

Notation and Terminology

A *set* is a collection of objects called *elements* of the set. Why not use the word “collection” or the word “set”, thereby having fewer words to worry about? “Collection” is a common word meaning is understood by most people. The use of the word “set” means that there is also a method to determine whether or not a particular object belongs in the set. We then say that the set is *well-defined*, it is easy to decide that the number 8 is not in the set consisting of the integers 1 through 10. In all, there are only five objects to consider and it is clear that 8 is not one of them by simply checking.

A basic problem here is now to indicate sets on paper and verbally. As seen above, a set could be defined with a phrase such as “the integers 1 through 5” and the speaker hopes that it is understood. So we use two common methods to write sets. The *roster notation* is a complete or implied listing of elements of the set. So $A = \{a, b, c, d\}$ and $B = \{2, 4, 6, 8, \dots, 40\}$ are examples of roster notation defining sets with 4 and 20 elements respectively. The ellipsis, “...”, is used to mean you fill in the missing elements in the obvious manner or pattern, as there are too many to actually list out on paper. The *set-builder notation* is used when the roster method is cumbersome or impossible. The set B above is described by $\{x | 2 \leq x \leq 40 \text{ and } x \text{ is even}\}$. The vertical bar, “|”, is read as “such that”. This notation is read aloud as “the set of x such that x is between 2 and 40 (inclusive) and x is even.” (A colon is used instead of |.) In set-builder notation, whatever comes after the bar describes the rule for determining whether or not an object is in the set. For the set $\{x | x \text{ is a real number}\}$, the roster notation would be impossible since there are too many reals to actually list out, explicitly or implicitly.

To discuss and manipulate sets we need a short list of symbols commonly used in print. We start with the symbols summarized in the following table.

Symbol	Meaning	Example	Read as:
\in	element of	$x \in A$	“ x an element of A ” or “ x in A ”
\subset	subset	$A \subset B$	“ A is a subset of B ” or “ A contained in B ”
\cup	union	$A \cup B$	“ A union B ”
\cap	intersection	$A \cap B$	“ A intersect B ”
$'$	complement	A'	“ A complement”

The first symbol, \in , indicates membership of an object in a particular set. The negation of this, nonmembership is often indicated by “ $x \notin A$,” (“ x is not in A ”). The subset relation, $A \subset B$, means every element of A is also an element of B . Logically, this would be: if $x \in A$ then $x \in B$. The union and intersection operators form new sets by the following rules. The set $A \cup B$ is defined to be $\{x | x \in A \text{ or } x \in B\}$ while $A \cap B$ is defined to be $\{x | x \in A \text{ and } x \in B\}$. Finally, the complement of a set consists of those objects that are not in the given set. This presents a minor problem. If $A = \{-3, \pi, \sqrt{2}\}$ then clearly I am not in A so should I be considered an element of A' ? No, I should not. I think. Underlying a discussion or argument involving sets is usually a large set called the *universe* of the discourse and is commonly denoted by U . This universe may be implied or stated. Operations involving union, intersection or complement are understood to be contained in this universe. For example, if we were discussing real numbers (so that our universe would be the set of reals) and the set A above with 3 elements, it is understood that A' consists of those real numbers not in A . This notation conveniently excludes me from the set A' .