# 185A Homework 8

**Question 1** Study the convergence of the sequence of functions $\{f_n\}$ with

$$f_n(z) = \sum_{k=0}^{n} \frac{1}{z^k}\,.$$

Hint, first describe the effect of the mapping $z \mapsto \frac{1}{z}$ on the unit disc.

**Question 2** Sterographic Projection and the Riemann Sphere: Consider the following mapping: Lay the complex plane $\mathbb{C}$ out on a tabletop and take a sphere $S^2$ with unit radius and put it on the origin. Call the top of this sphere the north pole. Now stand on the table and point a telescope at the north pole. To do this, you will have to sight through a point on the sphere (draw a picture) . Call this point the image of the stereographic projection $P : \mathbb{C} \to S^2$. Write a formula for a point $(X, Y, Z) \in S^2$ (where $X^2 + Y^2 + (Z-1)^2 = 1$) in terms of $z \in \mathbb{C}$. Also give the inverse, *i.e.* $z$ in terms of $(X, Y, Z)$. (A hint, look for similar triangles). Do you have an interpretation for $P(\infty)$? Try to show that circles and lines[1] in $\mathbb{C}$ are mapped to circles in $S^2$.

**Question 3** Develop and prove a ratio test and comparison test for the convergence of complex series.

**Question 4** Develop an asymptotic series expansion for the integral

$$I(t) = \int_0^\infty \frac{dx e^{-x}}{1 + (xt) + (xt)^2}\,,$$

valid when $t$ is small. You may compute just the first five terms in this series. Investigate how well your expansion works for explicit small values of $t$.

**Question 5** Find out what the group of angle preserving transformations for $\mathbb{R}^3$ is. Compare its dimensionality to the same group in two dimensions. (*Hint;* For "googlers", the word *conformal* might help.)

[1] The equation of a circle or line in $\mathbb{C}$ is $Az\bar{z} + \bar{B}z + B\bar{z} + C = 0$ for $B$ complex and $A$ and $C$ real. It helps to take $|B|^2 > C$.