1. (*pts*) Prove that $f(x) = 5x^2 - 7$ is continuous on the interval $(1, \infty)$ by verifying the $\epsilon\delta$-property.

2. (*pts*) Prove that $f(x) = 5x^2 - 7$ is not uniformly continuous on the interval $(1, \infty)$ by definition.

3. (*pts*) Let $f(x) = \lfloor x \rfloor$ be the floor function (i.e. $f(x)$ is the largest integer less than x, which can be defined as $\lfloor x \rfloor := \max\{p \in \mathbb{Z} : p \leq x\}$). Define the function $g(x) = x - \lfloor x \rfloor$ to be the fractional part of x.

   (a) Sketch both functions $f(x)$ and $g(x)$ over the interval $[-4, 4]$, and determine where each function is discontinuous on $\mathbb{R}$.

   (b) Prove $g(x)$ is discontinuous at $x_0 = 0$ using the definition.

4. (*pts*) Prove that $\ln(x+1) = 1 - x$ is solvable.

5. (*pts*) Give an example of a continuous function $f(x)$ bounded on $[0, \infty)$ that does not obtain it's maximum value (i.e. $\nexists x^* \in [0, \infty)$ such that $f(x) \leq f(x^*) \quad \forall x \in [0, \infty)$).

6. (*pts*) Which are the following functions on the indicated domain are continuous and/or uniformly continuous or neither? Briefly justify your answer, using any theorem (in the book) you wish.

   (a) $f(x) = 2^{x^3}$ on $[-7, 5]$.

   (b) $g(x) = \dfrac{1}{x^3}$ on $(0, 1)$.

   (c) $h(x) = \begin{cases} -1 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$ on $(-2, 2)$.

7. (*pts*) Let f be a continuous function with domain (a, b). Prove that if $f(r) = 0$ for each rational number r in (a, b), then $f(x) = 0$ for all $x \in (a, b)$.

The following extra credit problems are OPTIONAL and you are advised to finish the rest of the test before trying these problems.

1. (*pts*) Prove that for all $x_0 \in \mathbb{R} \setminus \mathbb{Q}$ (i.e. the irrationals) there exists a sequence $x_n \subseteq \mathbb{Q}$ which converges to x_0.