqrt{x^2+y^2}}\, dA = \iint_R \sqrt{2} \cdot dA$$

$$= \sqrt{2} \iint_R 1\, dA = \sqrt{2}\ (\text{area } R)$$

$$= \sqrt{2}\left(\pi(2)^2 - \pi(1)^2\right) = 3\sqrt{2}\,\pi \text{ , and}$$

$$\iint_{S} z\, dS = \iint_{R} z \cdot \sec\gamma \, dS$$

$$= \iint_{R} \cancel{z} \cdot \frac{\sqrt{x^2+y^2+z^2}}{\cancel{z}} \, dA$$

$$= \iint_{R} \sqrt{x^2+y^2+(x^2+y^2)} \, dA$$

$$= \iint_{R} \sqrt{2}\cdot\sqrt{x^2+y^2}\, dA$$

$$= \int_0^{2\pi}\int_1^2 \sqrt{2}\cdot\sqrt{r^2}\cdot r\, dr\, d\theta$$

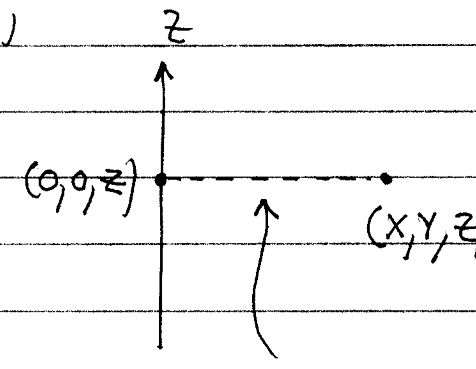
$$= \int_0^{2\pi}\int_1^2 \sqrt{2}\, r^2\, dr\, d\theta = \int_0^{2\pi}\left(\sqrt{2}\cdot\frac{1}{3}r^3\Big|_{r=1}^{r=2}\right) d\theta$$

$$= \int_0^{2\pi}\left(\frac{\sqrt{2}}{3}(8) - \frac{\sqrt{2}}{3}(1)\right) d\theta$$

$$= \int_0^{2\pi} \frac{7}{3}\sqrt{2}\, d\theta = \frac{7}{3}\sqrt{2}\,\theta\Big|_0^{2\pi}$$

$$= \frac{14}{3}\sqrt{2}\,\pi \; ;$$ then

$$\bar{z} = \frac{\frac{14}{3}\cancel{\sqrt{2}}\,\cancel{\pi}}{3\cancel{\sqrt{2}}\,\cancel{\pi}} = \frac{14}{9} \; ;$$



$$\text{M. of I.} = \iint_{S} (\text{distance})^2\, \delta\, dS$$

$$= \delta \iint_{S} (x^2+y^2)\, dS$$

distance $= \sqrt{x^2+y^2}$

$$= \delta \iint_R (x^2+y^2) \cdot \sec \gamma \, dA$$

$$= \delta \iint_R (x^2+y^2) \cdot \frac{\sqrt{x^2+y^2+z^2}}{z} \, dA$$

$$= \delta \iint_R (x^2+y^2) \frac{\sqrt{x^2+y^2+(x^2+y^2)}}{\sqrt{x^2+y^2}} \, dA$$

$$= \delta \iint_R (x^2+y^2) \cdot \sqrt{2} \frac{\cancel{\sqrt{x^2+y^2}}}{\cancel{\sqrt{x^2+y^2}}} \, dA$$

$$= \sqrt{2}\, \delta \int_0^{2\pi} \int_1^2 r^2 \cdot r \, dr \, d\theta = \sqrt{2}\, \delta \int_0^{2\pi} \int_1^2 r^3 \, dr \, d\theta$$

$$= \sqrt{2}\, \delta \int_0^{2\pi} \left(\frac{1}{4} r^4 \Big|_1^2 \right) d\theta$$

$$= \sqrt{2}\, \delta \int_0^{2\pi} \frac{15}{4} \, d\theta = \frac{15\sqrt{2}}{4} \delta \, \theta \Big|_0^{2\pi}$$

$$= \frac{15\sqrt{2}}{2} \delta \pi$$