1. (*pts*) Let $f(x) = \sin\frac{1}{x}$. Use the definition of the limit to prove that $\lim_{x\to 0} f(x)$ does not exist.

2. (*pts*) Find the interval of convergence for the following power series

$$\sum \frac{2^n}{n5^{n+1}}x^n$$

3. (*pts*) Let the sequence of functions $\{f_n\}$ be $f_n(x) = x - x^n$ for $x \in [0, 1]$.

   (a) Find $f(x)$ such that $\{f_n\} \to f$ on $[0, 1]$.

   (b) Using the definition, prove $\{f_n\}$ does not converge uniformly to f (found in part a) on $[0, 1]$.

4. (*pts*) Let the sequence of functions $\{f_n\}$ be $f_n(x) = \dfrac{1}{1+nx}$ for $x \in [2, \infty)$. Let $f(x) = 0$ for $x \in [2, \infty)$. Using the definition, prove $\{f_n\}$ converges uniformly to f on $x \in [2, \infty)$.

5. (*pts*) For $x \in [0,1]$, we have the following power series

$$\sqrt{1+x} = \sum \frac{(-1)^n (2n)!}{(1-2n)(n!)^2(4^n)} x^n.$$

Use this fact to build a power series for $\dfrac{1}{\sqrt{1-x^2}}$

6. (*pts*) Prove the following series converges uniformly on $\mathbb{R}$ to a continuous function

$$\sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$$

7. (*pts*) Use the definition of the derivative to prove the Quotient Rule.

8. (*pts*) Use the definition of the derivative to show $f(x) = |x| + |x+1|$ is not differentiable at $x = -1$.

9. (*pts*) Let the sequence of functions $\{f_n\}$ be $f_n(x) = \dfrac{nx}{1+n^2x^2}$ for $x \in [0,1]$. Let $f(x) = 0$ for $x \in [0,1]$. Prove $\{f_n\}$ does not converge uniformly to f on $x \in [0,1]$.

The following extra credit problem is OPTIONAL and you are advised to finish the rest of the test before trying this problem.

1. (*pts*) Prove that for all $x_0 \in \mathbb{R}$ there exists a sequence of rational numbers which converges to x_0. Also, there exists a sequence of irrational numbers which converges to x_0.