Kakimizu Complexes of Alternating Knots of 11 and 12 Crossings

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To my friends, family, and Real Madrid.

Contents

Abstract

The Kakimizu complexes have been studied for various classes of links. O.Kakimizu initially found the Kakimizu complexes for knots with crossing numbers less than or equal to 10*. Hatcher and Thurston found the 0-skeleton of the Kakimizu complexes of* 2*-bridge links, while Sakuma later generalized this finding for special arborescent links, describing the Kakimizu complexes for the same. Banks provided a comprehensive proof of results previously announced by Hirasawa and Sakuma, explicitly describing the Kakimizu complexes of non-split, prime special alternating links.*

It is established that the Kakimizu complexes of prime, non-split alternating links contain a finite number of vertices. In this dissertation, we compute the Kakimizu complexes for all 11*-crossing prime alternating knots, explicitly describing each and primarily using the methods described above. Some remaining Kakimizu complexes for* 11*-crossing knots were then determined using Murasugi sums and the sutured manifold theory developed by Gabai, Scharlemann, Kakimizu, and others. Additionally, we apply these computational techniques to the first* 1000 *knots with* 12*-crossings, discussing potential obstructions to existing methodologies.*

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CHAPTER 1

Introduction

1.1. Important Concepts

1.1.1. Knots and Links.

A *knot* (denoted by K) in a space X (where $X = \mathbb{R}^3$ or $X = \mathbb{S}^3$) is defined as a smooth embedding of S^1 in X. For our discussions, we primarily focus on the case where $X = \mathbb{S}^3$. More broadly, a *link* L is a disjoint union of one or more knots, specifically a disjoint union of embedded copies of S^1.

Two knots K_1 and K_2 are considered equivalent if there exists a diffeomorphism $h : X \to X$ such that $h(K_1) = K_2$. This means the pairs (X, K_1) and (X, K') are diffeomorphic through the map h. Similarly, two links are equivalent if there is a specific ordering of their components and a diffeomorphism of X that maps the corresponding components of one link to those of the other. For instance, let $L_1 = K_1 \sqcup K_2 \sqcup \cdots \sqcup K_n$ and $L_2 = J_1 \sqcup J_2 \sqcup \cdots \sqcup J_n$ be two ordered links, where K_i and J_i are their respective components. We say L_1 and L_2 are equivalent if a diffeomorphism $h : X \to X$ exists such that $h(K_i) = J_i$ for all i.

The equivalence classes of knots and links are called *knot types or link types*. In this dissertation, we often use the terms "knot" or "link" to refer to a specific knot type or link type, respectively.

A *regular projection* is a mapping $p : \mathbb{R}^3 \to \mathbb{R}^2$ that ensures that each link component is projected to a closed curve in $\mathbb{R}^2$ such that all intersections are transverse with exactly two strands crossing. The image of a link L in $\mathbb{R}^2$ or $\mathbb{S}^2$ under this projection, along with over and under information at each crossing, is called a *knot or link diagram*.

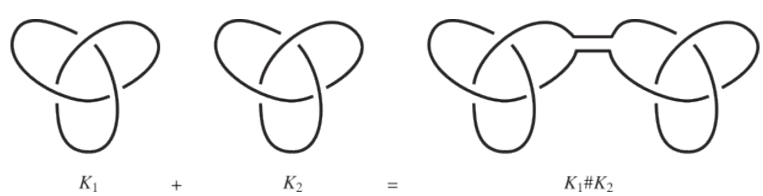


Figure 1.1

An *orientation of a knot K (or link L)* assigns a direction to K (or to each component of the link L). Two oriented knots (or links) can be summed, denoted by $K_1\#K_2$ (see Figure 1.1), and referred to as *the connected sum of* K_1 *and* K_2. This operation naturally provides an orientation on the resulting sum. A knot is called a *prime knot* if it is not the unknot (the knot with zero crossings) and if $K = K_1 \# K_2$ implies that either K_1 or K_2 is an unknot.

A link L, with at least two components, is said to be a *split link* if there exists a sphere $\mathbb{S}^2$ in $\mathbb{S}^3 - L$ separating $\mathbb{S}^3$ into two balls $\mathbb{B}^3$, each containing at least one component of L. A link L is called a *non-split* link if it is not a split link.

A knot diagram D of an oriented knot K is called *an alternating diagram* if, starting from any point on the diagram, every undercrossing is followed by an overcrossing, and vice versa. This alternating pattern continues as one traverses the knot. Similarly, a link diagram is *alternating* if the overcrossings and undercrossings alternate from any starting point in the link diagram. A knot K or a link L is said to be *alternating* if there exists an alternating knot or link diagram representing it.

A knot or link diagram D which realizes the minimum number of crossings (known as the *crossing number*) is referred to as a *reduced diagram.* A *special alternating diagram* D is a link diagram, such that applying Seifert's algorithm (see the following section and Figure 1.2) results in only innermost Seifert disks in $\mathbb{S}^2$. A link L is a *special alternating link* if it can be represented by a *special alternating diagram* D.

1.1.2. Seifert Surfaces.

A *Seifert surface* for an oriented link L is a connected compact oriented surface S in $\mathbb{S}^3$ such that the boundary of S is L. Every oriented link L has at least one corresponding Seifert surface. Indeed, for an oriented, non-split link L and a diagram D of that link, there exists an algorithm to construct a Seifert surface with L as its boundary.

The algorithm, known as *Seifert's algorithm*, proceeds from an oriented diagram D of a link L, produces disks, and connects them with twisted bands to form an oriented surface S with L as its boundary. The process is as follows:

- *Select a starting point*: Choose a point in the link diagram that is not at a crossing.
- *Traverse the link*: Move along the link according to its orientation until reaching a crossing.
- *Navigate crossings*: Instead of continuing along the link at each crossing, select the adjacent arc flowing away from the crossing.
- *Continue tracing*: Follow the newly chosen arc until the next crossing is encountered. Repeat this process until returning to the starting point.
- *Produce Seifert disks*: This traversal forms a closed oriented curve, called a *Seifert circle* that bounds an oriented disk in $\mathbb{S}^2$, known as a *Seifert disk*.

- *Continue creating disks*: Start from a point in the link diagram that hasn't been part of any previously formed disks, and continue forming disks until all parts of the diagram are accounted for.
- *Nested Seifert circles*: For nested Seifert circles, make the Seifert disks distinct by placing them at different heights relative to each other in $\mathbb{S}^3$.
- *Join the disks by bands*: Connect the disks by attaching twisted bands, replicating the overcrossings and undercrossings with either a clockwise or counterclockwise twist.
- *Resulting surface*: The final result is a Seifert surface with L as its boundary.

The *genus* g of a knot K (or a link L) is defined as the minimum genus of Seifert surfaces whose boundaries correspond to K (or L). The *canonical genus* g_c of a knot (or link) is the minimum genus of surfaces derived by applying Seifert's algorithm to oriented diagrams of the link L. Notably, Murasugi, [24], in 1958 and, independently, Crowell, [8], in 1959, proved that for an alternating link, $g = g_c$, with this genus realized by applying Seifert's algorithm on a reduced alternating diagram of L.

Concretely, for a non-split alternating link L, a minimal genus Seifert surface — a Seifert surface that realizes the genus g of the link — can be obtained by applying Seifert's algorithm on a reduced, oriented, alternating diagram. The genus g can be calculated using the Euler characteristic, and is given by the formula:

$$g = \frac{c - s - l + 2}{2},$$

where:

$c =$ number of crossings in the diagram D,

$s =$ number of Seifert disks,

$l =$ number of link components.

An illustration of Seifert's algorithm is provided in the diagram below (see Figure 1.2). For more information on knot theory and Seifert surfaces, see [1], [21], [27].

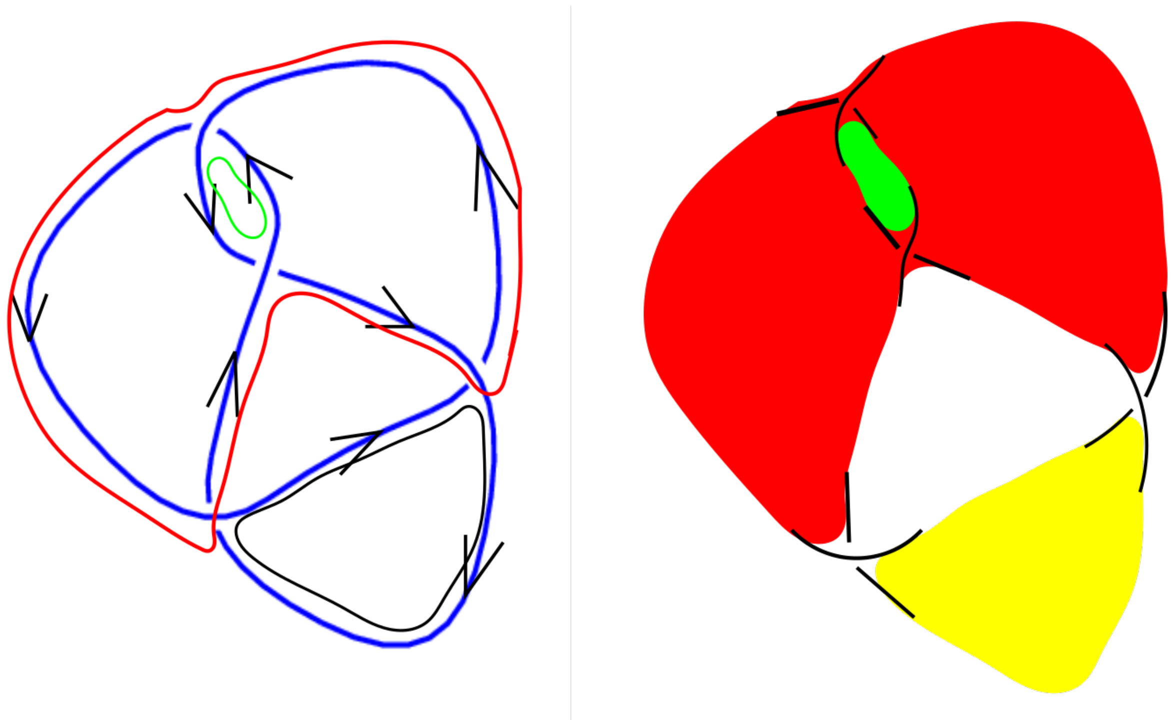


Figure 1.2

1.1.3. The Kakimizu complex of a link L.

Given an oriented non-split link L, applying Seifert's algorithm to a reduced oriented diagram of L yields a Seifert surface. We denote by $MS(L)$ the collection of minimal genus Seifert surfaces whose oriented boundaries correspond to the link L. Applying Seifert's algorithm guarantees the existence of a Seifert surface and ensures that there is at least one minimal genus Seifert surface for L. This confirms that $MS(L)$ is a non-empty set. Notably, when Seifert's algorithm is applied to a reduced oriented alternating diagram of L, it yields a minimal genus Seifert surface. Seifert surfaces are considered equivalent, if they are ambiently isotopic. Abusing notation, we will also denote the set of ambient isotopy classes of minimal genus Seifert surfaces by $MS(L)$.

The Kakimizu complex illustrates the structure of the collection of Seifert surfaces for a fixed link L in $\mathbb{S}^3$. It provides insights into whether two non-isotopic Seifert surfaces can be made disjoint in $\mathbb{S}^3$ while keeping the link L fixed throughout the isotopy. Let $N(L)$ be a regular neighborhood of L in $\mathbb{S}^3$, and define the link complement as $E(L) = \mathbb{S}^3 - N(L)$. For a non-split link L, the link complement $E(L)$ is an irreducible 3 manifold.

Further abusing notation, any Seifert surface S in $\mathbb{S}^3$ with L as its oriented boundary can be considered as a surface in $E(L)$ by $S \cap E(L) \subset E(L)$. *The Kakimizu complex* $\mathcal{MS}(L)$ *of a link* L is a finite-dimensional simplicial complex. Its 0-skeleton is $MS(L)$, the set of ambient isotopy classes of minimal genus Seifert surfaces of L. The 1-skeleton consists of vertices and edges, with an edge connecting two vertices v and v' if there exist representative surfaces S and S' (corresponding to v and v', respectively) such that S and S' are disjoint within $E(L)$. The Kakimizu complex is a flag complex: if there are $n+1$ vertices with edges connecting every pair, then there exists a n-simplex on these $n+1$ vertices. See [32, Theorem 5]. It is therefore determined by its 1-skeleton. The notation $\mathcal{I}S(L)$ is used for an analogous simplical complex in which vertices correspond to isotopy classes of incompressible Seifert surfaces of the link L.

1.2. Historical Background

1.2.1. Early developments.

1n 1966, Burde and Zieschang, [5], established that there exists a unique minimal genus Seifert surface (up to isotopy) for any fibered link. This result was reproven by W. Whitten in 1974, [35]. In the same paper, Whitten proved that the minimal genus Seifert surface for the double of a non-trivial non-cable knot is unique. In contrast, in 1977, Eisner, [9], showed that there exist infinitely many knots with infinitely many pairwise non-isotopic minimal genus Seifert surfaces. In 1978, Parris [25] demonstrated the existence of pretzel knots with infinitely many isotopy classes of incompressible Seifert surfaces.

In 1990, O.Kakimizu, [19, Theorem], showed that doubled knots have infinitely many non-isotopic incompressible Seifert surfaces of the same genus provided the knot is not fibered. Later, in 2012, Banks [4] computed Kakimizu complexes of connected sums of links, proving that $\mathcal{M}S(L_1\#L_2)$ is homeomorphic to $\mathcal{M}S(L_1)\times\mathcal{M}S(L_2)\times\mathbb{R}$, provided that L_1 and L_2 are non-split, non-fibered links, [4, Theorem 1.2]. Thus, for every such link $L=L_1\#L_2$, $\mathcal{M}S(L)$ is infinite.

1.2.2. Topological Properties of the Kakimizu complex.

Scharlemann and Thompson, in 1988 in the paper [30] showed that given two minimal genus Seifert surfaces S and S' for a knot K, there exists a sequence $S=S_0,S_1,\ldots,S_n=S'$ such that each S_i is a minimal genus Seifert surface and $S_i\cap S_{i-1}=\emptyset$ for all $i=1,2,\ldots,n$. By the definition of the Kakimizu complex, the isotopy classes of the surfaces $[S_i]$ are vertices in the Kakimizu complex. Moreover, $[S_i]$ and $[S_{i-1}]$ are either the same vertex (the surfaces are isotopic) or there is an edge between these vertices (since they can be made disjoint in the link exterior). Given any two vertices, v and v', in the Kakimizu complex, we can choose a representative of each vertex, say S and S', respectively and use the result of Scharlemann and Thompson to find a sequence of minimal genus Seifert surfaces, which are consecutively disjoint and “interpolate” between S and S'. This implies that there is a path in the Kakimizu complex that connects v to v', showing that the Kakimizu complex is connected.

In 1992, Kakimizu, [20], defined the Kakimizu complex of a knot or link and also proved that Kakimizu complexes are connected. Further, he conjectured that the Kakimizu complex is contractible for every knot. In 1994 Sakuma, [28], generalizing results of Hatcher and Thurston, [14], computed the Kakimizu complexes for a class of links called *special arborescent links.* Such links can be constructed from twisted bands via the operation of plumbing, which we will discuss later. What Sakuma showed was that for such a link every minimal genus Seifert surface can be isotoped to a plumbing of $2n$-twisted bands. He also proved that the Kakimizu complex is contractible for this class of links.

In 1997 Hirasawa and Sakuma, [15], showed how to compute the Kakimizu complex for special alternating links. They showed that the Kakimizu complex of a special arborescent link is homeomorphic to a ball, thus also proving that the Kakimizu complex is contractible for this class of links.

In 2010 Schultens, [32], proved that the Kakimizu complex of every knot is simply connected. This was the first breakthrough towards Kakimizu's contractibility conjecture.

In 2012 Schultens and Przytycki, [26], proved that the Kakimizu complex for a non-split oriented link is contractible. Subsequently, Johnson, Pelayo, and Wilson, [17] showed that the Kakimizu complex is *quasi-Euclidean.*

In 2009, Sakuma and Shackleton, [29], found that for atoroidal knots K, the distance $d(v, v')$ between two vertices in $\mathcal{MS}(K)$ (defined as the edge-length of the shortest path from v to v' in $\mathcal{MS}(K)$) is bounded by a quadratic function on the knot genus. Moreover, they found an upper bound for $i([S], [S'])$ (defined as the minimum number of components of $|S \cap S'|$ for Seifert surfaces isotopic to S, S' respectively). The number $i([S], [S'])$ is again bounded by a quadratic function on the knot genus.

In 2014, Wilson, [36], proved that the complement of a hyperbolic knot can contain only finitely many non-isotopic surfaces of a given genus. From this, it follows that the Kakimizu complex of a hyperbolic knot is finite. In 2019, Hass, Thompson, and Tsvietkova, [13], proved that, for alternating links, the number of genus g Seifert surfaces is bounded by a polynomial in g. This implies finiteness of Kakimizu complexes for alternating links. In 2022, Agol and Zhang, [2], proved that the Kakimizu complex exhibits a certain homogeneity, in that the dimension of maximal simplices in $\mathcal{MS}(K)$ is the same for all maximal simplices. (Any simplex lies in a maximal simplex, and all maximal simplices have the same dimension.) This dimension is thus a knot invariant.

1.2.3. Computations of Kakimizu complexes for various classes of knots.

In 1956 Seifert, [31], classified all 2-bridge knots in terms of rational numbers $\frac{p}{q}$, called the *2-bridge notation* of a knot. In 1985 Hatcher and Thurston, [14], described how to find all minimal genus Seifert surfaces for a 2-bridge knot. Using the continued fraction expansion of the 2-bridge notation, they showed that every Seifert surface for a 2-bridge knot is obtained as a sequence of plumbings of $2n$-twisted bands. The relevant twist numbers of 2-bridge knots can be found from the *even continued fraction expansion* of the 2-bridge notation p/q. For more detail, see [6].

In 1994, Sakuma, [28], provided an algorithm that completely described Kakimizu complexes of *special arborescent links*. We will not describe the computational tools developed by Sakuma here, but his algorithm underlies many of the calculations performed as part of this dissertation. As noted earlier, Sakuma also showed that the Kakimizu complex is contractible for special arborescent links.

In 2000, Kakimizu, [18], described Kakimizu complexes for all knots with at most ten crossings. For this purpose, he used the technique of *sutured manifold theory* developed by Gabai, Scharlemann, Kakimizu, and others. Kakimizu also used the methods of plumbing and Murasugi disks, and applied sutured manifold theory to determine Kakimizu complexes for more general links.

In 2022 Banks, [3], outlined an exact algorithm for computing Kakimizu complexes of special alternating links. Greene, Howie, Banks, and Hirasawa and Sakuma in [12], [16], independently showed that a minimal genus Seifert surface for a special alternating link L is isotopic to a surface obtained by applying Seifert's algorithm to a special alternating link diagram D. Thistlethwaite and Menasco, [23], proved that any two reduced, prime, oriented, alternating link diagrams of an alternating link could be connected by a sequence of *flypes*. An algorithm for computing the Kakimizu complex for a special alternating link was announced by Hirasawa and Sakuma in 1996, but only given explicitly by Banks in [3]. Recently, in 2023, Valdez-Sanchez, [33], determined the structure of Kakimizu complexes of genus 1 hyperbolic knots, $K \subset S^3$.

1.3. Overview

This dissertation concerns calculations of Kakimizu complexes for alternating 11-crossing knots and some 12-crossing knots. All calculations of Kakimizu complexes of 11 crossing knots and 12 crossing knots obtained in this dissertation are original. Several techniques are applied. Important concepts are introduced in Chapter 1. Chapter 2, entitled Preliminaries, covers the basics of sutured manifold theory which constitutes a key tool for calculating Kakimizu complexes and underlies several of the more specific techniques to calculate the same. Chapter 3, entitled Flypes and Plumbing, features an original result relating Seifert surfaces that differ by flypes to those that differ by plumbing.

Chapter 4, entitled The Kakimizu complexes of 11 crossing alternating knots, lists four classes of knots and tailored methods for calculating the Kakimizu complexes for each of these classes. The classes of knots considered are fibered knots, special alternating knots, 2-bridge knots, and knots arising as a plumbing of links with unique Seifert surfaces. The specific methods for calculating the Kakimizu complexes are only described in general terms, but the lists of knots falling into each of these classes are given, along with the results of the calculations. References are provided for background in each of these computational techniques. In Chapter 5, the general strategy used in this dissertation for computing all the Kakimizu complexes of 11 crossing alternating knots is summarized. The 11 crossing knots not already considered in Chapter 4 are listed and their Kakimizu complexes computed. This Chapter features original results pertaining to a general strategy for computing Kakimizu complexes along with ad hoc methods to compute the Kakimizu complexes of the remaining 11 crossing alternating knots.

Chapter 6 follows the general strategy used in the previous two chapters and considers 12 crossing alternating knots. The same four classes of knots (fibered, special alternating, 2-bridge, and those arising as a plumbing of two links with unique Seifert surfaces) are considered, along with the general strategy developed in Chapter 5 to compute Kakmizu complexes of several additional alternating 12 crossing knots.

CHAPTER 2

Preliminaries

2.0.1. Sutured Manifold Theory.

Much of this section is taken verbatim from the paper by Kakimizu, [18]. We refer to sections 1, 2, and 3 of this paper for details.

Below, we quote from [18, Section 1] for a brief explanation of the theory of *sutured manifolds* and *sutured surfaces*:

> A *sutured manifold* (M, γ) is a compact oriented 3 manifold M together with a subset $\gamma \subset \partial M$ which is a union of finitely many pairwise disjoint annuli. For each component of γ, a suture, i.e., an oriented core circle, is fixed, and $s(\gamma)$ denotes the set of sutures. Moreover, every component of $R(\gamma) = \partial M - \text{Int}\gamma$ is oriented so that the orientations on $R(\gamma)$ are coherent with respect to $s(\gamma)$. Let $R_+(\gamma)$ (resp. $R_-(\gamma)$) denote the union of those components of $R(\gamma)$ whose normal vectors point out of (resp. into) M. In the case that (M, γ) is homeomorphic to $(F \times [0, 1], \partial F \times [0, 1])$, where F is a compact oriented 2-manifold, (M, γ) is called a *product sutured manifold.*
>
> A properly embedded compact oriented 2-manifold (possibly disconnected) $S \subset M$ is said to be a γ*-surface* if S has no closed components, the oriented boundary ∂S is contained in $\text{Int}\gamma$ and isotopic to $s(\gamma)$ in γ. A γ-surface S is *parallel* to a surface in $R(\gamma)$ if there is an embedding $e : (S, \partial S) \times [0, 1] \to (M, \gamma)$ such that $e_0 = id : S \to S$ and $e_1(S) \subset R(\gamma)$: Note that $e_1(S)$ is a union of some components of $R(\gamma)$. A γ-surface S is *essential* if S is incompressible in M and not parallel to a surface in $R(\gamma)$. A γ*-isotopy* of M is an isotopy $\{h_t\}$ of M such that $h_0 = id$, $h_t|R(\gamma) = id$ and $h_t(\gamma) = \gamma$ for all $0 \leq t \leq 1$. Two γ-surfaces in M are *equivalent* if they are ambient isotopic to each other by a γ-isotopy. Let $\mathscr{E}(M, \gamma)$ denote the set of equivalence classes of essential γ-surfaces in M.

Lemma. (See [18, Lemma 1.1])

Let $(M,\gamma))$ be a sutured manifold, and let S be a γ-surface. Suppose that ∂M is connected and that S is parallel to a surface in $R(\gamma)$ by an embedding $e : (S, \partial S) \times [0,1] \to (M,\gamma)$ with $e_0 = id_S$ and $e_1(S) \subset R(\gamma)$. Then $e_1(S) = R_+(\gamma)$ or $e_1(S) = R_-(\gamma)$.

An important example of a sutured manifold is the *complementary sutured manifold*, we again quote from [18, Section 2, page 56]:

> Let $L \subset S^3$ be an (oriented) link and $S \subset E(L)$ a spanning surface for L. Let $(N(S),\delta) = (S \times [-1,1], \partial S \times [-1,1])$ be the product sutured manifold associated to S. The *complementary sutured manifold* for S is the sutured manifold $(M,\gamma) = (Cl(E(L) - N(S)), Cl(\partial E(L) - \delta))$ with $R_\gamma = R_+(\delta)$ and $R_+(\gamma) = R_-(\delta)$. If L is non-split, then $E(L)$ and M are irreducible. We also note that ∂M is connected if and only if so is S.

2.0.2. Product Decomposition. This section briefly explains an important operation on a sutured manifold called *product decomposition* and its relation to sutured surfaces. We again quote from [18, Section 1, page 51]:

> Let (M,γ) be a sutured manifold. A *product disk* $\Delta \subset M$ is a properly embedded disk such that $\partial\Delta$ intersects $s(\gamma)$ transversely in two points. For a product disk $\Delta \subset M$, we get a new sutured manifold (M',γ'), obtained by cutting M open along the disk Δ. This decomposition
>
> $$(M,\gamma) \xrightarrow{\Delta} (M',\gamma')$$
>
> is called a *product decomposition.*
>
> Let $S \subset M$ be an essential γ-surface. Suppose that M is irreducible. Then we can move S by a γ-isotopy so that $\partial S = s(\gamma)$ and that $S \cap \Delta$ is a single arc A connecting the two points of $\partial\Delta \cap s(\gamma)$. By cutting S along A, we obtain a γ'-surface $S_\Delta \subset M'$.

Lemma. (See [18, Lemma 1.3].)

Let $(M,\gamma) \xrightarrow{\Delta} (M',\gamma')$ be a product decomposition. Suppose that M is irreducible and $\partial M'$ is connected. Then for each essential γ-surface $S \subset M$, the γ'-surface $S_\Delta \subset M'$ is also essential. Moreover if two essential γ-surfaces S and $S' \subset M$ are equivalent, then so are S_Δ and S'_Δ.

Under the assumptions of this lemma, Kakimizu defines a map

$$\mathscr{E}_\Delta : \mathscr{E}(M,\gamma) \to \mathscr{E}(M',\gamma'), \qquad\qquad [S]_\gamma \to [S_\Delta]_{\gamma'}.$$

Lemma. (See [18, Proposition 1.4].)

Let $(M,\gamma) \xrightarrow{\Delta} (M',\gamma')$ be a product decomposition. Suppose that M is irreducible and $\partial M'$ is connected. Then the map $\mathscr{E}_\Delta : \mathscr{E}(M,\gamma) \to \mathscr{E}(M',\gamma')$ is bijective.

For a fibered link L with fiber S, the complementary sutured manifold (M_S,γ) is a product sutured manifold homeomorphic to $(S \times I, L \times I)$.

Lemma. (See [18, Proposition 1.5].)

Let $(M,\gamma) \xrightarrow{\Delta} (M',\gamma')$ be a product decomposition. Suppose that M is irreducible and that (M',γ') has two components (M_1,γ_1) and (M_2,γ_2). Suppose further that (M_2,γ_2) is a product sutured manifold and ∂M_1 is connected. Then the map $\mathscr{E}_{\Delta,1} : \mathscr{E}(M,\gamma) \to \mathscr{E}(M_1,\gamma_1)$ is bijective.

Lemma. (See [18, Lemma 1.7] as well as [34].)

Let X be a connected Haken 3-manifold such that ∂X is a union of incompressible tori. Let Y be a compact irreducible 3-submanifold of X (possibly disconnected) such that each component of $Fr(Y)$ is a properly embedded incompressible surface in X. Let F and F' be two properly embedded orientable incompressible surfaces in X (possibly disconnected), which satisfy the following properties (1) − (4). Then there is an isotopy $\{h_t\}$ of X keeping Y fixed so that $h_0 = id$ and $h_1(F) = F'$.

1. *$F \cup F' \subset X - Y$.*
2. *Each component of ∂X contains at most one component of ∂F and F has no closed components.*
3. *There is a homotopy $f : F \times [0,1] \to X$ such that $f_0 = id : F \to F$ and $f_1 : F \to F'$ is a homeomorphism and $f(\partial F \times [0,1]) \subset \partial X$.*
4. *There is no component of F which is parallel to a component of $Fr(Y)$.*

An important application of the above Lemma is stated in [18, Section 2, Proposition 2.1], where a *spanning surface* is simply a Seifert surface that is not necessarily of minimal genus:

Let L be a non-split link, S a connected incompressible spanning surface for L and (M, γ) the complementary sutured manifold for S. Let $\mathcal{JS}(L)$ denote the set of equivalence classes of incompressible spanning surfaces for L. Let $\mathcal{JS}(L, S)$ denote the set of $\eta \in \mathcal{JS}(L)$ such that $\eta \neq [S]$ and there is a representative $F \in \eta$ with $F \cap S = \emptyset$. Then the inclusion $M \subset E(L)$ induces a bijection

$$\mathcal{E}(M, \gamma) \to \mathcal{JS}(L, S), [F]_\gamma \mapsto [F].$$

2.0.3. Murasugi sums and Plumbings.

The following section, taken from [18, Section 2] introduces a fundamental operation on surfaces known as the *Murasugi sum*, along with the concept of *plumbing*:

An oriented surface $\Sigma \subset S^3$ is a *Murasugi sum* of compact oriented surfaces Σ_1 and $\Sigma_2 \subset S^3$ if there are 3-balls V_1 and $V_2 \subset S^3$ satisfying the following property:

$$V_1 \cup V_2 = S^3, \qquad V_1 \cap V_2 = \partial V_1 = \partial V_2, \qquad \Sigma_i \subset V_i \qquad (i = 1, 2),$$

$$\Sigma = \Sigma_1 \cup \Sigma_2 \qquad \text{and} \qquad D = \Sigma_1 \cap \Sigma_2 \text{ is a } 2n-\text{gon}.$$

When D is a 4-gon, the Murasugi sum is called a *plumbing* of Σ_1 and Σ_2. Put $L = \partial\Sigma$, $L_i = \partial\Sigma_i$, $S = \Sigma \cap E(L)$ and $S_i = \Sigma_i \cap E(L_i)$. Then we will also say that S is a *Murasugi sum* of S_1 and S_2. Note that $\Sigma' = (\Sigma - D) \cup D'$ is an oriented surface with $\partial\Sigma' = L$ where $D' = \partial V_1 - Int(D)$. We will say that Σ' (resp. $S' = \Sigma' \cap E(L)$) is the *dual* of Σ (resp. S') or the *outer plumbing*, in which case we refer to the original plumbing as the *inner plumbing*. Note that Σ' (resp. S') is also a Murasugi sum of Σ'_1 and Σ'_2 (resp. S'_1 and S'_2) where $\Sigma'_i = (\Sigma_i - D) \cup D'$ and $S'_i = \Sigma'_i \cap E(L_i)$ $(i = 1, 2)$. D' is also called a *dual* of D. Gabai showed that the Murasugi sum operations hold the following natural properties:

Lemma. [18, Proposition 2.3]

i) *S is of minimal genus if and only if so are both S_1 and S_2.*
ii) *L is a fibered link with fiber S if and only if both L_1 and L_2 are fibered links with fibers S_1 and S_2 respectively.*

Notation: For Seifert surfaces S_1, S_2 of links L_1, L_2 we will denote the Murasugi sum along a disk D by $S_1 \cup_D S_2$ and its dual by $S_1 \cup_{D^c} S_2$.

DEFINITION 2.0.1. If the surface $S = S_1 \cup_D S_2$ is given as a plumbing, then the *deplumbing* of S along D is the surface S_1.

CHAPTER 3

Flypes and Plumbing

Computing Kakimizu complexes of special alternating links fundamentally involves identifying what are called *essential flypes* on a reduced special alternating diagram of a special alternating link L. A *flype circle* in a link diagram D of a link L is a simple closed curve that intersects one crossing along with two additional points of L in the diagram D. A *flype* is a $180°$ rotation of the disk bounded by a flype circle. This produces an alternate diagram of L. See Figure 3.1, 3.2, and 3.3. Subtle variations exist concerning the definition of flype: In this dissertation we will specifically assume that a flype circle intersects a Seifert surface in a single arc, as in Figure 3.3.

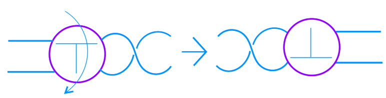


FIGURE 3.1

Given a link L, a flype on a diagram D of L is called an *essential flype* if applying Seifert's algorithm to the resulting diagram D' provides a surface S' that is not isotopic to the original surface S obtained from D via Seifert's algorithm. We say that the diagram D' is obtained by *applying a flype* to the diagram D.

The following definition of $*$-product below is taken from a paper by Cromwell, [7, Section 1]:

> The Seifert circles of a diagram can be separated into two kinds: a circle is of *type I* if it does not contain any other Seifert circles, otherwise it is of *type II*. Let $D \subset \mathbb{R}^2$ be a link diagram, and suppose that C is one of its type II Seifert circles. Then C separates $\mathbb{R}^2$ into components U, V such that $U \cup V = \mathbb{R}^2$ and $U \cap V = \partial U = \partial V = C$. Let D_1 and D_2 be the diagrams formed from $D \cap U$ and $D \cap V$ by adding suitable arcs from C. If both $(U - C) \cap D \neq \emptyset$ and $(V - C) \cap D \neq \emptyset$ then the type II Seifert circle C *decomposes* D *as a* $*$*-product* of the two diagrams D_1 and D_2. This is written as $D = D_1 * D_2$.

An alternating knot is *special* if no nested circles appear in Seifert's algorithm. Cromwell proves in [7, Theorem 1] that *every homogeneous link* (a class that includes alternating links as a subset) is a $*$-product of special alternating links. Moreover, Cromwell shows that applying Seifert's algorithm to a reduced alternating diagram of a link yields a minimal genus Seifert surface. This surface can be decomposed into a tower of spanning surfaces corresponding to special alternating links L_i. Consequently, the $*$-product of these links manifests as Murasugi sums of the spanning surfaces on L_i.

The diagrams with flype circles are inspired from [3, Figure 3].

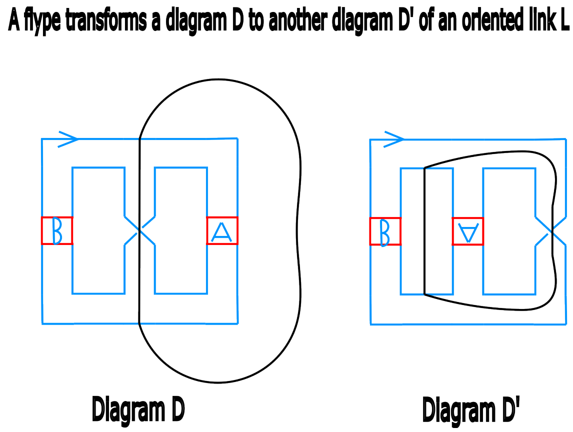


FIGURE 3.2

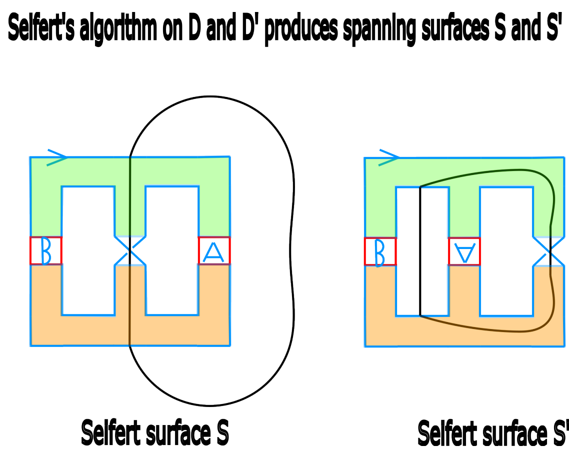


FIGURE 3.3

The following is an original result:

THEOREM 3.0.1. Let L be a link such that a diagram D of L contains an essential flype. Then there exists a plumbing disk E such that the surfaces S and S' (obtained by applying Seifert's algorithm before and after the flype) are dual surfaces with respect to E.

A motivating example for this result is the knot $K = 7_4$. This knot is a special alternating knot and a 2-bridge knot. The Kakimizu complex of the knot $K = 7_4$ consists of two vertices and one edge. Viewing it as a special alternating link, we obtain one minimal genus Seifert surface by applying Seifert's algorithm to a reduced alternating diagram D (say, the standard diagram) of 7_4, and the other by applying Seifert's algorithm to a diagram D' obtained from D after the application of the flype, see Figure 3.4, [3]. When the knot $K = 7_4$ is viewed as a 2-bridge knot, the two minimal genus Seifert surfaces correspond to inner and outer plumbings of two twisted bands (with 2 full twists), see Figure 3.5. The special alternating diagram D of 7_4 is isotoped to the 2-bridge diagram by introducing two Reidemeister-2 moves on the top left and the bottom right corners of the diagram. The three crossings, namely, the flype crossing and the two consecutive crossings introduced by the Reidemeister-2 moves, provide the plumbing disk's boundary. The isotopy of the knot 7_4 (special alternating diagram of 7_4 to the 2-bridge diagram of 7_4) extends to the surface S, making it isotopic to the surface with inner plumbing (with respect to the 2-bridge diagram) of 7_4, see Figure 3.6.

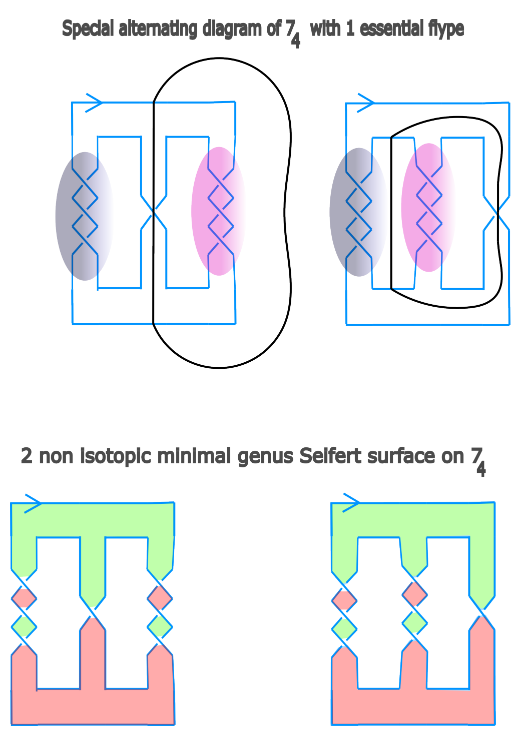


FIGURE 3.4

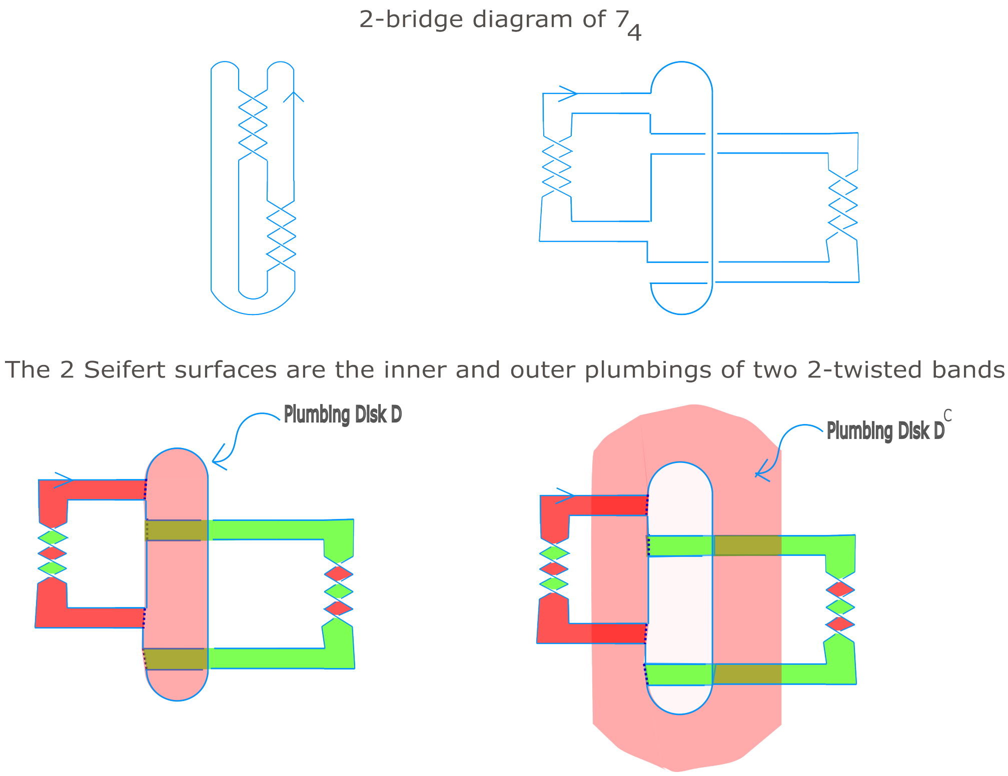


FIGURE 3.5

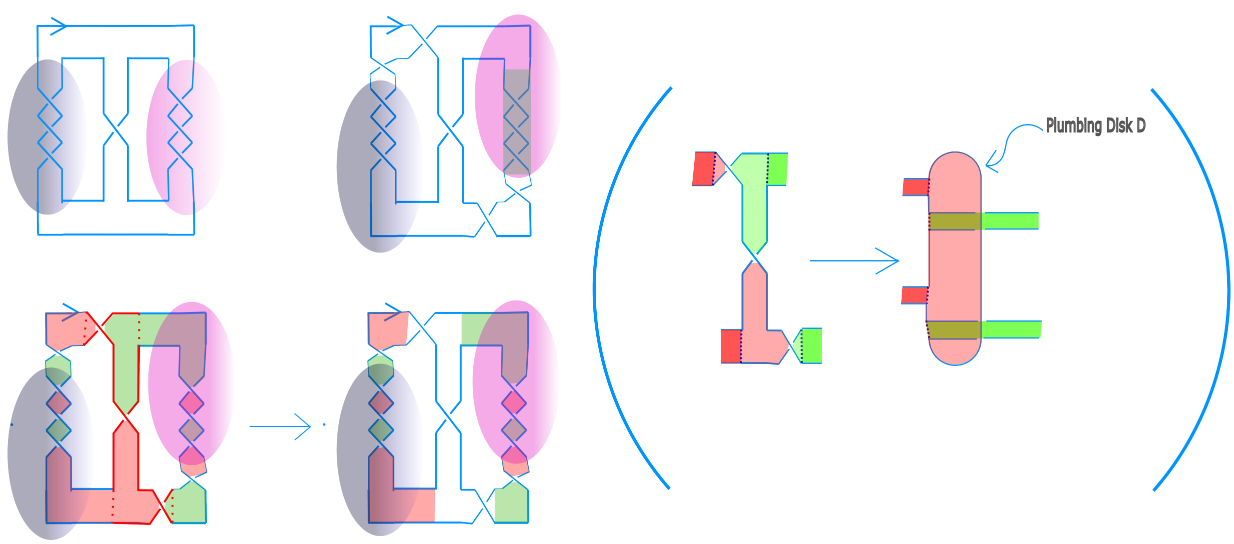


FIGURE 3.6

While the example of 7_4 is distinctive and unique, it highlights the general correspondence between flype circles and plumbing spheres.

PROOF. To prove the theorem, we generalize the above example of $K = 7_4$. We obtain, as in the example, a diagram as shown in Figures 3.7, 3.8, 3.9, 3.10, below, which illustrate the proof of the theorem. The first diagram shows a generic flype, with a flype crossing along with the corresponding Seifert surfaces obtained by applying Seifert's algorithm to the respective diagrams (Figure 3.7). Denote the components of L inside and outside the flype circle by A and B. The difference between the two surfaces is that the Seifert surface S obtained before the flype is the union of A and B with the front of A matching up to the front of B. Whereas the Seifert surface S' obtained after the flype is also the union of A and B, but with B rotated by $180°$, and so the front of A matches up with the back of B.

By applying a pair of Reidemeister-2 moves, as pictured in Figure 3.8, we establish the existence of the plumbing disk E, a portion of the Seifert surface "between" the locations for the Reidemeister-2 moves. This exhibits S as the sum $S = S_1 \cup_E S_2$ where E is the plumbing disk. In this inner plumbing the front of A is matching up with the front of B. See Figure 3.8.

Likewise, considering S' as pictured on the top left in Figure 3.9, we can reverse the flype and isotope S' along. This results in portions of S' being layered on top of each other. However, we now see that an analogous pair of Reidemeister moves in the central portions of the diagrams in the bottom row of Figure 3.9 exhibits S' as the corresponding outer plumbing $S' = S_1 \cup_{E^c} S_2$. In the outer plumbing, the front of A matches up with the back of B. □

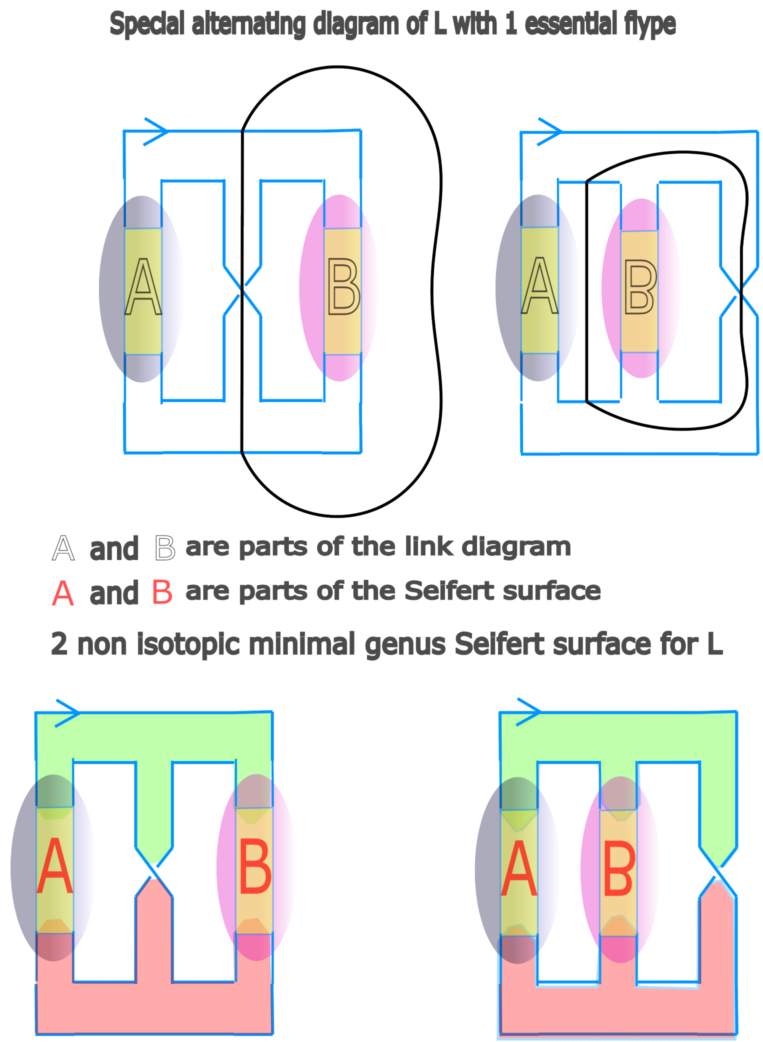

Special alternating diagram of L with 1 essential flype
A
B
A
B
A and B are parts of the link diagram
A and B are parts of the Seifert surface
2 non isotopic minimal genus Seifert surface for L
A
B
A
B

FIGURE 3.7

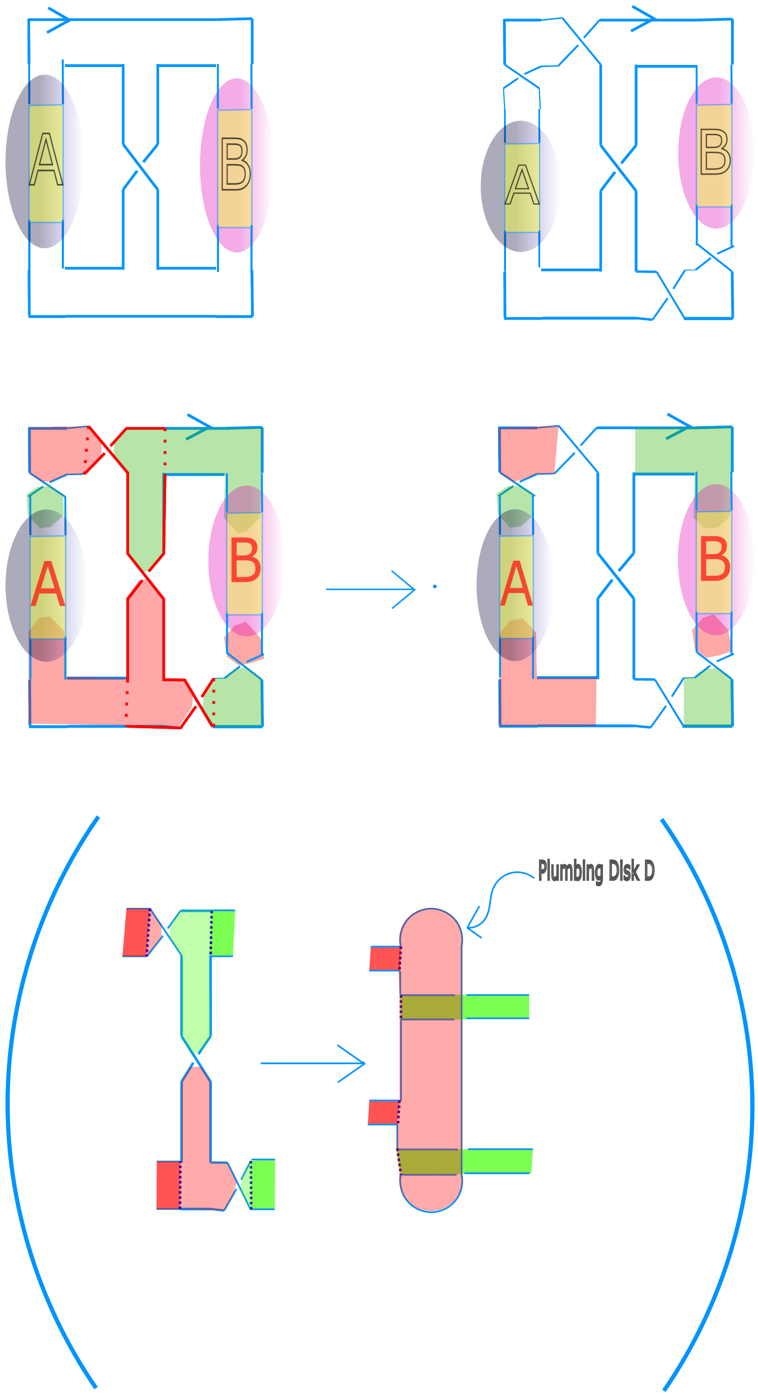


FIGURE 3.8

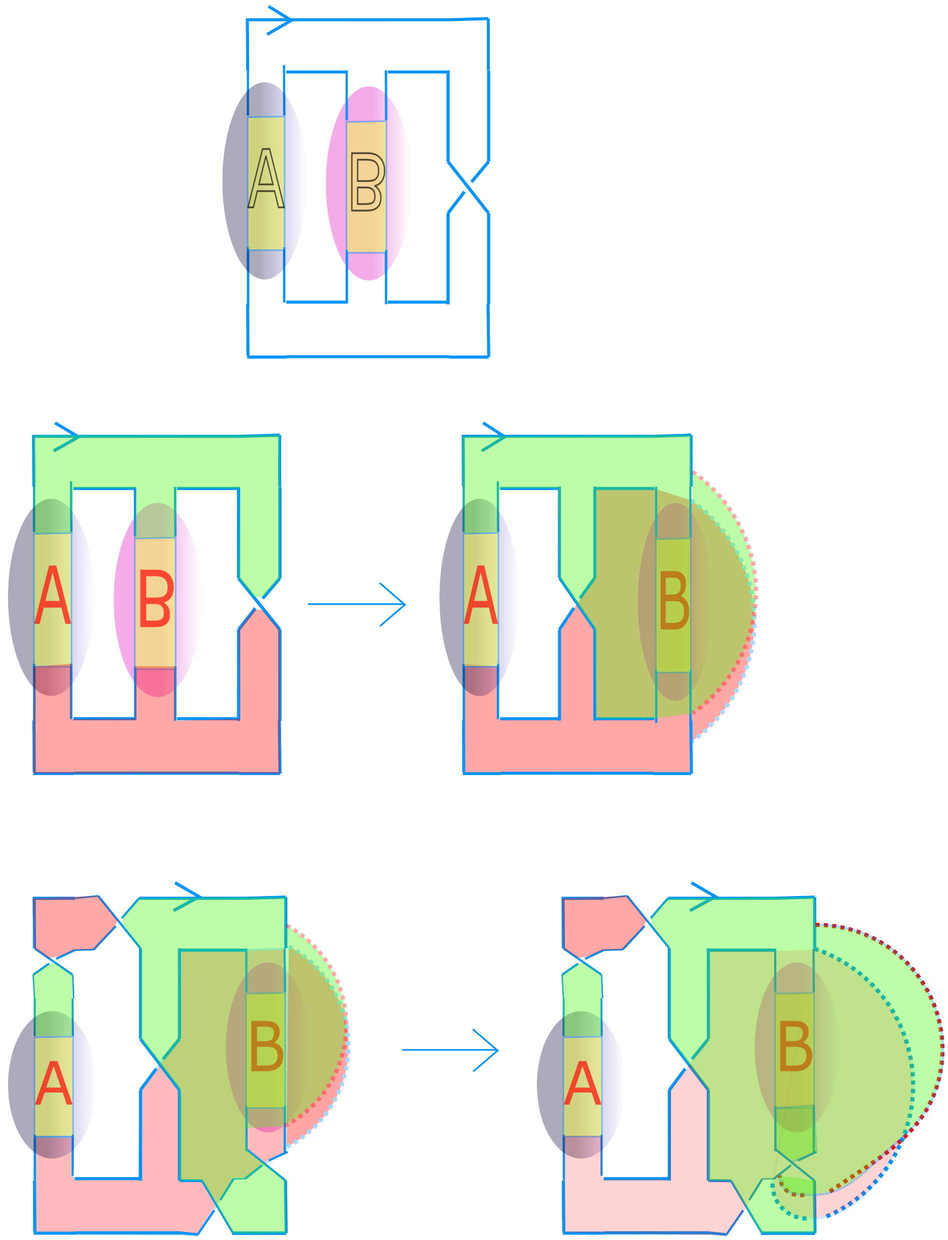


Figure 3.9

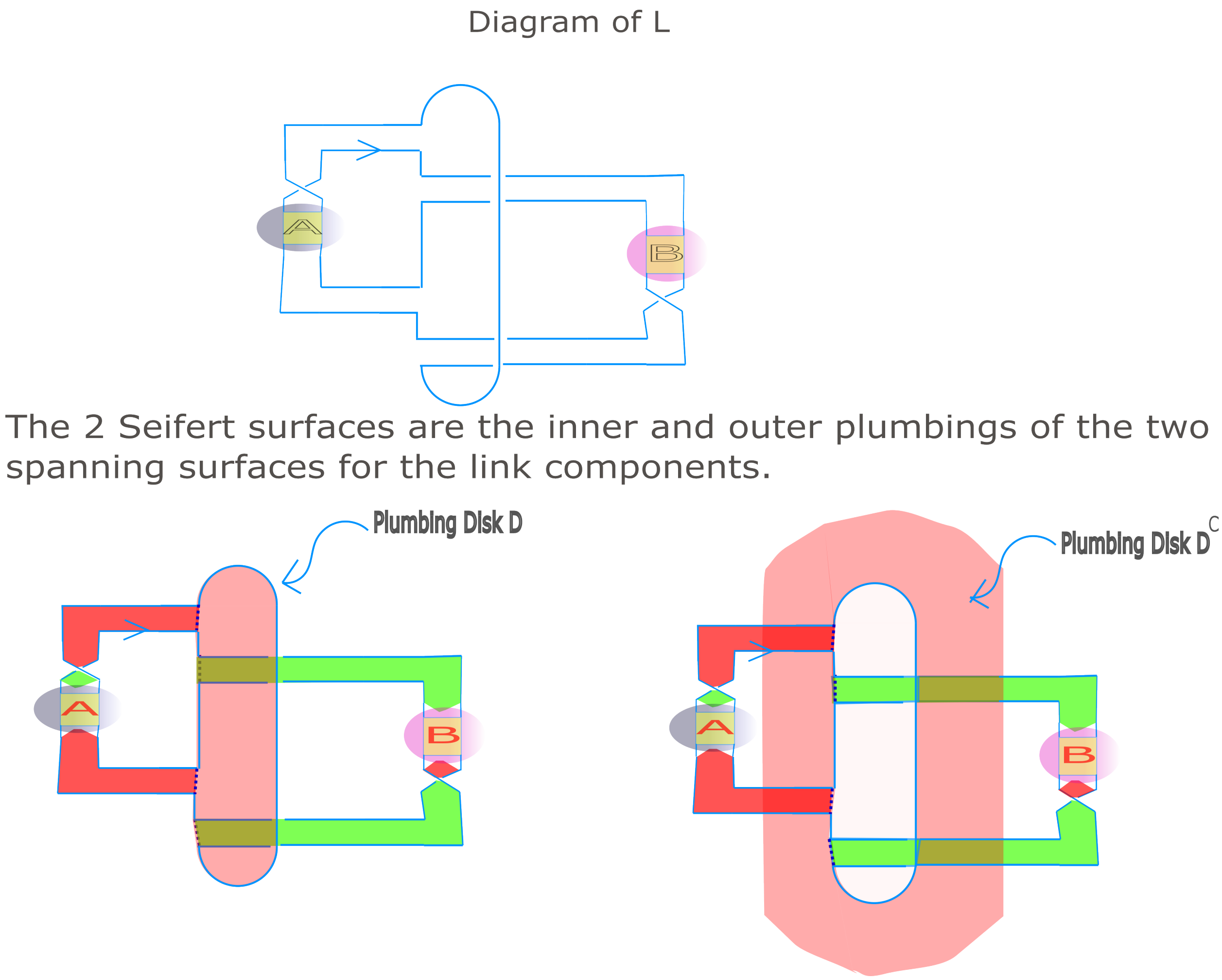


Diagram of L
A
B
The 2 Seifert surfaces are the inner and outer plumbings of the two spanning surfaces for the link components.
Plumbing Disk D
A
B
Plumbing Disk D^C
A
B

FIGURE 3.10

CHAPTER 4

The Kakimizu complexes of 11 crossing alternating knots

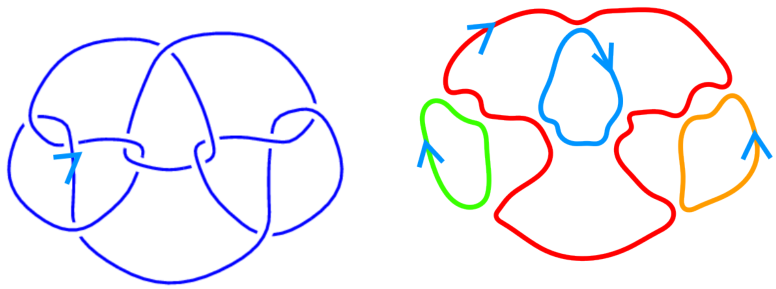
4.1. The Kakimizu complex of a fibered knot

In 1972, Whitten demonstrated in [35] that for a fibered link L, there exists a unique (up to isotopy) incompressible spanning surface, which can be identified as the unique vertex of the Kakimizu complex. Consequently, we have $\mathcal{M}S(L) = \{[S]\}$, indicating that the Kakimizu complex of a fibered knot is a singleton. In 1983, Gabai, [11], showed that the Murasugi sum of two links, L_1 and L_2, is fibered if and only if both L_1 and L_2 are fibered. See also [18, Proposition 2.3]. Cromwell's work, [7], on homogeneous links, cited above, therefore reduces the problem of deciding whether or not an alternating link is fibered to deciding whether or not a special alternating link is fibered. An algorithm to do so was given by Banks, in [3, Corollary 5.11]. Here is a description of her algorithm:

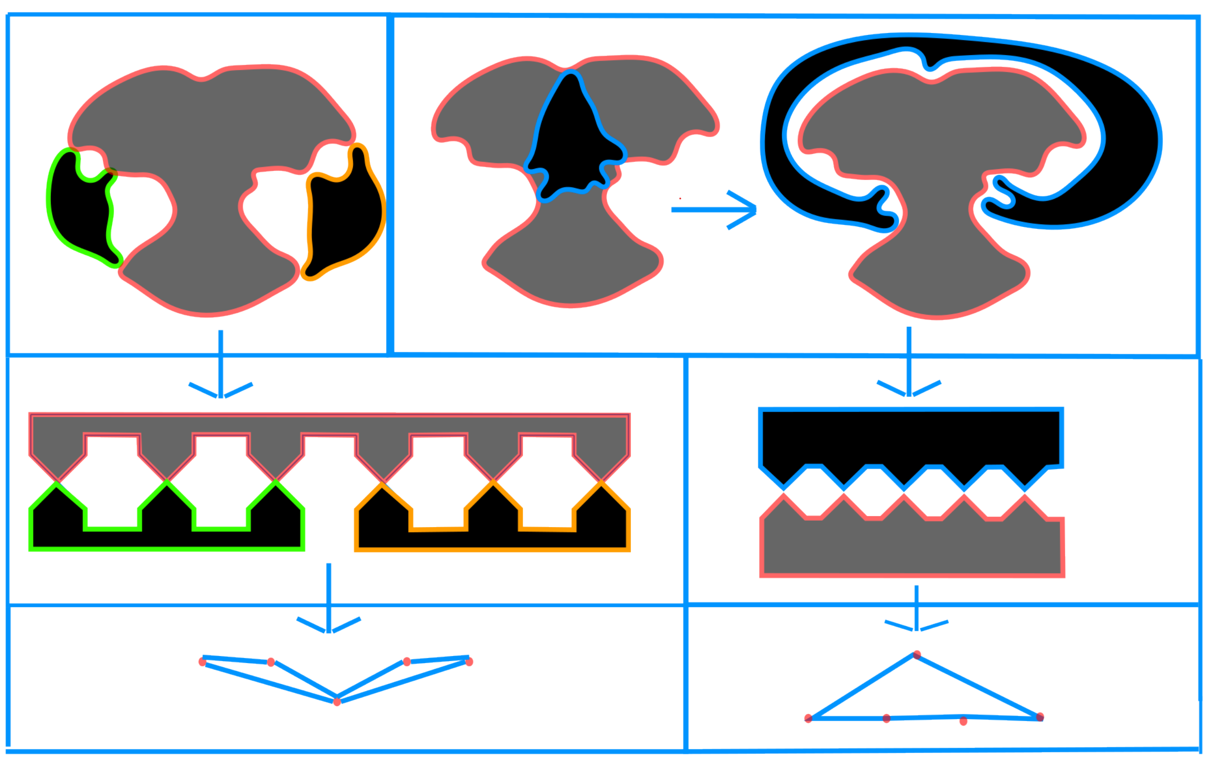
Let D be a reduced special alternating diagram of a special alternating link L. After applying the first stages of Seifert's algorithm, choose $S(D)$ to be a coloring of $S^2 - L$ into black and white regions, such that each Seifert disk corresponds to a black region. Define $G(D)$ to be the planar graph where each white region corresponds to a vertex, and edges represent crossings. A special alternating link L is fibered if we can simplify the graph $G(D)$ to a single vertex using the following moves:

- Delete all simple loops (edges with both endpoints on the same vertex).
- Repeatedly contract all edges for which if one of their endpoints is a vertex of valence 2.

A demonstration to determine whether an alternating knot is fibred. For the alternating knot $K = 11_{14}$, applying Seifert's algorithm reveals that the Murasugi summands of K are two special alternating links. We apply the fibredness algorithm to each summand, and if each is fibred, then by Gabai's lemma, [18, Proposition 2.3], the knot K is fibred.



The two Murasugi summands are special alternating links. We apply the fibredness algorithm to each summand.



Every edge has a vertex with valence 2. Contracting each edge results in a single point •

Since every component is fibered, the Murasugi sum is fibered so 11_{14} is a fibered knot

FIGURE 4.1

We derive the set of fibered knots from KnotInfo [22].

The list of fibered links for 11 crossing alternating links is as follows:

$$11_3, 11_5, 11_7, 11_9, 11_{14}, 11_{15}, 11_{17}, 11_{19}, 11_{22}, 11_{24}, 11_{25}, 11_{26}, 11_{28}, 11_{33}, 11_{34}, 11_{35}, 11_{40}, 11_{42},$$

$$11_{44}, 11_{47}, 11_{51}, 11_{53}, 11_{55}, 11_{57}, 11_{62}, 11_{66}, 11_{68}, 11_{71}, 11_{72}, 11_{73}, 11_{74}, 11_{76}, 11_{79}, 11_{80}, 11_{81},$$

$$11_{82}, 11_{83}, 11_{86}, 11_{88}, 11_{92}, 11_{96}, 11_{99}, 11_{106}, 11_{108}, 11_{109}, 11_{112}, 11_{113}, 11_{121}, 11_{125}, 11_{126}, 11_{127},$$

$$11_{128}, 11_{129}, 11_{131}, 11_{139}, 11_{142}, 11_{146}, 11_{147}, 11_{151}, 11_{156}, 11_{157}, 11_{158}, 11_{159}, 11_{160}, 11_{162}, 11_{163},$$

$$11_{164}, 11_{170}, 11_{171}, 11_{174}, 11_{175}, 11_{176}, 11_{177}, 11_{179}, 11_{180}, 11_{182}, 11_{184}, 11_{189}, 11_{194}, 11_{196}, 11_{203},$$

$$11_{206}, 11_{209}, 11_{215}, 11_{216}, 11_{217}, 11_{218}, 11_{221}, 11_{223}, 11_{228}, 11_{231}, 11_{232}, 11_{233}, 11_{239}, 11_{248}, 11_{250},$$

$$11_{251}, 11_{252}, 11_{253}, 11_{254}, 11_{255}, 11_{257}, 11_{259}, 11_{261}, 11_{264}, 11_{266}, 11_{267}, 11_{268}, 11_{269}, 11_{274}, 11_{277},$$

$$11_{281}, 11_{282}, 11_{284}, 11_{286}, 11_{287}, 11_{288}, 11_{289}, 11_{293}, 11_{300}, 11_{301}, 11_{302}, 11_{305}, 11_{306}, 11_{308}, 11_{314},$$

$$11_{315}, 11_{316}, 11_{326}, 11_{330}, 11_{332}, 11_{346}, 11_{348}, 11_{350}, 11_{351}, 11_{367}.$$

The Kakimizu complex of a fibered link is a single vertex.

4.2. The Kakimizu complex of a special alternating knot

In 2012, Banks [3] presented her explicit algorithm for computing the Kakimizu complex of a prime, non-split, oriented, special alternating link L. Banks' algorithm rests on the work of Thistlethwaite and Menasco, [23], who proved the *flyping conjecture*, which states: "Given any two reduced alternating diagrams D_1 and D_2 of an oriented, prime link L, the diagram D_1 can be transformed to D_2 by applying a sequence of flypes." It also draws on a result of Greene, [12, Corollary 1.3], who demonstrated the following: "A Seifert surface for a special alternating link L has minimal genus if and only if it is obtained by applying Seifert's algorithm to a special alternating diagram of L."

The algorithm was originally announced in 1990, by Hirasawa and Sakuma [15]. Banks, [3], Greene, [12], and others have since independently characterized Seifert surfaces for special alternating knots. Since the Kakimizu complex is a flag complex, the 1-skeleton completely determines the Kakimizu complex of the link L.

Banks' algorithm provides an intricate way to capture all possible essential flypes in a graph so that complementary regions correspond to flypes that can be performed independently of each other (the graph is called the "θ-graph"). Her algorithm applies to all special alternating knots (by showing that it equals a "maximal simplex"). We provide an illustration of this algorithm for the knot 11_{237} and a list of 11 crossing alternating knots that fall into this class along with their Kakimizu complexes. See Figure 4.2.

$K = 11_{237}$ is a special alternating knot. We compute the Kakimizu complex to illustrate the algorithm. We begin with the vertex represented by $(0,1,0)$. See the diagram in Figure 4.2, where the θ-graph is computed. The maximal simplex containing the vertex $(0,1,0)$ is given by:

$$(0,1,0) \to (0,0,1) \to (1,0,0) \to (0,1,0).$$

Thus for $K = 11_{237}$, the Kakimizu complex is a triangle with these three vertices.

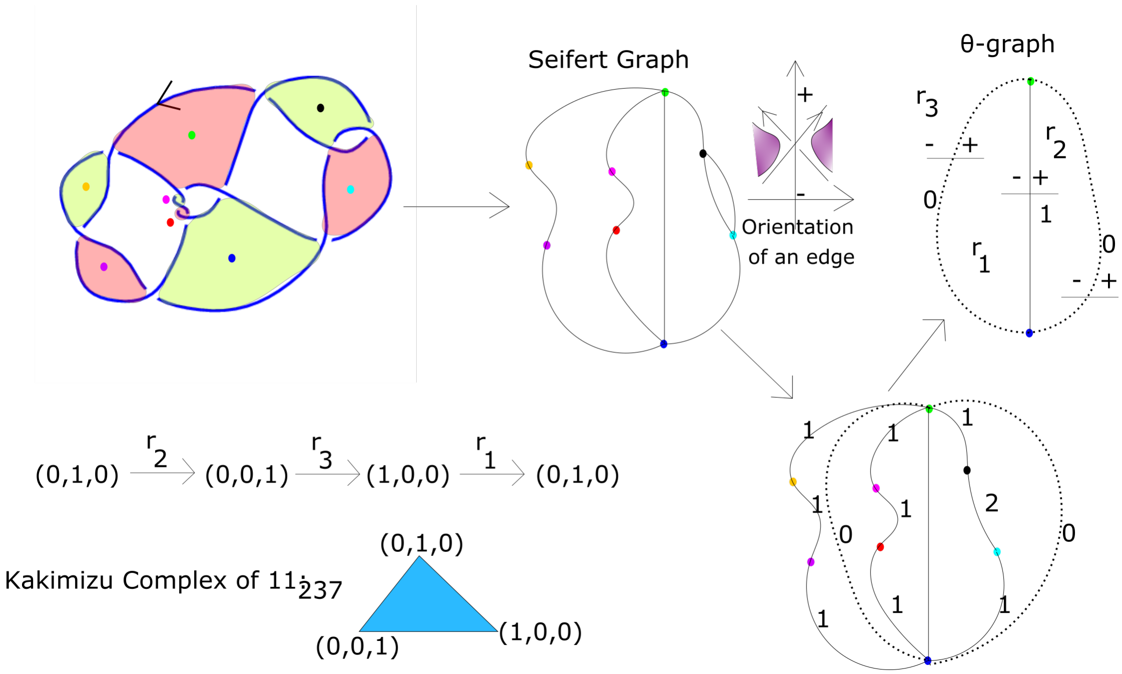


FIGURE 4.2. The Kakimizu complex of 11_{237}. The knot 11_{237} is a special alternating knot.

The list of special alternating links among 11-crossing alternating knots (together with their Kakimizu complexes) is as follows:

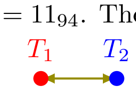
- $$K = 11_{43}, 11_{123}, 11_{124}, 11_{200}, 11_{227}, 11_{240}, 11_{241}, 11_{244}, 11_{245}, 11_{263}, 11_{291}, 11_{292},$$

  $$11_{298}, 11_{299}, 11_{318}, 11_{319}, 11_{320}, 11_{329}, 11_{338}, 11_{354}, 11_{361},$$

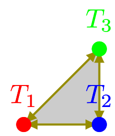
  are spanned by a minimal genus Seifert surface, unique up to isotopy.

  The Kakimizu complexes of these knots consist of a single vertex.

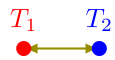
- $K = 11_{94}$. The θ-graph contains 2 regions. The Kakimizu complex is



- $K = 11_{237}$. The θ-graph contains three regions.



- $K = 11_{340}$. The θ-graph contains 2 regions. The Kakimizu complex is:



4.3. The Kakimizu complex of a 2-bridge knot

Hatcher and Thurston, [14] provide an algorithm to find all Seifert surfaces of a given 2-bridge knot. The algorithm also provides information that, using work of Sakuma, [28, Proposition 4.7], to decide whether or not two such surfaces can be isotoped to be disjoint. The algorithm builds on Conway's, [6], correspondence between 2-bridge knots and rational numbers.

Example:

Consider the knot $K = 11_{13}$. Without going into detail, we will mention that K is a 2-bridge knot with bridge notation 28/61 and the following continued fraction expansion:

$$28/61 = \cfrac{1}{2 - \cfrac{1}{-6 - \cfrac{1}{-2 - \cfrac{1}{2}}}}$$

We use the notation:

$$28/61 = [2, -6, -2, 2].$$

The knot can be isotoped to a 2-bridge diagram (with respect to the height function), appearing as a plumbing of three Hopf links and one link with three full twists, corresponding to the continued fraction $[2, -6, -2, 2]$. And the algorithm devised by Hatcher and Thurston shows that this plumbing provides the only minimal genus Seifert surface for K.

The knot 11_{192} is also a 2-bridge knot. It too corresponds to a rational number with the continued fraction expansion indicated in Figure 4.3. Figure 4.3 exhibits the knot as a plumbing, but this inner plumbing admits outer plumbings, corresponding to alternate Seifert sufaces:

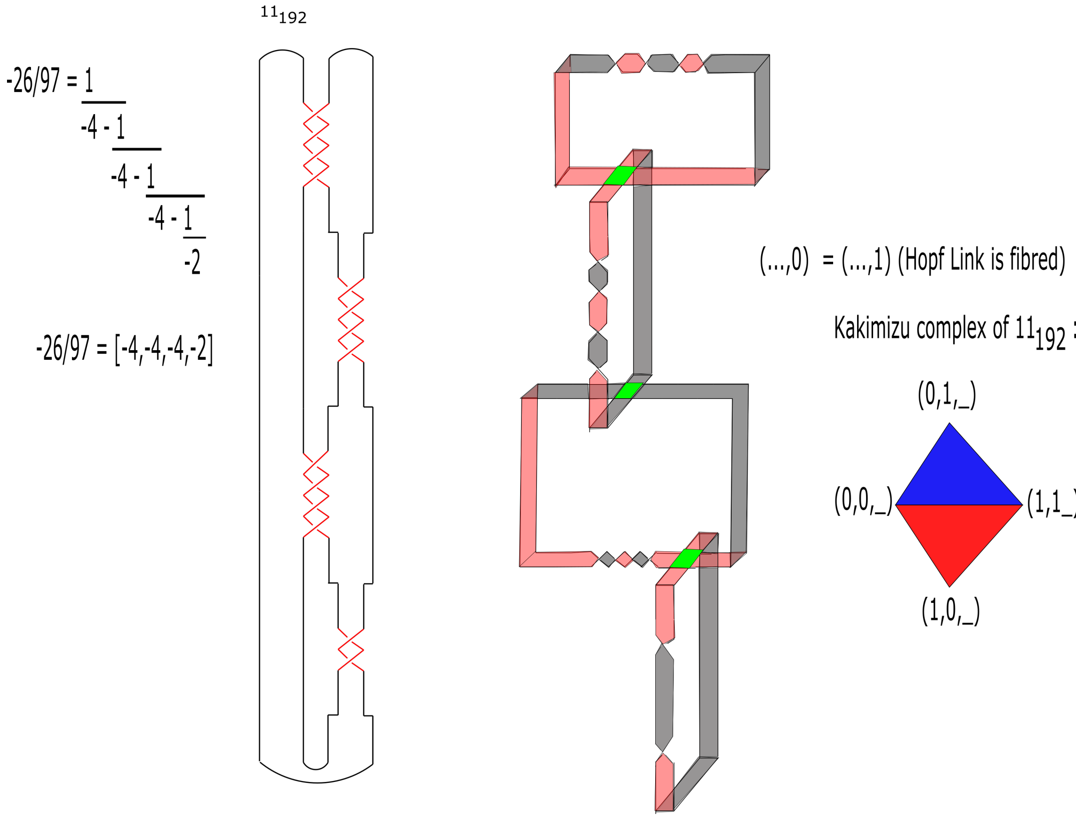


FIGURE 4.3. The Kakimizu complex of 11_{192}. The knot 11_{192} is a 2-bridge knot.

The list of the Kakimizu complexes of 2-bridge 11-crossing knots is as follows:

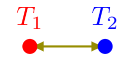
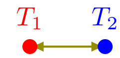
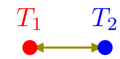
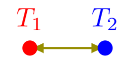
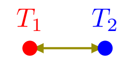
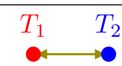
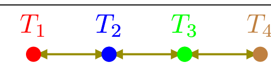
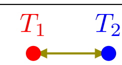
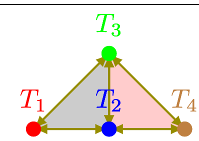
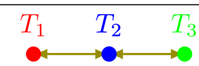
- $$K = 11_{13}, 11_{59}, 11_{65}, 11_{75}, 11_{77}, 11_{84}, 11_{85}, 11_{89}, 11_{90}, 11_{91}, 11_{93}, 11_{110}, 11_{111},$$
  $$11_{117}, 11_{120}, 11_{140}, 11_{144}, 11_{178}, 11_{183}, 11_{185}, 11_{188}, 11_{190}, 11_{193}, 11_{195}, 11_{204}, 11_{205},$$
  $$11_{207}, 11_{208}, 11_{211}, 11_{220}, 11_{224}, 11_{225}, 11_{230}, 11_{234}, 11_{242}, 11_{246}, 11_{247}, 11_{307}, 11_{309},$$
  $$11_{334}, 11_{339}, 11_{342}, 11_{355}, 11_{358}, 11_{364},$$

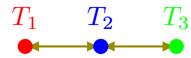
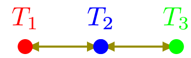
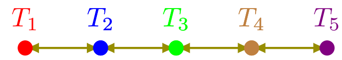
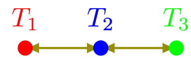
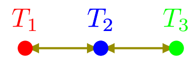
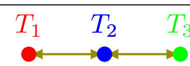
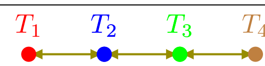
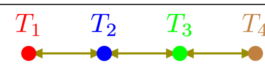
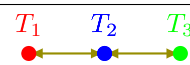
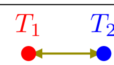
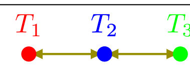
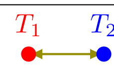
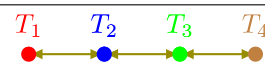
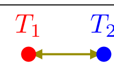
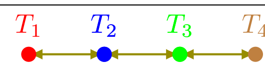
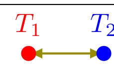
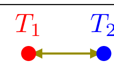
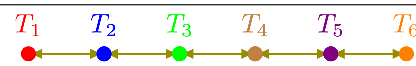
  are spanned by a unique (up to isotopy) minimal genus Seifert surface.

  The Kakimizu complexes of these knots consist of a single vertex.

  T

  
- The list of 2-bridge 11-crossing knots with non-trivial Kakimizu complexes.

Knots	2-bridge notation	Even continued fraction expansion	Kakimizu complex
11_{95}	$33/73 = -40/73$	$[-2, -6, -4, -2]$	
11_{98}	$18/47$	$[4, -4, -2, 2]$	
11_{119}	$64/109$	$[2, -4, -4, 2]$,	
11_{145}	$22/83$	$[4, 4, -2, 2]$,	
11_{154}	$37/67 = -30/67$	$[-2, 4, -4, -2]$	
11_{166}	$45/59 = -14/59$	$[-4, 4, -2, -2]$	
11_{186}	$39/95 = -56/95$	$[-2, -4, -2, -2, -4, -2]$	
11_{191}	$19/83 = -64/83$	$[-2, -2, -2, -4, -4, -2]$,	
11_{192}	$71/97 = -26/97$	$[-4, -4, -4, -2]$	
11_{210}	$16/73$	$[4, -2, -4, -2]$,	

Knots	2-bridge notation	Continued fraction expansion	Kakimizu complex
11_{226}	$20/71$	$[4, 2, -4, 2]$,	
11_{229}	$55/71 = -16/71$	$[-4, 2, -4, -2]$,	
11_{235}	$49/71 = -22/71$	$[-4, -2, -2, -2, -4, -2]$,	
11_{236}	$29/99 = -70/99$	$[-2, -2, -4, -2, -4, -2]$,	
11_{238}	$53/65 = -12/65$	$[-6, -2, -4, -2]$,	
11_{243}	$49/69 = -20/69$	$[-4, -2, -6, -2]$,	
11_{311}	$61/79 = -18/79$	$[-4, 2, -2, -4]$,	
11_{333}	$14/65$	$[4, -2, -2, 4]$,	
11_{335}	$17/75 = -58/75$	$[-2, -2, -2, -4, -2, -4]$,	
11_{336}	$11/59 = -48/59$	$[-2, -2, -2, -2, -4, -4]$,	
11_{337}	$63/89 = -16/89$	$[-6, -2, 4, 2]$,	
11_{343}	$27/31 = -4/31$	$[-8, -4]$,	
11_{356}	$55/79 = -24/79$	$[-4, -2, -2, -4, -2, -2]$,	
11_{357}	$27/91 = -64/91$	$[-2, -2, -4, -4, -2, -2]$,	
11_{359}	$43/53 = -10/79$	$[-6, -2, -2, -4]$,	
11_{360}	$47/57 = -10/59$	$[-6, -4, -2, -2]$,	
11_{363}	$29/35 = -6/35$	$[-6, -6]$,	
11_{365}	$35/51 = -16/51$	$[-4, -2, -2, -2, -2, -4]$,	

4.4. The Kakimizu complex of the plumbing of two links with unique spanning surfaces

Kakimizu uses sutured manifold theory to describe plumbings. To employ his techniques, we provide relevant portions of [18, Section 3]:

> A *marked sutured manifold* (M, γ, A) is a sutured manifold (M, γ) together with a properly embedded arc $A \subset R(\gamma)$, and we call A *a mark on* (M, γ). If there is a product disk $\Delta \subset M$ with A as an edge, then (M, γ, B) is also a marked sutured manifold, where B is the opposite edge of A, and we call B an *opposite mark* of A relative to Δ.

Kakimizu in [18, Lemma 3.3] of his paper also proves:

> *Let (M, γ, A) be a marked sutured manifold. Suppose that M is irreducible and each component of $R(\gamma)$ is incompressible. If there is a product disk with A as an edge, then the ambient isotopy types of such disks are unique, and hence so are the isotopy types in $R(\gamma)$ of opposite marks of A.*

In this context, we note that if L is a non-split link, then $\mathcal{E}(L)$ and the complementary sutured manifold, (M, γ) associated with a Seifert surface S, are irreducible 3-manifolds. Moreover, if S is incompressible, it follows that $R(\gamma)$, the boundary of the complementary sutured manifold (M, γ), is also incompressible.

Kakimizu further observes:

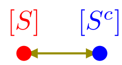
> Suppose that S is a plumbing of S_1 and S_2 where S_i is a spanning surface for a link L_i $(i = 1, 2)$. We call $D = S_1 \cap S_2$ the *plumbing disk*. Let (M_1, γ_1) and (M_2, γ_2) be the complementary sutured manifolds for S, S_1 and S_2 respectively. Let I_1 be a core arc of D relative to the embedding $D \subset S_1$, i.e. I_1 is a properly embedded arc in S_1 such that D is a regular neighborhood of I_1 in S_1. Push out I_1 from S_1 to the side on which S_2 is attached, and consider this arc A_1 to be properly embedded in $R(\gamma_1)$. Thus we get marked sutured manifolds (M_1, γ_1, A_1). In the same way we also get (M_2, γ_2, A_2). These markings correspond to the way of plumbing S_1 and S_2.

The following theorem is adapted from [18].

Theorem 4.4.1. [18, Theorems 3.12 and 3.15]

Let L be a non-split, prime, alternating link, and D be a reduced alternating, oriented diagram of L. Let S be the Seifert surface obtained by applying Seifert's algorithm on the diagram D. Let S be plumbing of S_1 and S_2, unique minimal genus Seifert surfaces for links L_1 and L_2, respectively. Assume that S_1 and S_2 are not fibered. Let (M_i, γ_i, A_i) and (M_i, γ_i, A'_i) $(i = 1, 2)$ be the marked sutured manifolds for $S = S_1 \cup_{D_0} S_2$ and $S^c = S_1 \cup_{D_0^c} S_2$, the dual of S respectively. Then

- $MS(L) = \{[S], [S^c]\}$ and the Kakimizu complex is



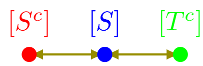
provided that the following conditions hold:

– There is no product disk with A_1 or A'_1 in M_1.

– There is no product disk with A_2 or A'_2 in M_2.

- Let us assume that there is a product disk in (M_1, γ_1, A_1) with A_1 as an edge. Let B_1 be the opposite mark of the product disk. Let $S = S_1 \cup_{D_0} S_2$. Suppose $T = S_1 \cup_{E_0} T_2$ is the plumbing with respect to the marking B_1. (Note that $S = T$.)

  Then $MS(L) = \{[S], [S^c], [T^c]\}$ and the Kakimizu complex is



provided that the following conditions hold:

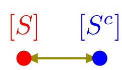
– there is no product disk with A'_1 or B'_1 in M_1.

– There is no product disk with A_2 or A'_2 in M_2.

Using this theorem we obtain the list of the Kakimizu complexes of 11-crossing alternating knots K, each spanning a surface S which is a plumbing of links with unique incompressible surfaces. There are three such knots:

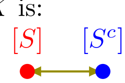
- $K = 11_{45}$. Let $S = S_1 \cup_{D_0} S_2$ on links L_1 and L_2.

  The link L_1 is a special alternating link with a unique minimal genus Seifert surface S_1, and S_2 is the 4-half twisted band, a unique minimal genus Seifert surface. There are no product disks with the markings as an edge. Therefore, the Kakimizu complex of K is:

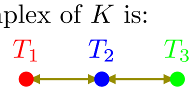


- $K = 11_{280}$.

  $S = S_1 \cup_{D_0} S_2$ with $L_1 = (2, 2, 2)$ and $L_2 = (3, 1, 1)$ are Pretzel knots. Both S_1 and S_2 are unique minimal genus spanning surfaces for L_1 and L_2 respectively. Then the Kakimizu complex of K is:



- $K = 11_{325}$. $S = S_1 \cup_{D_0} S_2$ with S_1 a 4-half twisted band and S_2 is a unique spanning surface on a special alternating link L_2. There is a product disk with the marking as an edge. The Kakimizu complex of K is:



CHAPTER 5

The Kakimizu complexes of some special knots

5.1. General Strategy

Recall that the Kakimizu complex is a flag complex, hence determining the 1-skeleton of the Kakimizu complex is sufficient to construct the entire complex. Additionally, the Kakimizu complex is connected, therefore, starting from a minimal genus Seifert surface S for the knot K, we can proceed as follows: Identify all vertices adjacent to $[S]$, decide whether they are adjacent to each other, and repeat this process for all newly found vertices, deciding, at each step, whether a newly found vertex is adjacent to any of the previously considered vertices.

Let L be a non-split prime alternating link with $n > 0$ crossings. A surface constructed via Seifert's algorithm applied to a n-crossing link has genus at most $n/2$, so this is an upper bound for the minimal genus, g, of a Seifert surface for such an L. Hass, Thompson, and Tsvietkova, [13], obtained an explicit (polynomial in g) bound for the number of genus g Seifert surfaces for such an L. This implies that the Kakimizu complex for a link L is finite and that this process will eventually terminate.

Moreover, any alternating link K represented by an alternating diagram D can be expressed as a $*$-product of special alternating links, denoted as $K = *(L_i)$. When we apply Seifert's algorithm to an oriented reduced diagram D of K, the resulting surface S can be viewed as a Murasugi sum of spanning surfaces S_i corresponding to special alternating links L_i.

Our general method for computation of the Kakimizu complexes of 11 crossing alternating knots is as follows:

- Firstly, check the table KnotInfo [22] to see whether or not the given knot K is fibered. If it is, its Kakimizu complex is a single vertex. See Section 4.1.
- Second, check the table KnotInfo[22]) to see if the given 11-crossing alternating link is a 2-bridge knot. If this is the case, we proceed as in Section 4.3.

  If K is neither fibered, nor a 2-bridge knot, choose a diagram D of S to construct a Seifert surface S for K via Seifert's algorithm.
- If K is a special alternating link, apply Banks' algorithm to compute the Kakimizu complex of K as in Section 4.2.

- If K is a Murasugi sum of two links with unique minimal genus Seifert surfaces, proceed as in Section 4.4.
- If K is neither fibered, nor a 2-bridge knot, nor a special alternating knot, identify all fibered components on S (if any) that are Murasugi sums with a surface S_1. If S_1 is not fibered, has a unique Seifert surface, and is Murasugi summed with the fibered surfaces identified earlier, then the Kakimizu complex of K is a single vertex, by Theorem 5.1.2 proved below.
- After excluding all cases above, the only remaining 11 crossing alternating knots are 11_{61}, 11_{103}, and 11_{201}. Calculations of the Kakimizu complexes of these knots are included below.

Let L be a non-split prime link and let S be a minimal genus Seifert surface for L. As before, $\mathcal{IS}(L, S)$ denotes the set of isotopy classes of surfaces that can be made disjoint from S in the link complement. In the context of the Kakimizu complex, surfaces in $\mathcal{IS}(L, S)$ (up to isotopy) correspond to vertices in the complex that share an edge with the vertex $[S]$.

Kakimizu, [18, Proposition 2.4], proves the following (as a consequence of a Theorem of Gabai, [10, Theorem 1.9]):

Theorem 5.1.1. *Let L be a non-split oriented link and S a connected incompressible spanning surface for L. Suppose that S is a Murasugi sum of S_1 and S_2, where each S_i is a spanning surface for an oriented link L_i (for $i = 1, 2$). Suppose further that L_2 is a fibered link with fiber S_2. Then L_1 is non-split, and S_1 is connected and incompressible. Moreover, there is a bijection*

$$\phi : \mathcal{IS}(L, S) \to \mathcal{IS}(L_1, S_1).$$

The following is an original result:

Theorem 5.1.2. Let L be a non-split, oriented link, and let S be a connected minimal genus Seifert surface for L. Suppose that S is a Murasugi sum $S = S_1 \cup_{D_1} F_1 \cup_{D_2} F_2 \cdots \cup_{D_n} F_n$, where S_1 is the unique minimal genus Seifert surface of a link $L^1 = \partial S_1$, and each F_i is a spanning surface for an oriented link L_i (for $i = 1, \ldots, n$). Suppose further that the F_i are fibered surfaces, and each D_i intersects S_1. Then we have $\mathcal{MS}(L) = [S]$.

Proof. $\mathcal{MS}(L^1, S_1) = \emptyset$.

Consider the surface

$$S_1^1 = S_1 \cup_{D_1} F_1.$$

Since F_1 is a fibered surface for L_1, Theorem 5.1.1 (by discarding incompressible spanning surfaces that are not minimal genus) gives us

$$\mathcal{M}S(\partial S_1^1, S_1^1) = \mathcal{M}S(\partial L_1, S_1) = \emptyset.$$

Next recursively define the surfaces,

$$S_1^k = S_1 \cup_{D_1} F_1 \cup_{D_2} F_2 \cup \cdots \cup_{D_k} F_k$$

for every $1 \le k \le n$. By repeating the same argument, we obtain that

$$\mathcal{M}S(\partial S_1^k, S_1^k) = \mathcal{M}S(\partial S_1^{k-1}, S_1^{k-1}) = \emptyset$$

for each k.

For $k = n$, this leads to

$$\mathcal{M}S(L, S) = \emptyset \implies \mathcal{M}S(L) = [S].$$

□

The list of the Kakimizu complexes of 11-crossing alternating knots K that arise in this way and hence have a unique incompressible spanning surface, and therefore a trivial Kakimizu complex, is given here:

- The following knots K span a unique (up to isotopy) minimal genus Seifert surface.

$$K = 11_1, 11_2, 11_4, 11_6, 11_8, 11_{10}, 11_{11}, 11_{12}, 11_{16}, 11_{18}, 11_{20}, 11_{21}, 11_{23}, 11_{27}, 11_{29}, 11_{30}, 11_{31},$$

$$11_{32}, 11_{36}, 11_{37}, 11_{38}, 11_{39}, 11_{41}, 11_{46}, 11_{48}, 11_{49}, 11_{50}, 11_{52}, 11_{54}, 11_{56}, 11_{58}, 11_{60}, 11_{63}, 11_{64},$$

$$11_{67}, 11_{69}, 11_{70}, 11_{78}, 11_{87}, 11_{97}, 11_{100}, 11_{101}, 11_{102}, 11_{104}, 11_{105}, 11_{107}, 11_{114}, 11_{115}, 11_{116},$$

$$11_{118}, 11_{122}, 11_{130}, 11_{132}, 11_{133}, 11_{134}, 11_{135}, 11_{136}, 11_{137}, 11_{138}, 11_{141}, 11_{143}, 11_{148}, 11_{149}, 11_{150},$$

$$11_{152}, 11_{153}, 11_{155}, 11_{161}, 11_{165}, 11_{167}, 11_{168}, 11_{169}, 11_{172}, 11_{173}, 11_{181}, 11_{187}, 11_{197}, 11_{198}, 11_{199},$$

$$11_{202}, 11_{212}, 11_{213}, 11_{214}, 11_{219}, 11_{222}, 11_{249}, 11_{256}, 11_{258}, 11_{260}, 11_{262}, 11_{265}, 11_{270}, 11_{271}, 11_{272},$$

$$11_{273}, 11_{275}, 11_{276}, 11_{278}, 11_{279}, 11_{283}, 11_{285}, 11_{290}, 11_{294}, 11_{295}, 11_{296}, 11_{297}, 11_{303}, 11_{304}, 11_{312},$$

$$11_{313}, 11_{317}, 11_{321}, 11_{322}, 11_{323}, 11_{324}, 11_{327}, 11_{328}, 11_{331}, 11_{344}, 11_{345}, 11_{347}, 11_{349}, 11_{352}.$$

The Kakimizu complex of such a knot K is:

Let T, T' be surfaces in a 3-manifold M. A *product region* between subsurfaces $E \subset T$ and $E' \subset T'$, is an embedding $\theta : F \times [0,1] \to M$ for an appropriate compact orientable surface F, with $\theta(F \times \{0\}) = E$ and $\theta(F \times \{1\}) = E'$. Let T, T', T'' be three surfaces in M and E, E', E" be subsurfaces of T, T', T'' and suppose that there are product regions between E and E' and between E and E''. We say that the two product regions are *aligned* if they either combine to give a product region between E' and E'' or restrict to such a product region.

The following is an original result:

THEOREM 5.1.3. Let K be a knot, and S a minimal genus Seifert surface of K that is realized as a plumbing $S = S_1 \cup_D S_2$. Using the notation $L_i = \partial S_i$, we will assume that L_2 is a fibered link, S_2 a fiber, and, moreover, that every minimal genus Seifert surface S' of K can be isotoped so that S_2 and a subsurface of S' cobound a product region. Then $\mathcal{MS}(K) = \mathcal{MS}(L_1)$.

PROOF. Let S' be a minimal genus Seifert surface of K. The product region between a subsurface of S' and S_2 tells us whether S' is "above" or "below" $D \subset S$ near D. This allows for a deplumbing of S' producing a surface S_1' such that $S' = S_1' \cup_D S_2$. Moreover, S and S' are disjoint if and only if S_1 and S_1' are disjoint.

We use surfaces such as S_1' to define a map

$$f : vert\mathcal{MS}(K) \to vert\mathcal{MS}(L_1).$$

Specifically, for each vertex of $\mathcal{MS}(K)$, choose a representative S' and assign $f(v) = [S_1']$. Seeing that this function is well-defined is somewhat subtle. Suppose that $S' = S_1' \cup_D S_2$ and $T = T_1 \cup_D S_2$ are isotopic. Standard cut-and-paste techniques can be used to ensure that the product regions between the two copies of S_2 are aligned. A sequence of product decompositions of these product regions in the complement of, say, S', produces a copy of T_1 in the complement of S_1. If S', T are isotopic, successive applications of Lemma 2.0.2 therefore establish that S_1' and T_1 are isotopic. Hence f is well-defined on $vert\mathcal{MS}(K)$.

Moreover, f has a natural inverse function

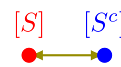
$$f^{-1} : vert\mathcal{MS}(L_1) \to vert\mathcal{MS}(K)$$

given by $f^{-1}([R]) = [R \cup_D S_2]$. It follows that f is $1-1$ and onto between $vert\mathcal{MS}(K)$ and $vert\mathcal{MS}(L_1)$.

Standard cut-and-paste techniques can be used to ensure that if S' and S'' are two minimal genus Seifert surface of K, the product regions between subsurfaces of S', S'' and S_2 are aligned. This allows for a deplumbing of S' producing a surface S_1' such that $S' = S_1' \cup_D S_2$, and a deplumbing of S'' producing a surface S_1'' such that $S'' = S_1'' \cup_D S_2$, with the property that S' and S'' are disjoint if and only if S_1'' and S_1'' are disjoint.

This tells us that f extends to the edges of $\mathcal{M}S(K)$ and, likewise, f^{-1} extends to the edges of $\mathcal{M}S(L_1)$. Therefore f is an isomorphism on 1-skeleta. Since Kakimizu complexes are flag, $\mathcal{M}S(K) = \mathcal{M}S(L_1)$. □

The knot $K = 11_{61}$ is depicted in Figure 5.1. It is a Murasugi sum of a special alternating link and a Hopf band, and satisfies the hypotheses of Lemma 5.1.3.

$\mathcal{M}S(K) = \{[S_1], [S_2]\}$ and the Kakimizu complex is: 

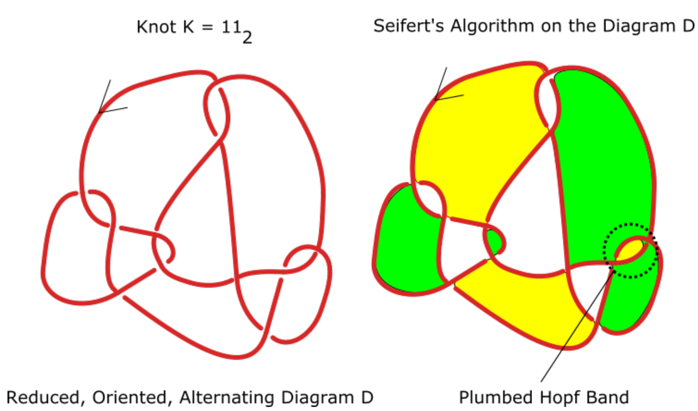


FIGURE 5.1. Knot $K = 11_2$ with a Seifert surface of K

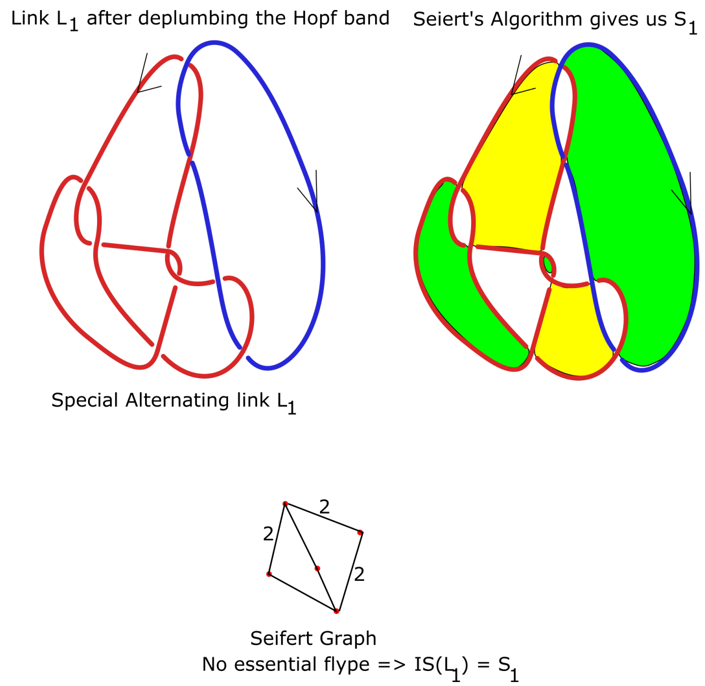


FIGURE 5.2. Deplumb the Hopf band to obtain a special alternating link L_1. There is a unique, up to isotopy, (minimal genus) Seifert surface for L_1.

5.2. Special Cases: The links 11_{103} and 11_{201}

5.2.1. Special case: The link 11_{103}. The calculations in this section are original.

Let D be a reduced oriented alternating diagram of $L = 11_{103}$. See Figure 5.3.

Applying Seifert's algorithm to this diagram, we obtain a surface T_1, as illustrated in Figure 5.3.

$$T_1 = S_1(7_4) \cup_{D_1} H_1 \cup_{D_2} H_2,$$

where $S_1(7_4)$ and $S_2(7_4)$ are the two distinct surfaces (up to isotopy) depicted, and H_i (dotted circles) represent Hopf bands plumbed onto $S_1(7_4)$.

Applying Theorem 5.1.1, we obtain

$$\mathcal{M}S(11_{103}, T_1) \xrightarrow{\phi_1} \mathcal{M}S(L_0, S_1(7_4) \cup_{D_1} H_1) \xrightarrow{\phi_2} \mathcal{M}S(7_4, S_1(7_4)) = [S_2(7_4)],$$

where ϕ_1 and ϕ_2 are bijections, and

$$L_0 = \partial(S_1(7_4) \cup_{D_1} H_1).$$

Thus

$$\mathcal{M}S(11_{103}, T_1) = [T_2] = \phi_1^{-1} \circ \phi_2^{-1}([S_2(7_4)]).$$

Thus, we establish the following Lemma:

Lemma 5.2.0.1.

$$\mathcal{M}S(11_{103}, T_1) = [T_2]$$

where T_2 is a surface in $\mathcal{E}(11_{103})$ disjoint from T_1 and not isotopic to T_1, obtained from $S_2(7_4)$ by attaching two Hopf bands.

Since the link of $[T_1]$ in $\mathcal{M}S(11_{103})$ is a single vertex, $[T_2]$, our next task is to compute the link of $[T_2]$. Note that the surfaces T_1 and T_2 are parallel along H_1 (see Figure 5.3) but not H_2 (see Figure 5.4).

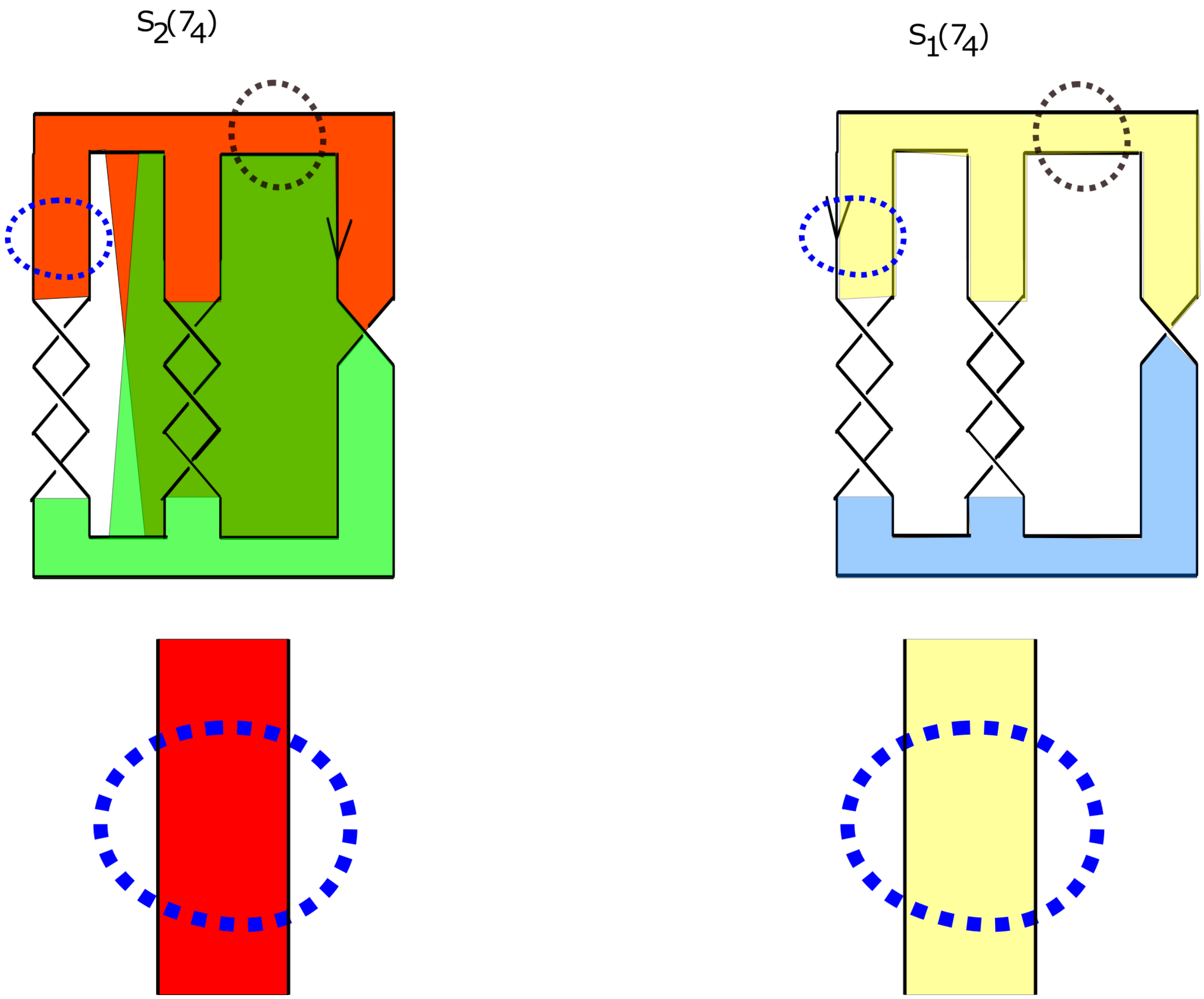


FIGURE 5.3

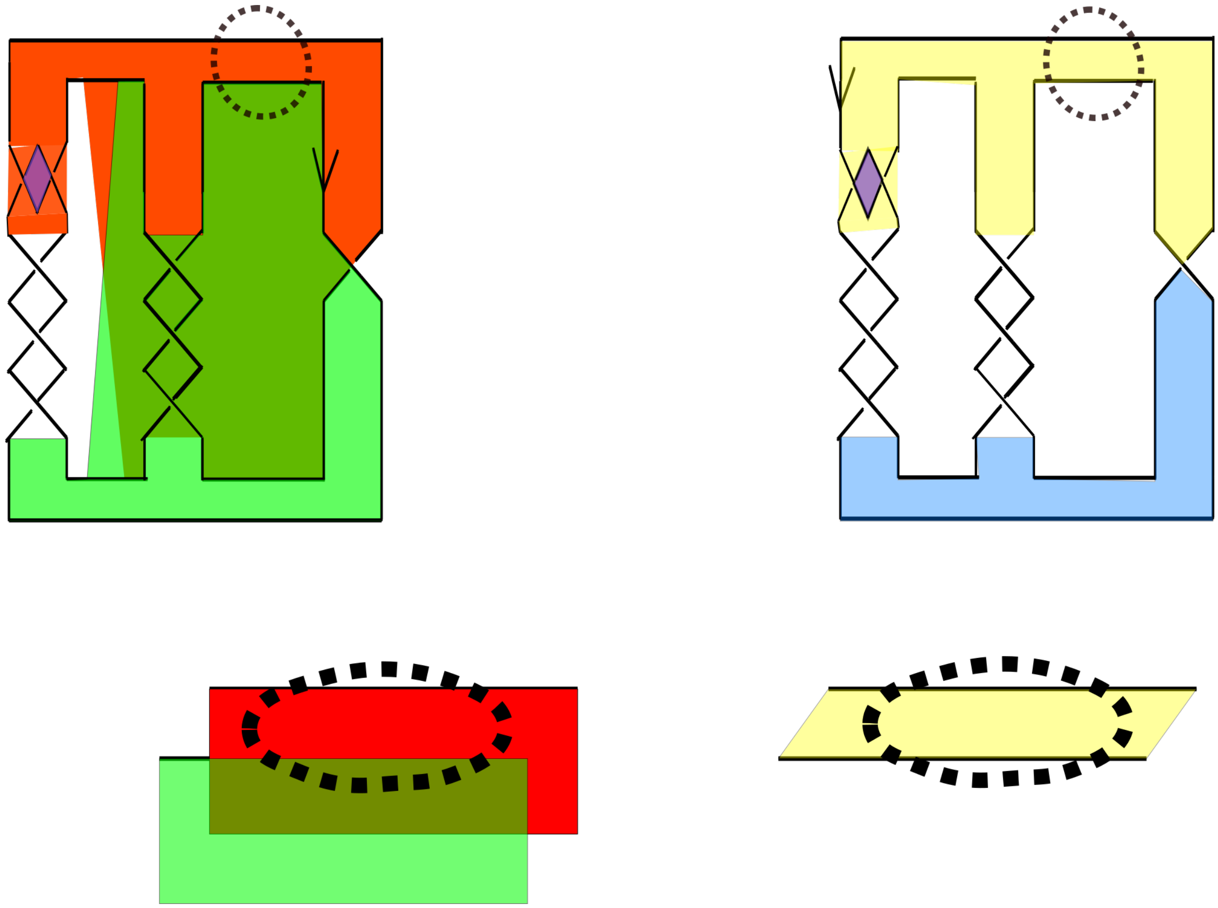


FIGURE 5.4

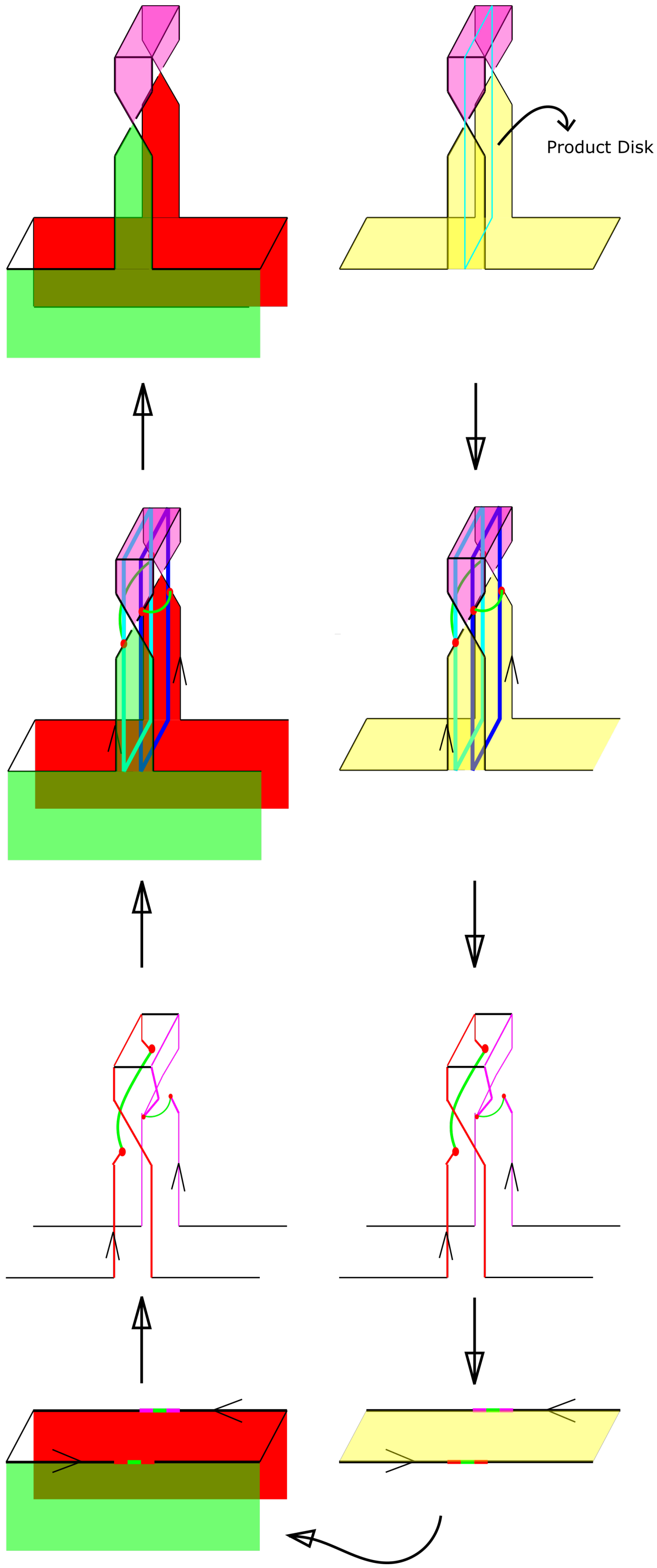


Figure 5.5

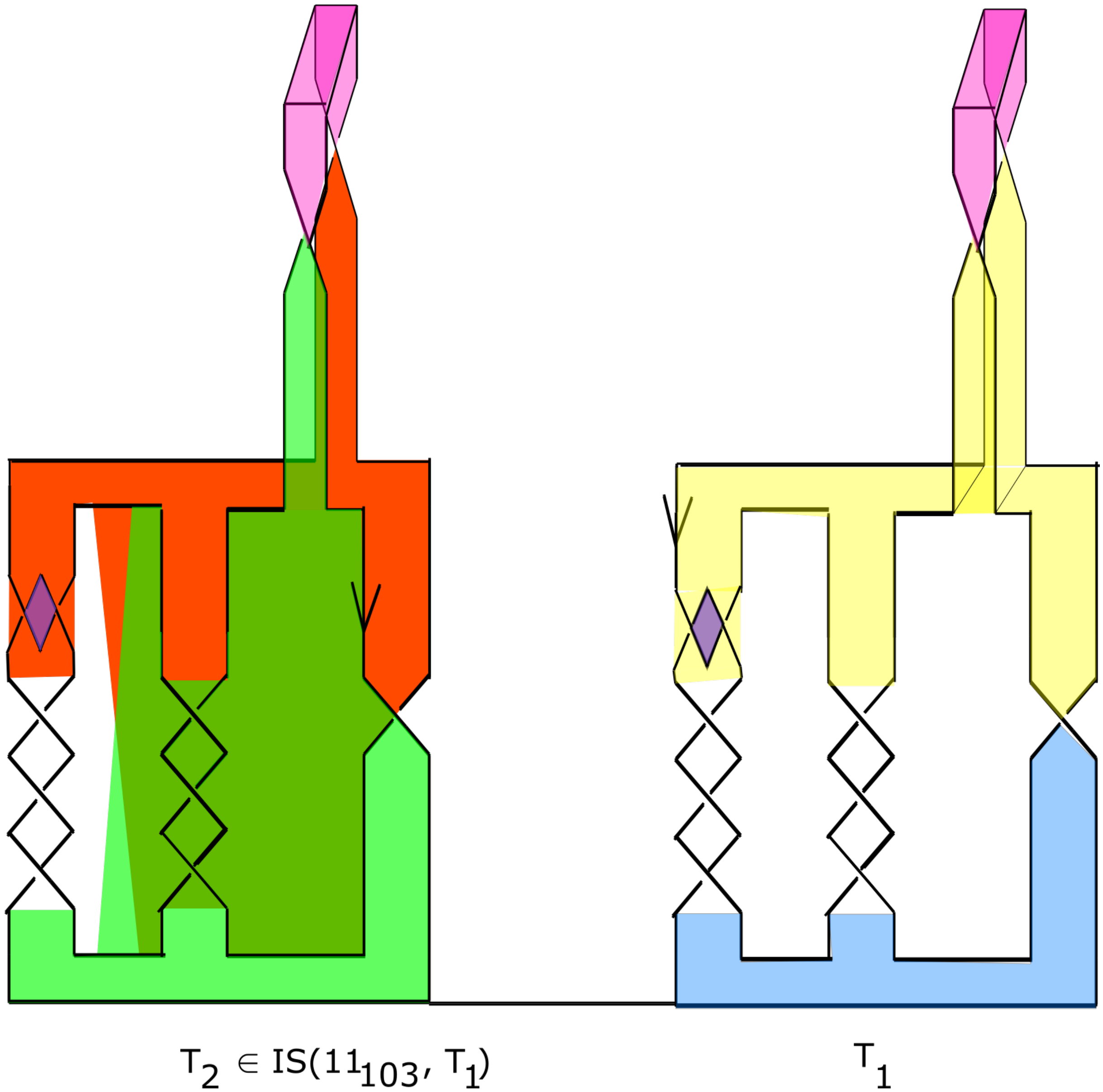


FIGURE 5.6

Set $\tilde{L} = \partial S_1(7_4) \cup_{D_2} (H_2)$ and denote the result of deplumbing H_1 from T_i by $S_i(\tilde{L})$, *i.e.* $S_(\tilde{L}) = S_i(7_4) \cup_{D_2} H_2$. Then, applying Theorem 5.1.1, we obtain

$$\mathcal{M}S(11_{103}, T_2) \xrightarrow{\psi_1} \mathcal{M}S(\tilde{L}, S_2(\tilde{L}))$$

Where ψ_1 is a bijection. Thus it suffices to prove:

Lemma 5.2.0.2.

$$MS(\tilde{L}, S_2(\tilde{L})) = [T_1]$$

Proof.

Let $\eta \in \mathcal{M}S(\tilde{L}, S_2(\tilde{L}))$ and let F be a representative of η.

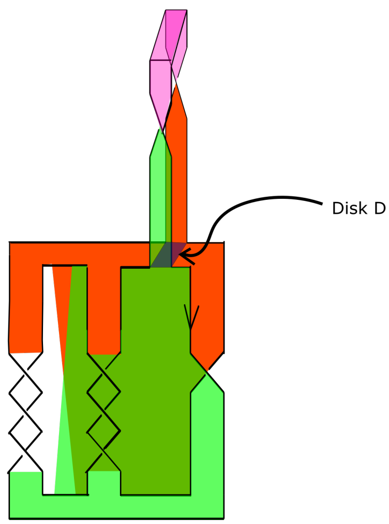


Figure 5.7

Choose two balls V and W such that

$$V \cup W = S^3, V \cap W = S^2$$

with

$$V \cap (S_2(\tilde{L})) = H_2 - E \text{ and } W \cap (S_2(\tilde{L}) - E) = S_2(7_4),$$

where E is 4-gon disk, the boundary of which is a rectangle with two opposite sides being $\partial V \cap S_2(\tilde{L}) = \partial W \cap S_2(\tilde{L})$. For illustrations, see Figures 5.7 and 5.8, where we assume the ball on the top is V and the one at the bottom is W.

We consider $F \cap S^2 = F \cap \partial V = F \cap \partial W$. After isotopy, if necessary, this intersection consists of two arcs. There are three cases to consider, as shown in Figure 5.8:

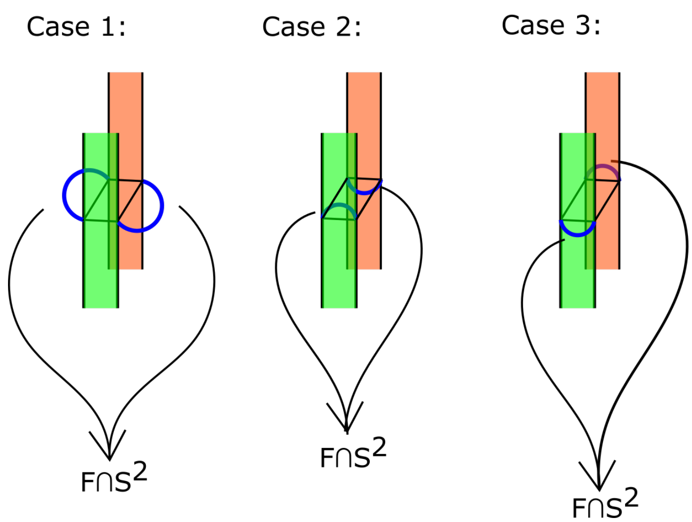


FIGURE 5.8

Case 1: Interpreting $V - S_2(\tilde{L})$ as the complementary sutured manifold, (M_1, γ_1) of the Hopf band H_2, we see that $F \cap M_1$ is a γ_1-surface and hence parallel into $R^+(\gamma_1)$ (and also into $R^-(\gamma_1)$). It follows that there is a boundary reducing disk for $F \cap M_1$ between $F \cap M_1$ and $R^+(\gamma_1)$ that describes an isotopy of $F \cap V - S_2(\tilde{L})$ after which we are in Case 2. (There is also a boundary reducing disk for $F \cap M_1$ between $F \cap M_1$ and $R^-(\gamma_1)$ that describes an isotopy of $F \cap V - S_2(\tilde{L})$ after which we are in Case 3.)

Case 2: In this case, $\partial F \cap M_1$ is an inessential simple closed curve in ∂M_1. Since $F \cap M_1$ is incompressible, $F \cap M_1$ is a boundary parallel disk. Since $(\partial F) \cap V$ runs along $(\partial S_2(\tilde{L})) \cap V$, the two surfaces are parallel in V. Interpreting $W - S_2(\tilde{L})$ as the complementary sutured manifold, (M_2, γ_2), of $S_2(7_4)$ and noting that $F \cap M_2$ is a γ_2-surface, we see that $F \cap M_2$ is either boundary parallel (a copy of $S_2(7_4)$) or essential (a copy of $S_1(7_4)$). If it is boundary parallel, then $F = (F \cap (V - S_2(\tilde{L}))) \cup (F \cap (W - S_2(\tilde{L})))$ is a copy of $S_2(\tilde{L})$, a contradiction. Therefore $F \cap (W - S_2(\tilde{L}))$ is a copy of $S_1(7_4)$ and hence F is isotopic to $S_1(\tilde{L})$.

Case 3: This case is identical to the previous case with $R^-(\gamma_2)$ replacing $R^+(\gamma_2)$.

Consequently, we have

$$\mathcal{M}S(L, T_2) = \{[T_1]\}.$$

□

To conclude, by Lemmas 5.2.0.1 and 5.2.0.2:

$$\mathcal{M}S(11_{103}) = \{[T_1], [T_2]\}.$$

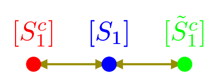
5.2.2. Special Case: 11_{201}. The calculations in this section are original.

Let D be an oriented alternating diagram of $L = 11_{201}$ and let S be the surface obtained by applying Seifert's algorithm to D. See Figures 5.9 and 5.10. Then $S = S_1 \cup S_2$, where S_2 is a Hopf band. Since the Hopf link is fibred, Theorem 5.1.2 yields

$$\mathcal{M}S(L, S) \cong \mathcal{M}S(L_1, S_1).$$

The surface S_1 is itself a plumbing of two surfaces, $S_1 = T_1 \cup_D T_2$, where T_1 and T_2 are the unique Seifert surfaces for special alternating links $\tilde{L}_1$, and $\tilde{L}_2$ respectively, see Figures 5.11 and 5.12.

If (M, γ) is the complementary sutured manifold for S_1 and (M_1, γ_1, A_1) along with (M_2, γ_2, A_2) are the marked complementary sutured manifolds for T_1 and T_2 respectively, then M_2 contains a product disk with A_2 as an edge. By Theorem 4.4.1 we obtain:

$\mathcal{I}S(L_1) =$ 

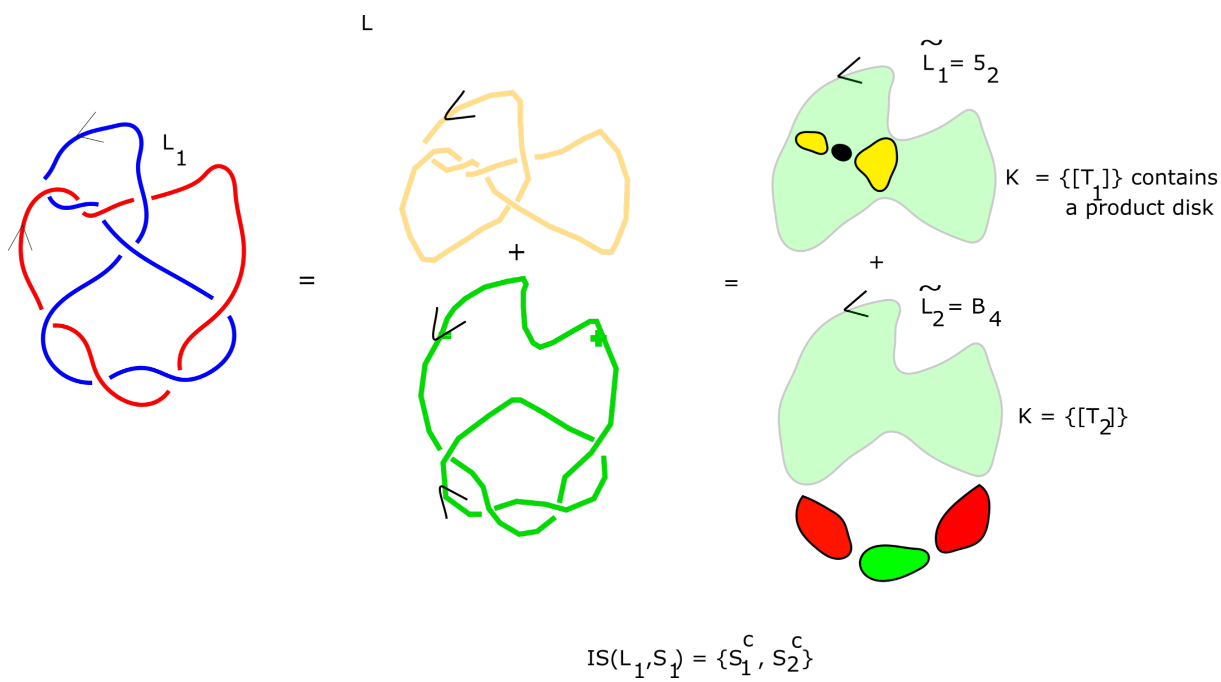


FIGURE 5.9

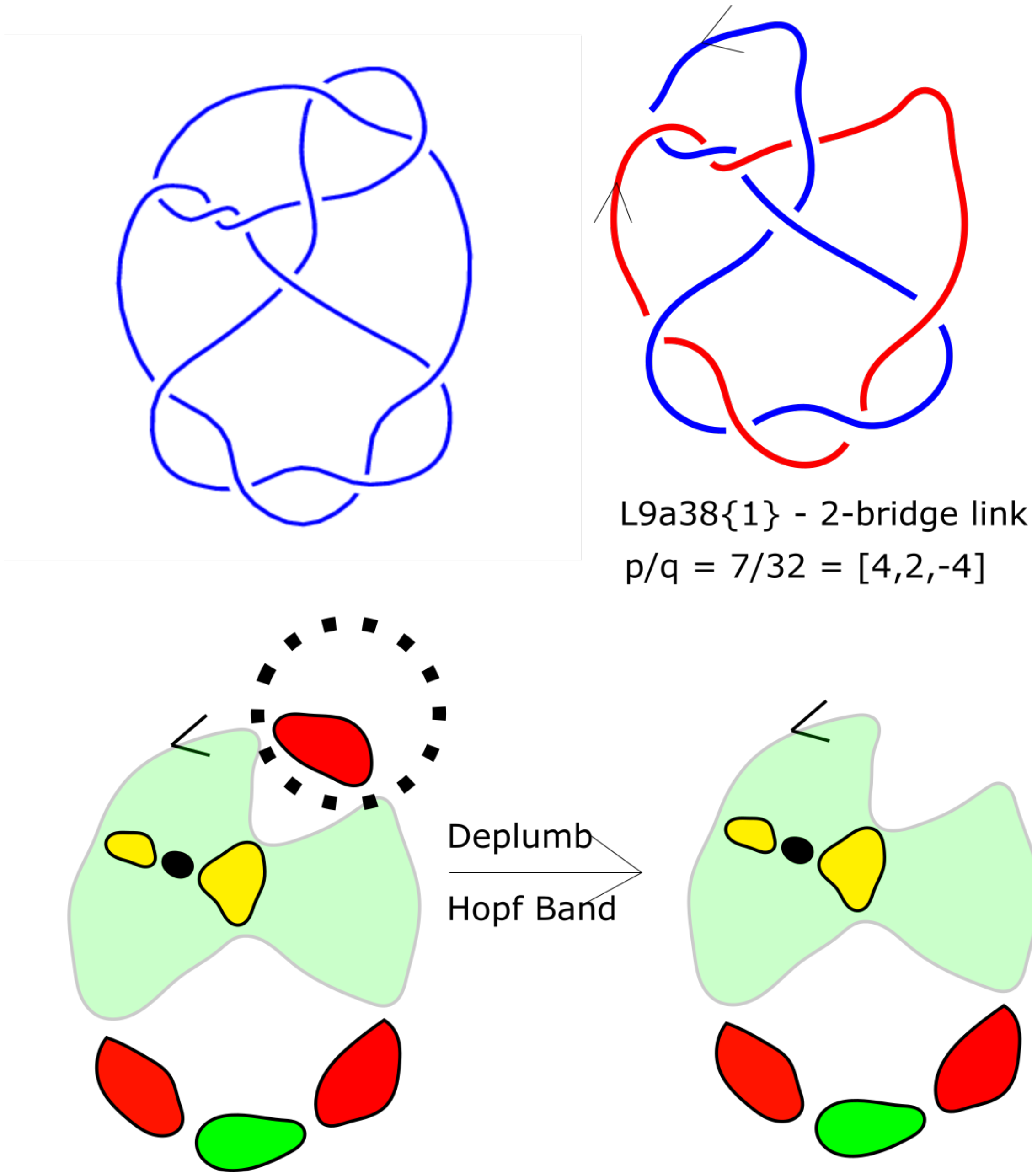


FIGURE 5.10

In particular, $MS(L_1, S_1) = \{[S_1^c], [S_2^c]\}$. Furthermore, $\mathcal{MS}(L, S) \cong \mathcal{MS}(L_1, S_1)$ and we use the notation $\mathcal{MS}(L, S) = \{[R_1], [R_2]\}$. Under this isomorphism, we can identify one of the surfaces, say, R_1, as a plumbing $S_1^c \cup_D S_2$ and the other surface, R_2, as a surface that is parallel to S near the plumbing disk of the Hopf band. (The surface R_2 is similar to the surface T_2 encountered in the calculation for 11_{103}.)

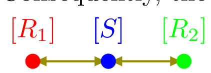
Given that $R_1 = S_1^c \cup_D S_2$ and S_2 is a Hopf band, we have $\mathcal{MS}(L, R_1) \cong \mathcal{MS}(L_1, S_1^c) = \{[S_1]\}$. Thus, we conclude that $\mathcal{MS}(L, R_1) = \{[S_1], [S_2]\}$. Since S_2 is a Hopf band and R_2 is a surface parallel to S_2 and L_1, we can apply the same reasoning as in Lemma 5.2.0.2 to determine $\mathcal{M}S(L, T_2)$, leading to:

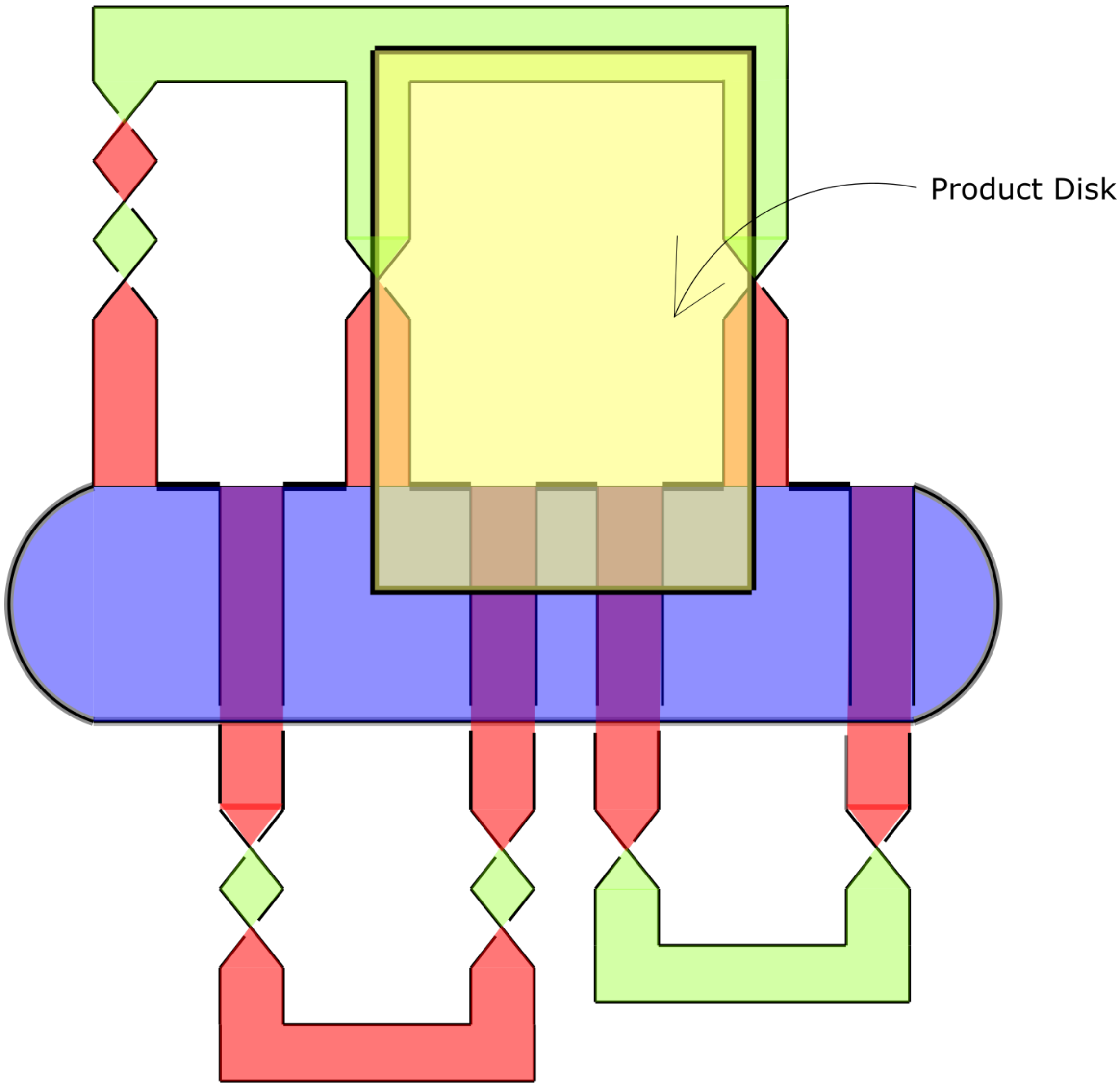
Lemma 5.2.0.3.

$$\mathcal{M}S(L, R_2) = \{[S]\}.$$

The three surfaces are illustrated in Figures 5.11, 5.12 and 5.13.

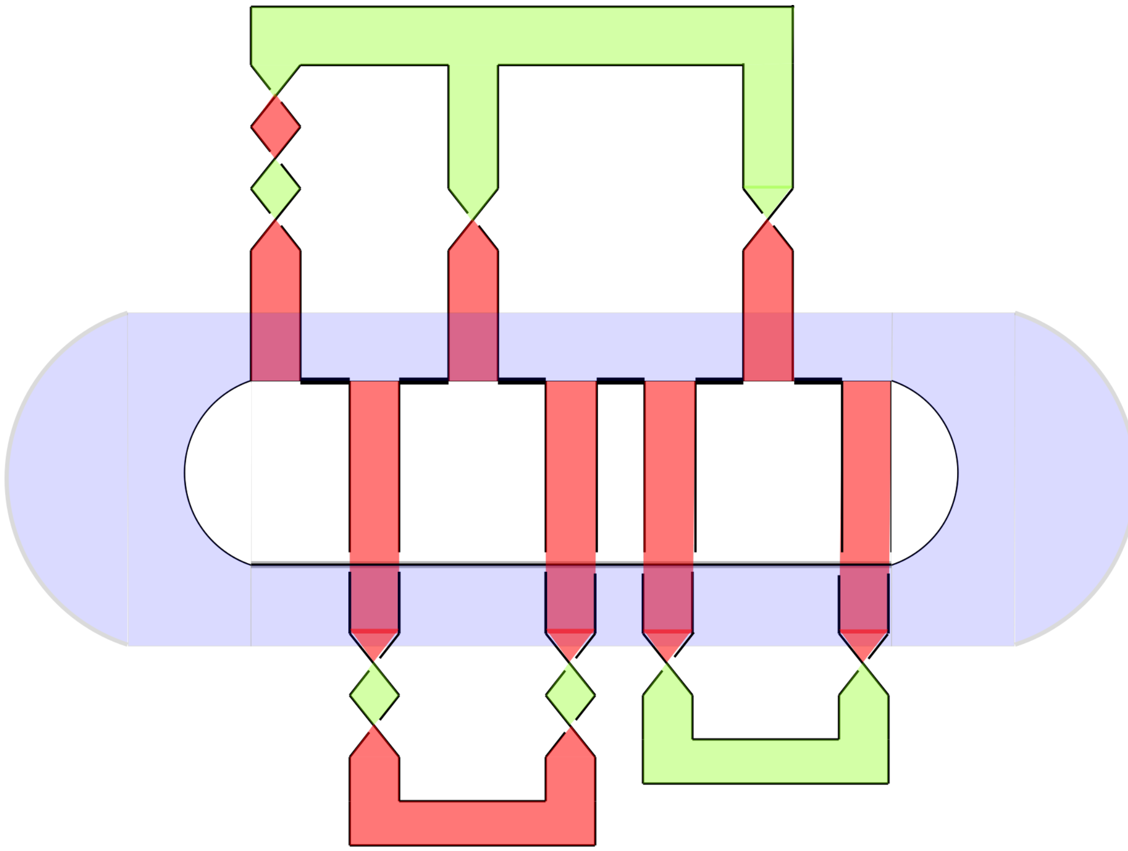
Consequently, the Kakimizu complex of $K = 11_{201}$ is:





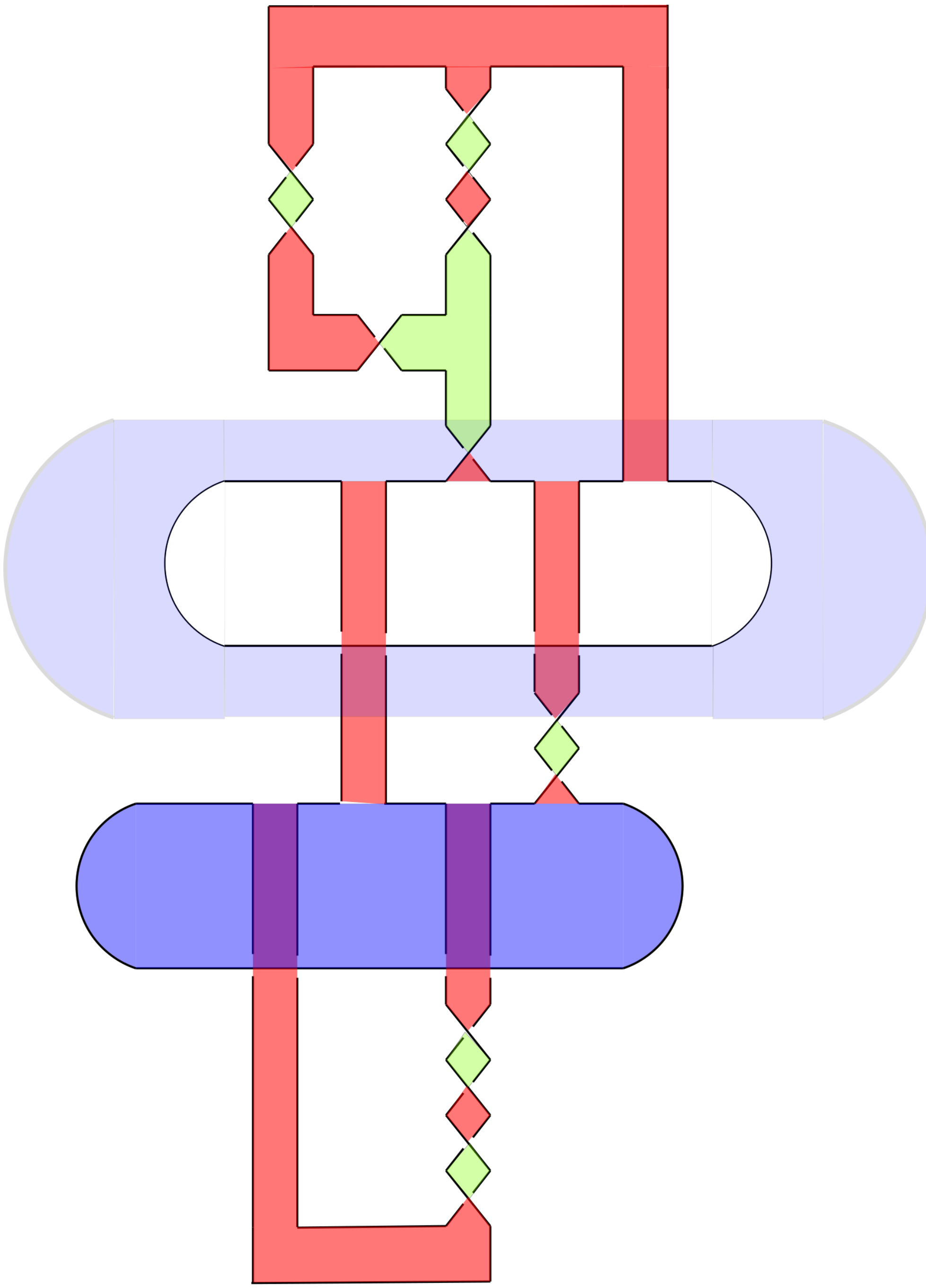
The surface "S"

FIGURE 5.11



The Surface: $R_1 = S_1^c \cup S_2$

Figure 5.12



The Surface: $R_2 = S_2^c // S_2$

FIGURE 5.13

CHAPTER 6

The Kakimizu complexes of 12 crossing alternating knots

6.1. The Kakimizu complex of a fibered knot

We derive the set of fibered knots from KnotInfo [22].

The list of fibered links for 12 crossing alternating links is as follows :

$K = 12_1, 12_2, 12_4, 12_6, 12_7, 12_8, 12_{11}, 12_{13}, 12_{14}, 12_{15}, 12_{16}, 12_{17}, 12_{18}, 12_{19}, 12_{20}, 12_{24}, 12_{26}, 12_{29}, 12_{30}, 12_{33}, 12_{39},$

$12_{40}, 12_{45}, 12_{48}, 12_{50}, 12_{58}, 12_{59}, 12_{60}, 12_{61}, 12_{63}, 12_{65}, 12_{66}, 12_{67}, 12_{69}, 12_{70}, 12_{71}, 12_{74}, 12_{77}, 12_{79}, 12_{80}, 12_{84}, 12_{87},$

$12_{88}, 12_{90}, 12_{91}, 12_{92}, 12_{98}, 12_{99}, 12_{101}, 12_{105}, 12_{108}, 12_{111}, 12_{112}, 12_{113}, 12_{115}, 12_{116}, 12_{119}, 12_{120}, 12_{122}, 12_{123}, 12_{125},$

$12_{126}, 12_{129}, 12_{131}, 12_{132}, 12_{133}, 12_{134}, 12_{136}, 12_{137}, 12_{138}, 12_{139}, 12_{141}, 12_{142}, 12_{146}, 12_{149}, 12_{157}, 12_{158}, 12_{164}, 12_{166},$

$12_{172}, 12_{173}, 12_{174}, 12_{179}, 12_{181}, 12_{182}, 12_{184}, 12_{185}, 12_{186}, 12_{188}, 12_{189}, 12_{190}, 12_{191}, 12_{193}, 12_{195}, 12_{202}, 12_{203}, 12_{207},$

$12_{209}, 12_{213}, 12_{214}, 12_{215}, 12_{216}, 12_{217}, 12_{219}, 12_{220}, 12_{222}, 12_{224}, 12_{225}, 12_{228}, 12_{233}, 12_{242}, 12_{243}, 12_{245}, 12_{246}, 12_{250},$

$12_{258}, 12_{260}, 12_{261}, 12_{262}, 12_{264}, 12_{265}, 12_{268}, 12_{271}, 12_{278}, 12_{280}, 12_{281}, 12_{282}, 12_{283}, 12_{284}, 12_{285}, 12_{287}, 12_{288}, 12_{292},$

$12_{298}, 12_{299}, 12_{304}, 12_{305}, 12_{310}, 12_{316}, 12_{318}, 12_{323}, 12_{324}, 12_{325}, 12_{328}, 12_{331}, 12_{333}, 12_{334}, 12_{335}, 12_{341}, 12_{342}, 12_{349},$

$12_{351}, 12_{352}, 12_{358}, 12_{359}, 12_{361}, 12_{362}, 12_{363}, 12_{364}, 12_{369}, 12_{373}, 12_{374}, 12_{377}, 12_{382}, 12_{383}, 12_{386}, 12_{387}, 12_{388}, 12_{389},$

$12_{396}, 12_{398}, 12_{402}, 12_{413}, 12_{415}, 12_{416}, 12_{417}, 12_{418}, 12_{419}, 12_{426}, 12_{427}, 12_{434}, 12_{435}, 12_{436}, 12_{438}, 12_{439}, 12_{445}, 12_{446},$

$12_{451}, 12_{452}, 12_{453}, 12_{455}, 12_{456}, 12_{457}, 12_{458}, 12_{462}, 12_{464}, 12_{465}, 12_{466}, 12_{467}, 12_{468}, 12_{469}, 12_{470}, 12_{473}, 12_{474}, 12_{475},$

$12_{476}, 12_{477}, 12_{478}, 12_{479}, 12_{480}, 12_{483}, 12_{484}, 12_{485}, 12_{486}, 12_{487}, 12_{488}, 12_{493}, 12_{497}, 12_{498}, 12_{499}, 12_{500}, 12_{501}, 12_{503},$

$12_{505}, 12_{506}, 12_{512}, 12_{515}, 12_{516}, 12_{517}, 12_{521}, 12_{528}, 12_{535}, 12_{536}, 12_{541}, 12_{561}, 12_{565}, 12_{569}, 12_{576}, 12_{579}, 12_{583}, 12_{584},$

$12_{621}, 12_{627}, 12_{629}, 12_{630}, 12_{637}, 12_{649}, 12_{651}, 12_{662}, 12_{664}, 12_{673}, 12_{674}, 12_{681}, 12_{683}, 12_{693}, 12_{695}, 12_{696}, 12_{697}, 12_{698},$

$12_{699}, 12_{700}, 12_{702}, 12_{703}, 12_{704}, 12_{705}, 12_{706}, 12_{707}, 12_{708}, 12_{709}, 12_{710}, 12_{711}, 12_{712}, 12_{716}, 12_{722}, 12_{747}, 12_{754}, 12_{756},$

$12_{765}, 12_{766}, 12_{768}, 12_{776}, 12_{777}, 12_{778}, 12_{785}, 12_{788}, 12_{793}, 12_{805}, 12_{815}, 12_{818}, 12_{819}, 12_{824}, 12_{835}, 12_{838}, 12_{844}, 12_{846},$

$12_{847}, 12_{850}, 12_{859}, 12_{864}, 12_{867}, 12_{868}, 12_{869}, 12_{878}, 12_{884}, 12_{885}, 12_{886}, 12_{887}, 12_{888}, 12_{893}, 12_{898}, 12_{901}, 12_{906}, 12_{909},$

$12_{913}, 12_{916}, 12_{918}, 12_{920}, 12_{928}, 12_{932}, 12_{933}, 12_{935}, 12_{948}, 12_{951}, 12_{961}, 12_{962}, 12_{964}, 12_{965}, 12_{968}, 12_{981}, 12_{984}, 12_{990},$

$12_{991}, 12_{992}, 12_{999}, 12_{1002}, 12_{1011}, 12_{1013}, 12_{1019}, 12_{1020}, 12_{1021}, 12_{1027}, 12_{1039}, 12_{1045}, 12_{1047}, 12_{1047}, 12_{1049},$

$12_{1050}, 12_{1051}, 12_{1054}, 12_{1065}, 12_{1067}, 12_{1070}, 12_{1074}, 12_{1076}, 12_{1080}, 12_{1081}, 12_{1082}, 12_{1084}, 12_{1087}, 12_{1088}, 12_{1089}, 12_{1092},$

$12_{1093}, 12_{1096}12_{1102}, 12_{1104}, 12_{1105}, 12_{1114}, 12_{1120}, 12_{1122}, 12_{1123}, 12_{1124}, 12_{1128}, 12_{1134}, 12_{1141}, 12_{1150}, 12_{1152},$

$12_{1153}, 12_{1156}, 12_{1167}, 12_{1168}, 12_{1176}, 12_{1188}, 12_{1190}, 12_{1191}, 12_{1195}, 12_{1199}, 12_{1203}, 12_{1209}, 12_{1210}, 12_{1211}, 12_{1212}, 12_{1213},$

$12_{1214}, 12_{1215}, 12_{1218}, 12_{1219}, 12_{1220}, 12_{1221}, 12_{1222}, 12_{1223}, 12_{1225}, 12_{1226}, 12_{1227}, 12_{1229}, 12_{1230}, 12_{1231}, 12_{1233}, 12_{1235},$

$12_{1238}, 12_{1246}, 12_{1248}, 12_{1249}, 12_{1250}, 12_{1253}, 12_{1254}, 12_{1255}, 12_{1258}, 12_{1260}, 12_{1273}, 12_{1280}, 12_{1283}, 12_{1288}.$

The Kakimizu complex of every fibered links is a single vertex.

6.2. The Kakimizu complex of a 2-bridge knot

We derive the set of non-fibered 12 crossing 2-bridge knots from KnotInfo [22]. They are as follows:

- Knots which are spanned by a unique (up to isotopy) minimal genus Seifert surface:

$$K = 12_{38}, 12_{169}, 12_{197}, 12_{204}, 12_{206}, 12_{221}, 12_{241}, 12_{243}, 12_{247}, 12_{251}, 12_{254}, 12_{257}, 12_{259}, 12_{300},$$

$$12_{303}, 12_{306}, 12_{307}, 12_{378}, 12_{379}, 12_{384}, 12_{385}, 12_{406}, 12_{437}, 12_{447}, 12_{454}, 12_{511}, 12_{519}, 12_{533}, 12_{537},$$

$$12_{538}, 12_{550}, 12_{552}, 12_{580}, 12_{585}, 12_{595}, 12_{597}, 12_{652}, 12_{682}, 12_{684}, 12_{691}, 12_{713}, 12_{714}, 12_{717}, 12_{718},$$

$$12_{720}, 12_{723}, 12_{724}, 12_{726}, 12_{728}, 12_{732}, 12_{733}, 12_{738}, 12-740, 12_{744}, 12_{745}, 12_{758}, 12_{759}, 12_{760}, 12_{762},$$

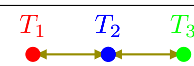
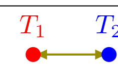
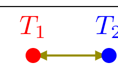
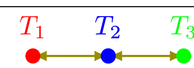
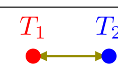
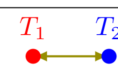
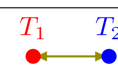
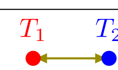
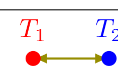
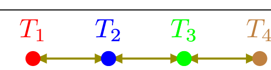
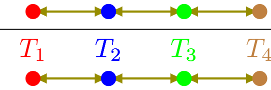
$$12_{796}, 12_{802}, 12_{803}, 12_{1023}, 12_{1029}, 12_{1040}, 12_{1125}, 12_{1129}, 12_{1130}, 12_{1131}, 12_{1135}, 12_{1136}, 12_{1138}, 12_{1140},$$

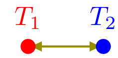
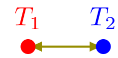
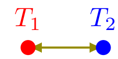
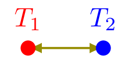
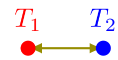
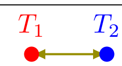
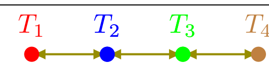
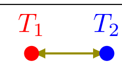
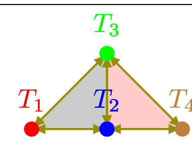
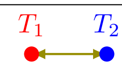
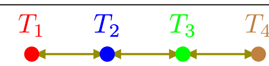
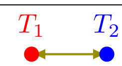
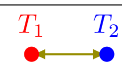
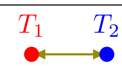
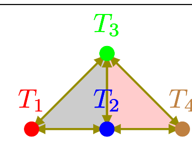
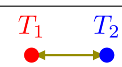
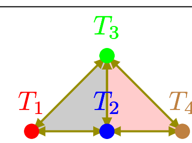
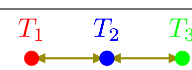
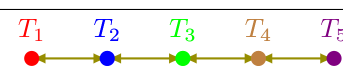
$$12_{1148}, 12_{1149}, 12_{1157}, 12_{1274}, 12_{1276}, 12_{1278}.$$

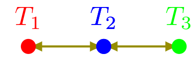
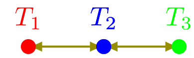
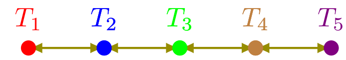
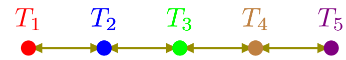
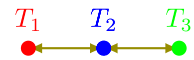
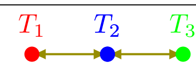
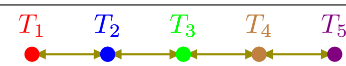
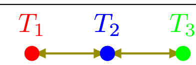
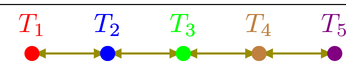
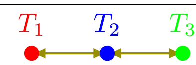
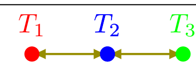
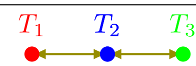
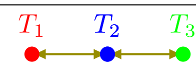
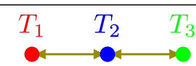
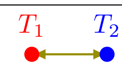
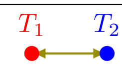
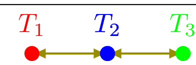
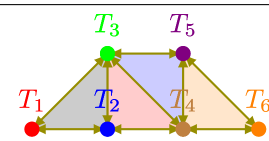
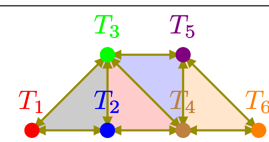
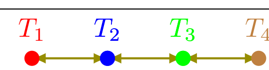
  The Kakimizu complexes of the knots which are a single vertex.

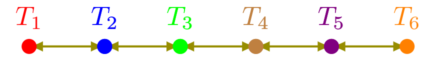
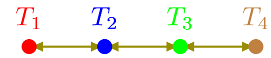
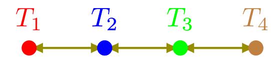
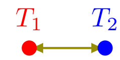
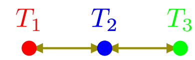
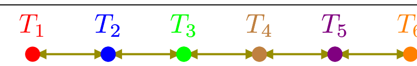
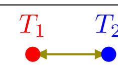
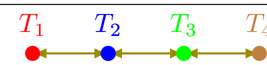
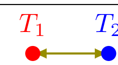
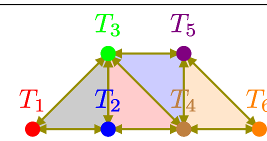
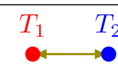
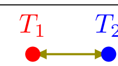
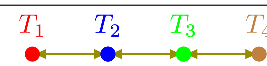
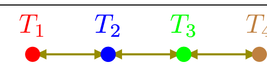
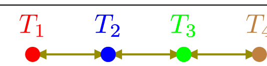
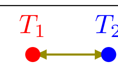
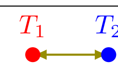
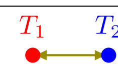
  

- The list of 2-bridge knots with non-trivial Kakimizu complexes.

Knots	2-bridge notation	Even continued fraction expansion	The Kakimizu complex
12_{226}	$70/181$	$[2, -2, -4, -2, -4, -2]$	
12_{239}	$37/87 = -50/87$	$[-2, -4, -6, 2]$,	
12_{255}	$42/107$	$[2, -2, -6, -4]$,	
12_{302}	$53/147 = -94/147$	$[-2, -2, 4, 2, 4, 2]$	
12_{330}	$42/95$	$[2, -4, -6, -2]$	
12_{380}	$20/77$	$[4, 6, -2, -2]$	
12_{425}	$35/81 = -46/81$	$[-2, -4, 6, 2]$,	
12_{471}	$38/85$	$[2, -4, 4, -2]$	
12_{482}	$38/93$	$[2, -2, 4, -4]$	
12_{508}	$53/129 = -76/129$	$[-2, -4, -2, -2, -4, 2]$,	
12_{510}	$81/193 = -112/193$	$[-2, -4, -2, 2, 4, 2]$	

Knots	2-bridge notation	Even continued fraction expansion	The Kakimizu complex
12_{514}	$71/187 = 116/187$	$[2, 2, -2, -4, -4, -2]$	
12_{518}	$60/157$	$[-2, -2, -2, -4, -4, -2]$,	
12_{520}	$48/133$	$[2, -2, -2, -2, -4, -4]$,	
12_{522}	$64/173$	$[2, -2, -2, -4, -4, -2]$,	
12_{532}	$53/125 = -72/125$	$[-2, -4, -4, 2, 2, 2]$,	
12_{534}	$44/163$	$[4, 4, 2, 2, -2, -2]$,	
12_{539}	$56/145$	$[2, -2, -4, -2, -2, -4]$	
12_{540}	$49/165 = -116/165$	$[-2, -2, -4, -4, -2, 2]$	
12_{549}	$26/111$	$[4, -4, -4, -2]$,	
12_{551}	$40/103$	$[2, -2, -4, -6]$	
12_{581}	$36/119$	$[4, 2, 2, 4, -2, -2]$,	
12_{582}	$47/131 = -84/131$	$[-2, -2, 4, 4, 2, 2]$	
12_{596}	$14/81$	$[6, 4, -2, -2]$,	
12_{600}	$25/109 = -84/109$	$[2, 2, 2, 4, 4, -2]$,	
12_{601}	$56/127$	$[2, -4, -4, -4]$,	
12_{643}	$43/99 = -56/99$,	[-2,-4,4,2,2,2]	
12_{644}	$30/113$	$[4, 4, -4, -2]$	
12_{690}	$40/89$	$[2, -4, 2, -4]$	
12_{715}	$50/169$	$[4, 2, 2, -2, -4, -2]$,	

Knots	2-bridge notation	Even continued fraction expansion	The Kakimizu complex
12_{721}	50/171	$[4, 2, 4, 2, -2, -2]$,	
12_{727}	58/157	$[2, -2, -2, -4, -2, -4]$	
12_{729}	46/167	$[4, 2, -2, -2, -4, -2]$,	
12_{731}	22/105	$[4, -2, -2, -2, -4, -2]$	
12_{736}	$59/141 = -82/141$	$[-2, -4, -2, 4, 2, 2]$,	
12_{743}	12/79	$[6, -2, -4, -2]$,	
12_{760}	34/111,	[4,2,2,2,-4,-2]	
12_{761}	$61/139 = -78/139$	$[-2, -4, 2, 4, 2, 2]$	
12_{763}	42/97	$[2, -4, -2, -2, -2, -4]$,	
12_{764}	$39/133 = -94/133$	$[-2, -2, -4, -2, -4, 2]$,	
12_{773}	20/91	$[4, -2, -6, -2]$	
12_{774}	16/89	$[6, 2, -4, -2]$	
12_{775}	38/87	$[2, -4, -2, -6]$	
12_{792}	24/85	$[4, 2, -6, -2]$	
12_{1024}	108/149	$[2, 2, 2, -2, -4, -4]$,	
12_{1030}	$19/91 = -72/91$	$[-2, -2, -2, -2, 4, 4]$	
12_{1033}	$77/107 = -30/107$	$[-4, -2, 4, 2, 2, 2]$	
12_{1034}	32/121	$[4, 4, -2, -4]$,	
12_{1126}	26/119	$[4, -2, -4, -4]$	
12_{1127}	22/97	$[4, -2, 2, -4]$	

Knots	2-bridge notation	Even continued fraction expansion	The Kakimizu complex
12_{1132}	$40/131$	$[4, 2, 2, 2, -2, -4]$,	
12_{1133}	$112/159$	$[2, 2, 4, 2, -2, -4]$,	
12_{1139}	$18/101$	$[6, 2, -2, -4]$	
12_{1145}	$15/79 = -64/79$	$[2, -4, -4, 2]$,	
12_{1146}	$83/117 = -34/117$	$[-4, -2, -4, 2, 2, 2]$,	
12_{1158}	$16/77$	$[4, -2, -2, -2, -2, -4]$,	
12_{1159}	$24/113$	$[-4, 4, -2, -2]$	
12_{1161}	$16/75$	$[4, -2, -2, -6]$,	
12_{1162}	$13/69 = -56/69$	$[-2, -2, -2, -2, -4, 4]$,	
12_{1163}	$24/103$	$[4, -4, -2, -4]$,	
12_{1165}	$16/67$	$[4, -6, -2, -2]$,	
12_{1166}	$8/33$	$[4, -8]$,	
12_{1275}	$44/194$,	[4,-2,2,-4]	
12_{1277}	$36/121$	$[4, 2, 2, -4, -2, -2]$	
12_{1279}	$20/67$	$[4, 2, 2, -6]$,	
12_{1281}	$33/109 = -76/109$	$[-2, -2, -4, 4, 2, 2]$	
12_{1282}	$44/63$	$[2, 2, 4, -6]$	
12_{1287}	$6/37$	$[6, -6]$	

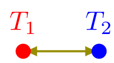
6.3. The Kakimizu complex of plumbings of two links with unique spanning surfaces

For knots that are plumbings of two links with unique minimal genus spanning surfaces the Kakimizu complex can be computed based on the existence of product disks as in Theorem 4.4.1. One needs to check, on each side of the plumbing, whether product disks exist with respect to the marking and dual marking of the complementary sutured manifold. The Kakimizu complexes of the following knots were determined in this manner.

$$K = 12_{183}, 12_{212}, 12_{424}, 12_{429}, 12_{548}, 12_{612}, 12_{642}, 12_{655}, 12_{772}, 12_{790}, 12_{882}, 12_{939}, 12_{945}.$$

6.4. The Kakimizu complex derived from general strategies

The Kakimizu complexes of knots K with a plumbed Hopf band or a fibered surface on a spanning surface S of a special alternating link with a rather special property, namely that the surface S has a unique flype passing through the plumbing disk, can be computed using methods similar to those already considered and have the following Kakimizu complex:



The following knots have this property:

$$K = 12_{267}, 12_{269}, 12_{322}, 12_{336}, 12_{327}, 12_{353}, 12_{395}, 12_{556}, 12_{563}, 12_{568}, 12_{611}, 12_{619}, 12_{623}, 12_{624}, 12_{633}, 12_{638},$$

$$12_{748}, 12_{749}, 12_{753}, 12_{845}, 12_{892}, 12_{947}.$$

We have also obtained the Kakimizu complexes of the following non-fibered, non 2-bridge knots with non-trivial Kakimizu complex using the general strategies described in the previous chapters:

$$K = 12_{21}, 12_{35}, 12_{43}, 12_{53}, 12_{86}, 12_{94}, 12_{97}, 12_{104}, 12_{144}, 12_{145}, 12_{152}, 12_{161}, 12_{210}, 12_{237}, 12_{238}, 12_{253}, 12_{270},$$

$$12_{295}, 12_{329}, 12_{412}, 12_{421}, 12_{443}, 12_{575}, 12_{767}, 12_{810}, 12_{814}, 12_{823}, 12_{834}, 12_{849}, 12_{853}, 12_{877}, 12_{880}, 12_{905}, 12_{924}.$$

6.5. The Kakimizu complexes not determined

- If K is a Murasugi sum of two knots with a 3-Murasugi disk, we do not have any information on the Kakimizu complex of K.

$$K = 12_{273}, 12_{347}, 12_{366}, 12_{593}, 12_{603}, 12_{606}, 12_{631}, 12_{658}, 12_{678}, 12_{821}, 12_{856}, 12_{872}, 12_{890}, 12_{908}, 12_{927}, 12_{936},$$

$$12_{942}, 12_{943}, 12_{955}, 12_{957}, 12_{960}, 12_{971}.$$

- The Kakimizu complexes are not known for knots K with a plumbed Hopf band or a fibered surface on a spanning surface S of a special alternating link with the property that it has more than one flype.

$$K = 12_{89}, 12_{150}, 12_{232}, 12_{275}, 12_{291}, 12_{313}, 12_{372}, 12_{376}, 12_{410}, 12_{441}, 12_{504}, 12_{513}, 12_{524}, 12_{559}, 12_{608}, 12_{632},$$

$$12_{634}, 12_{661}, 12_{677}, 12_{685}, 12_{719}, 12_{730}, 12_{735}, 12_{750}, 12_{752}, 12_{841}, 12_{862}, 12_{883}, 12_{931}, 12_{938}, 12_{953}, 12_{959}, 12_{989}.$$

- Murasugi sum of two knots with non-unique spanning surfaces.

$$K = 12_{787}.$$

- A plumbed surface S on a knot K with each component link having a unique spanning surface with the condition that there exist product disks with respect to the marking and the dual marking of the complementary sutured manifold of S.

$$K = 12_{235}, 12_{423}, 12_{1000}.$$

- The Kakimizu complexes of non-fibered, non-2-bridge knots, $12_{1001} - 12_{1288}$, were not determined.

CHAPTER 7

List of the Kakimizu complexes of 11 and 12 crossing alternating knots [22]

Table 7.1. The Kakimizu complexes of 11 crossing alternating knots

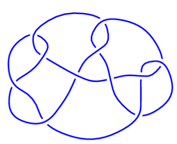
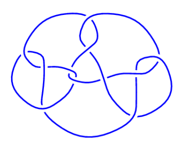
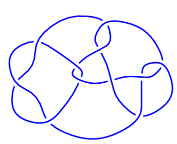
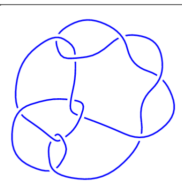
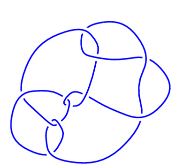
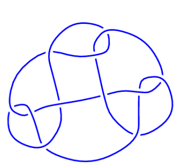
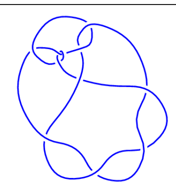
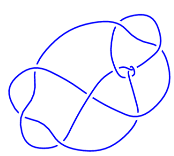
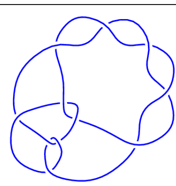
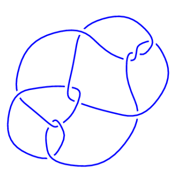
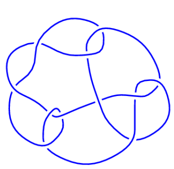
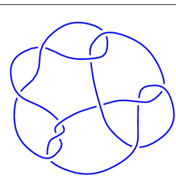
Knot	Diagram	Complex
11_1		
11_2		
11_3		
11_4		
11_5		
11_6		
11_7		
11_8		
11_9		
11_{10}		
11_{11}		
11_{12}		

Table 7.2. The The Kakimizu complexeses of 11 crossing alternating knots

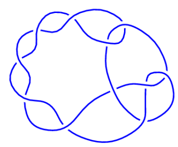
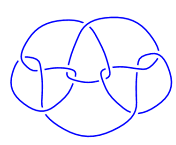
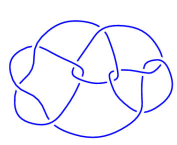
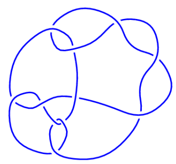
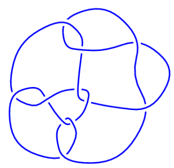
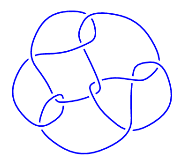
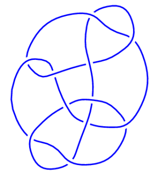
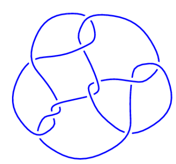
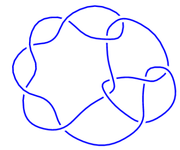
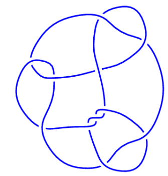
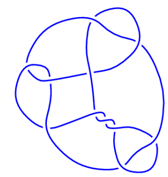
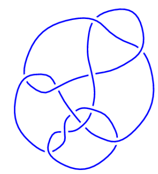
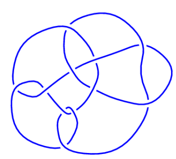
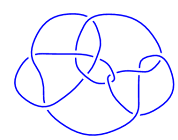
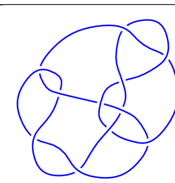
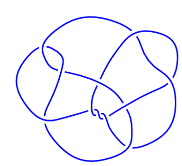
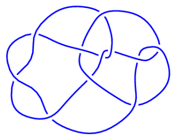
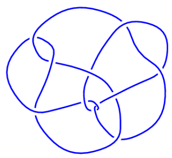
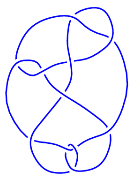
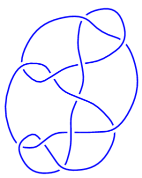
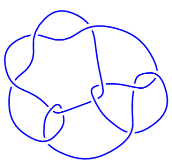
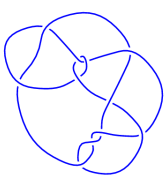
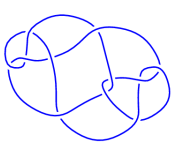
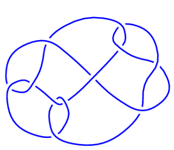
Knot	Diagram	Complex
11_{13}		
11_{14}		
11_{15}		
11_{16}		
11_{17}		
11_{18}		
11_{19}		
11_{20}		
11_{21}		
11_{22}		
11_{23}		
11_{24}		
11_{25}		
11_{26}		
11_{27}		
11_{28}		

Table 7.3. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{29}		T
11_{30}		T
11_{31}		T
11_{32}		T
11_{33}		T
11_{34}		T
11_{35}		T
11_{36}		T

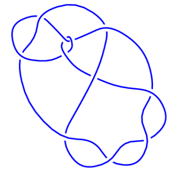
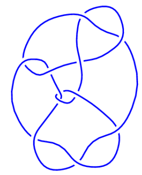
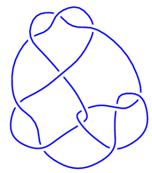
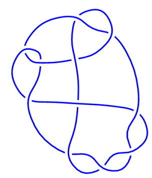
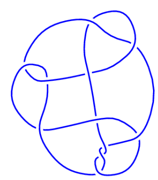
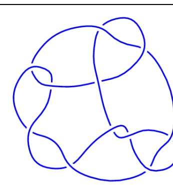
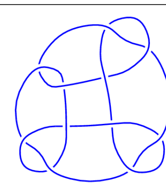
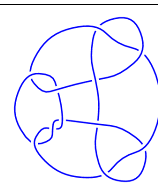
Knot	Diagram	Complex
11_{37}		T
11_{38}		T
11_{39}		T
11_{40}		T
11_{41}		T
11_{42}		T
11_{43}		T
11_{44}		T

Table 7.4. The Kakimizu complexes of 11 crossing alternating knots

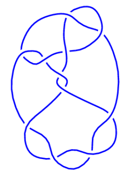
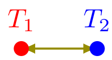
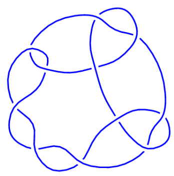
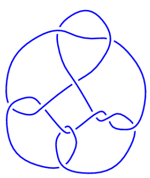
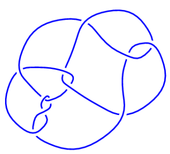
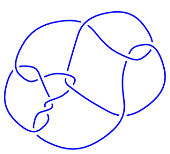
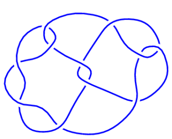
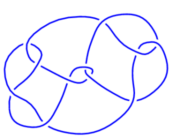
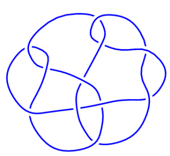
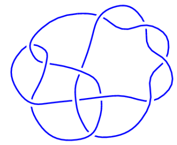
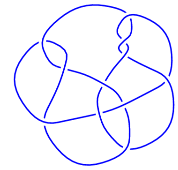
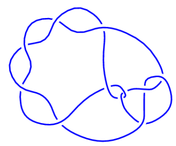
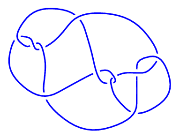
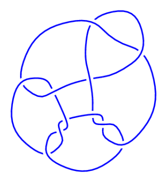
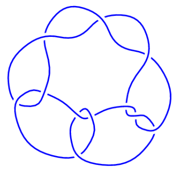
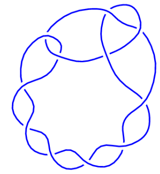
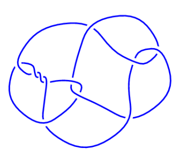
Knot	Diagram	Complex
11_{45}		
11_{46}		
11_{47}		
11_{48}		
11_{49}		
11_{50}		
11_{51}		
11_{52}		
11_{53}		
11_{54}		
11_{55}		
11_{56}		
11_{57}		
11_{58}		
11_{59}		
11_{60}		

TABLE 7.5. The Kakimizu complexes of 11 crossing alternating knots

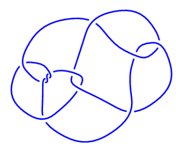
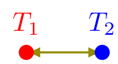
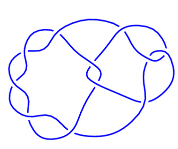
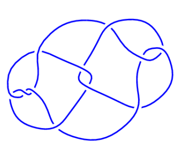
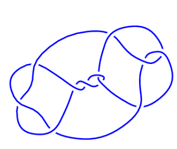
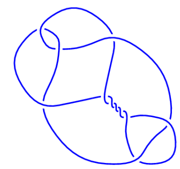
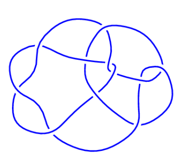
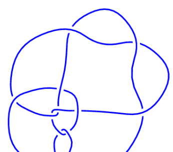
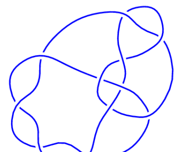
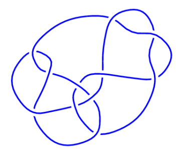
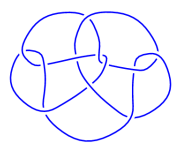
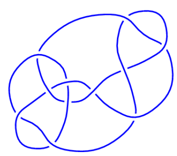
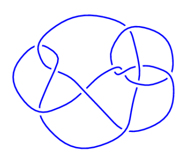
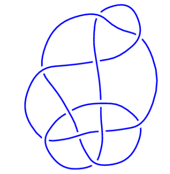
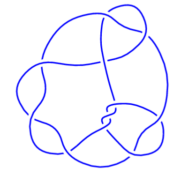
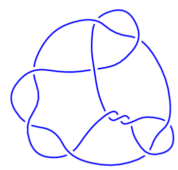
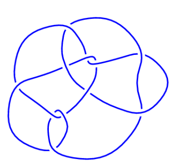
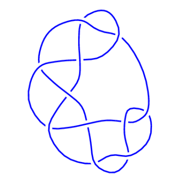
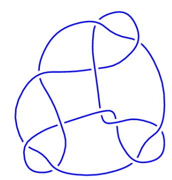
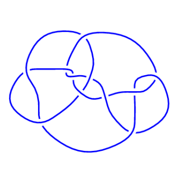
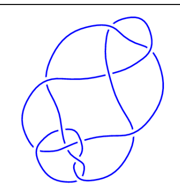
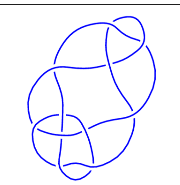
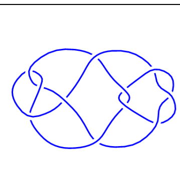
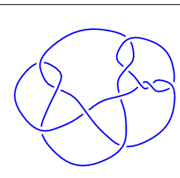
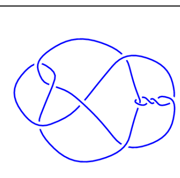
Knot	Diagram	Complex
11_{61}		
11_{62}		
11_{63}		
11_{64}		
11_{65}		
11_{66}		
11_{67}		
11_{68}		
11_{69}		
11_{70}		
11_{71}		
11_{72}		
11_{73}		
11_{74}		
11_{75}		
11_{76}		

Table 7.6. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{77}		
11_{78}		
11_{79}		
11_{80}		
11_{81}		
11_{82}		
11_{83}		
11_{84}		

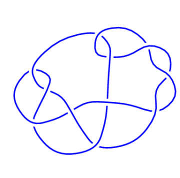
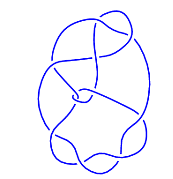
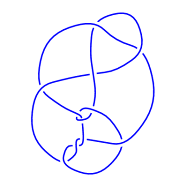
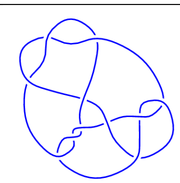
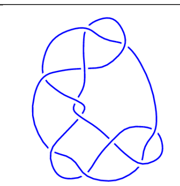
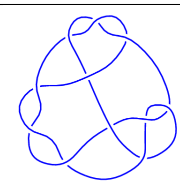
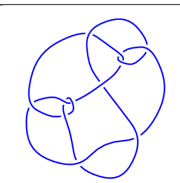
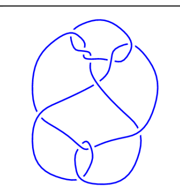
Knot	Diagram	Complex
11_{85}		
11_{86}		
11_{87}		
11_{88}		
11_{89}		
11_{90}		
11_{91}		
11_{92}		

Table 7.7. The Kakimizu complexes of 11 crossing alternating knots

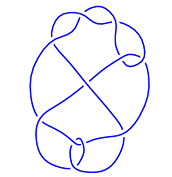
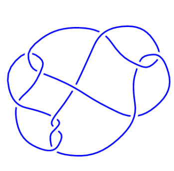
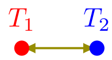
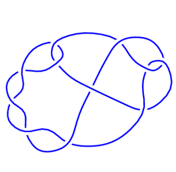
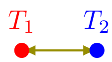
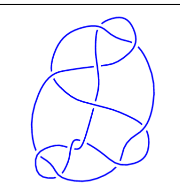
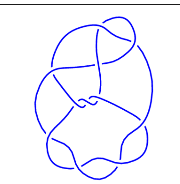
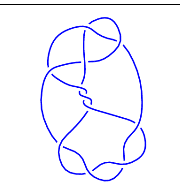
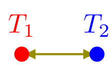
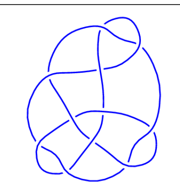
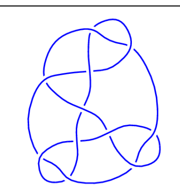
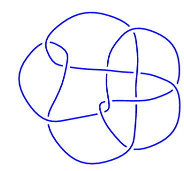
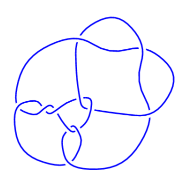
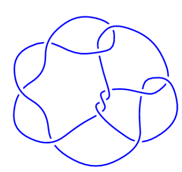
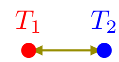
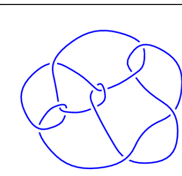
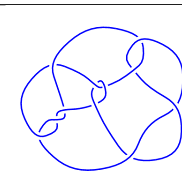
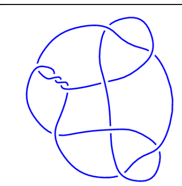
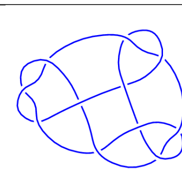
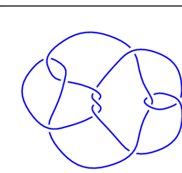
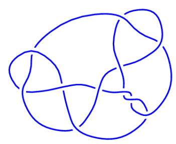
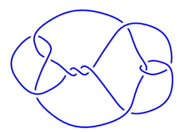
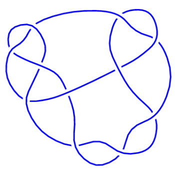
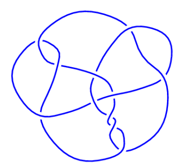
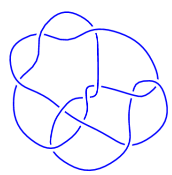
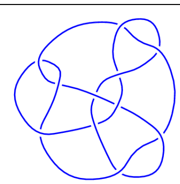
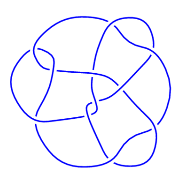
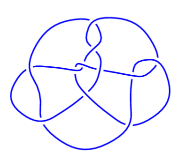
Knot	Diagram	Complex
11_{93}		
11_{94}		
11_{95}		
11_{96}		
11_{97}		
11_{98}		
11_{99}		
11_{100}		
11_{101}		
11_{102}		
11_{103}		
11_{104}		
11_{105}		
11_{106}		
11_{107}		
11_{108}		

TABLE 7.8. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{109}		
11_{110}		
11_{111}		
11_{112}		
11_{113}		
11_{114}		
11_{115}		
11_{116}		

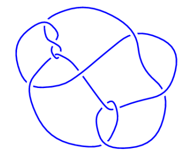
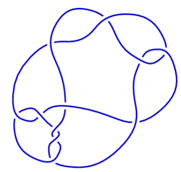
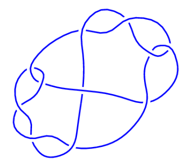
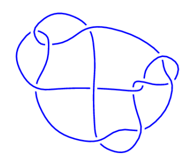
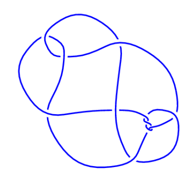
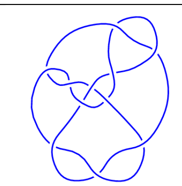
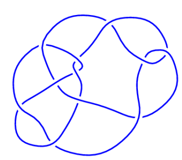
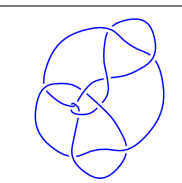
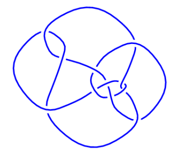
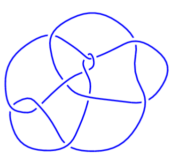
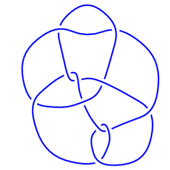
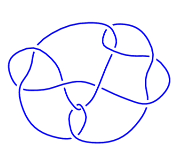
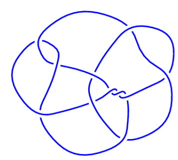
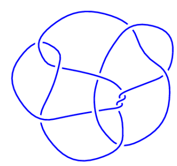
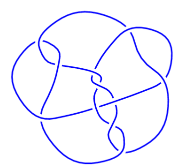
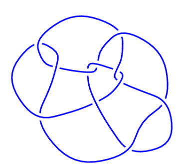
Knot	Diagram	Complex
11_{117}		
11_{118}		
11_{119}		
11_{120}		
11_{121}		
11_{122}		
11_{123}		
11_{124}		

Table 7.9. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{125}		
11_{126}		
11_{127}		
11_{128}		
11_{129}		
11_{130}		
11_{131}		
11_{132}		

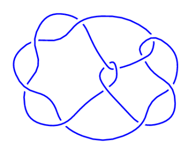
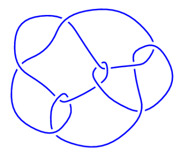
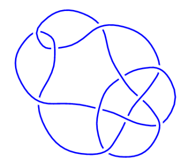
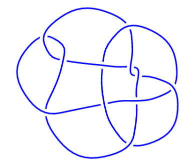
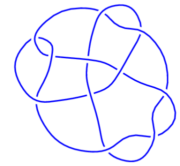
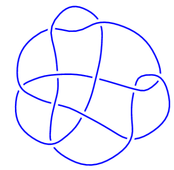
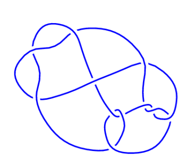
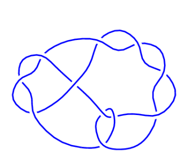
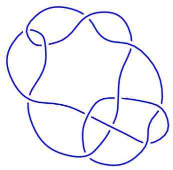
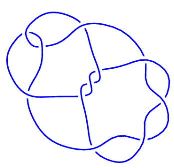
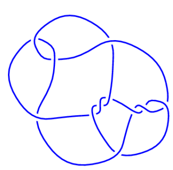
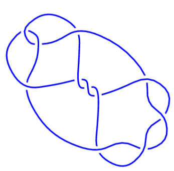
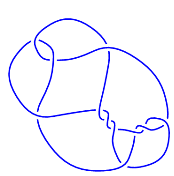
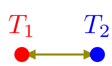
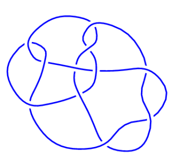
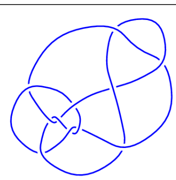
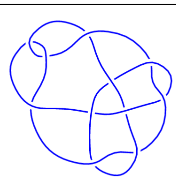
Knot	Diagram	Complex
11_{133}		
11_{134}		
11_{135}		
11_{136}		
11_{137}		
11_{138}		
11_{139}		
11_{140}		

TABLE 7.10. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{141}		
11_{142}		
11_{143}		
11_{144}		
11_{145}		
11_{146}		
11_{147}		
11_{148}		

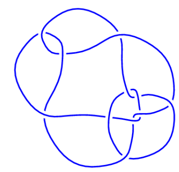
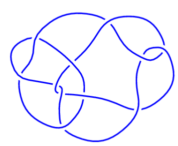
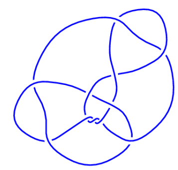
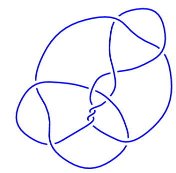
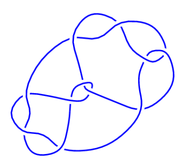
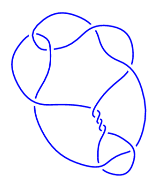
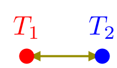
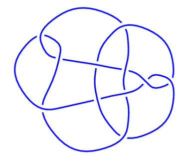
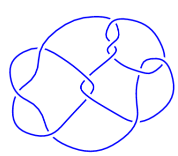
Knot	Diagram	Complex
11_{149}		
11_{150}		
11_{151}		
11_{152}		
11_{153}		
11_{154}		
11_{155}		
11_{156}		

Table 7.11. The Kakimizu complexes of 11 crossing alternating knots

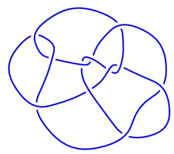
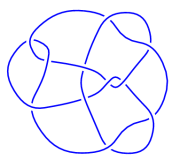
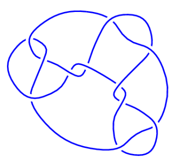
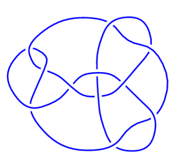
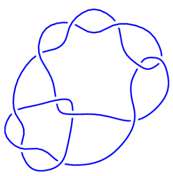
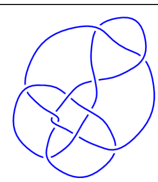
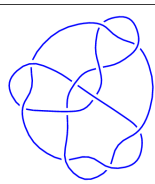
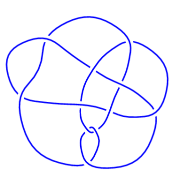
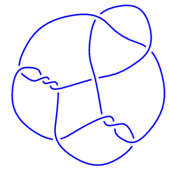
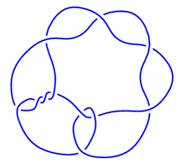
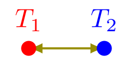
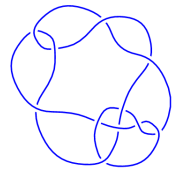
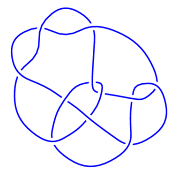
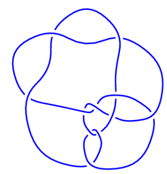
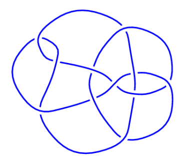
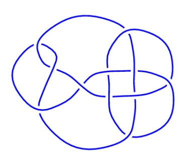
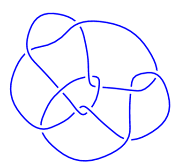
Knot	Diagram	Complex
11_{157}		
11_{158}		
11_{159}		
11_{160}		
11_{161}		
11_{162}		
11_{163}		
11_{164}		
11_{165}		
11_{166}		
11_{167}		
11_{168}		
11_{169}		
11_{170}		
11_{171}		
11_{172}		

Table 7.12. The Kakimizu complexes of 11 crossing alternating knots

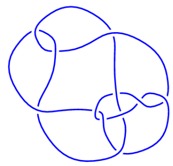
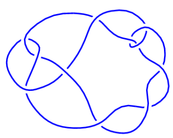
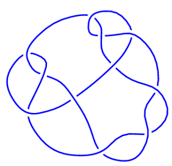
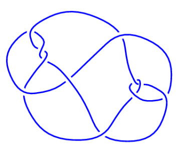
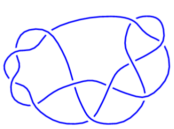
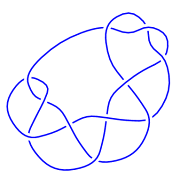
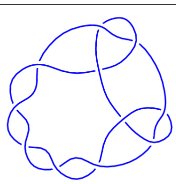
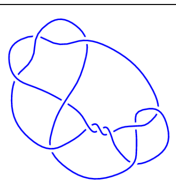
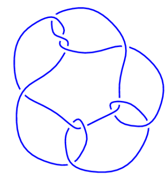
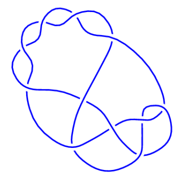
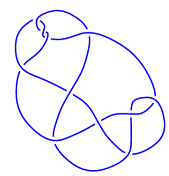
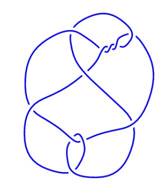
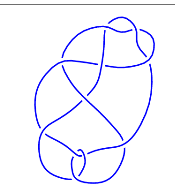
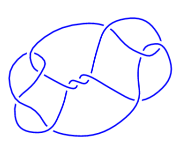
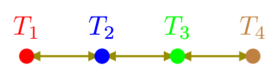
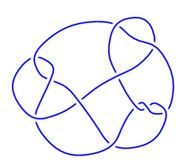
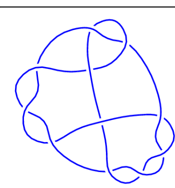
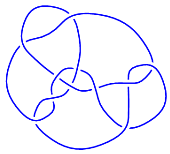
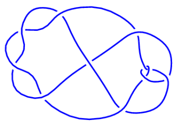
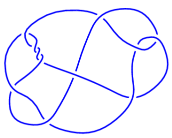
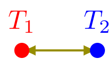
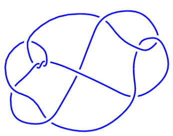
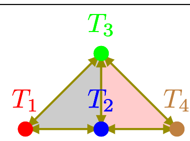
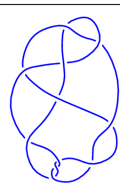
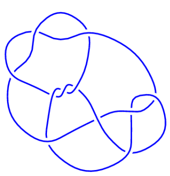
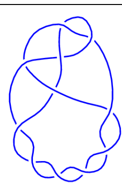
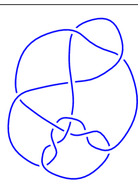
Knot	Diagram	Complex
11_{173}		
11_{174}		
11_{175}		
11_{176}		
11_{177}		
11_{178}		
11_{179}		
11_{180}		
11_{181}		
11_{182}		
11_{183}		
11_{184}		
11_{185}		
11_{186}		
11_{187}		
11_{188}		

TABLE 7.13. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{189}		
11_{190}		
11_{191}		
11_{192}		
11_{193}		
11_{194}		
11_{195}		
11_{196}		

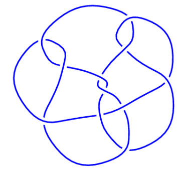
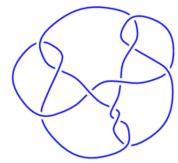
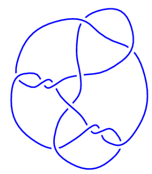
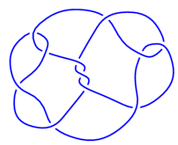
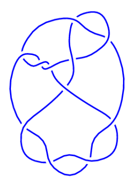
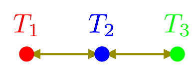
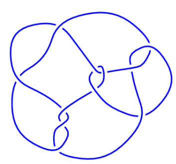
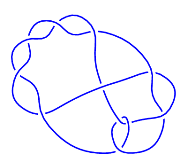
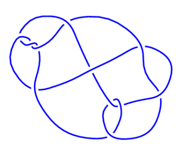
Knot	Diagram	Complex
11_{197}		
11_{198}		
11_{199}		
11_{200}		
11_{201}		
11_{202}		
11_{203}		
11_{204}		

TABLE 7.14. The Kakimizu complexes of 11 crossing alternating knots

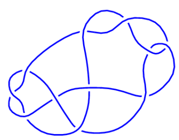
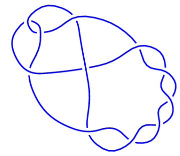
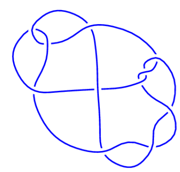
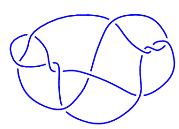
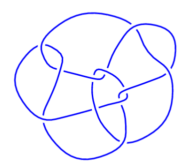
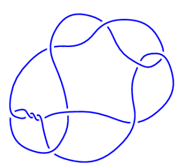
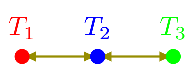
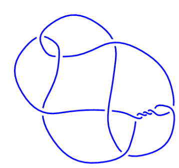
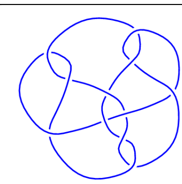
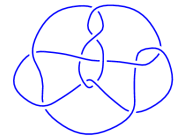
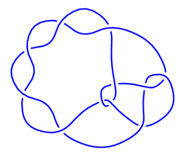
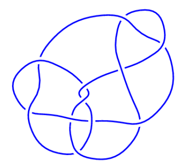
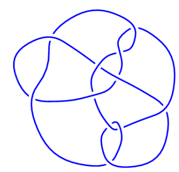
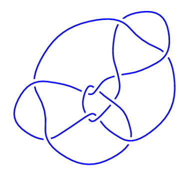
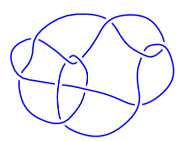
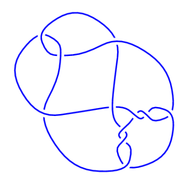
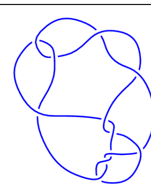
Knot	Diagram	Complex
11_{205}		
11_{206}		
11_{207}		
11_{208}		
11_{209}		
11_{210}		
11_{211}		
11_{212}		
11_{213}		
11_{214}		
11_{215}		
11_{216}		
11_{217}		
11_{218}		
11_{219}		
11_{220}		

Table 7.15. The Kakimizu complexes of 11 crossing alternating knots

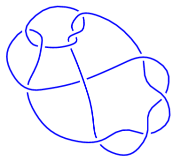
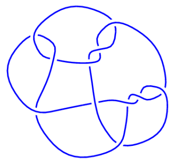
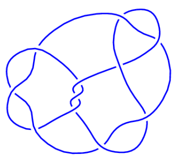
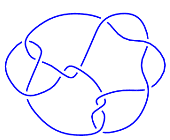
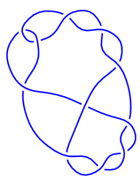
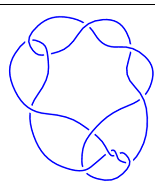
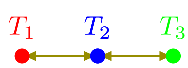
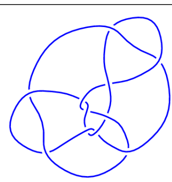
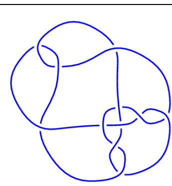
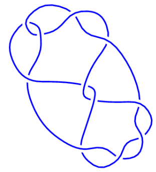
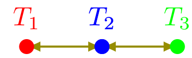
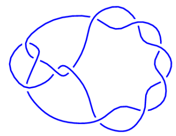
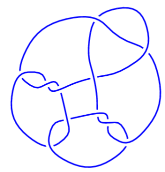
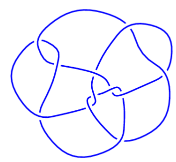
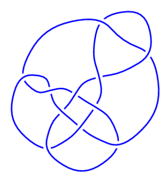
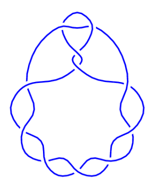
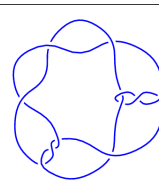
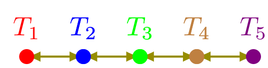
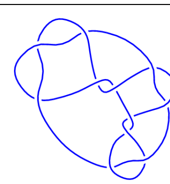
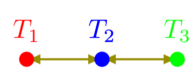
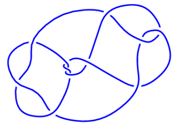
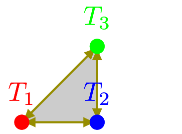
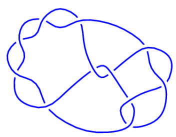
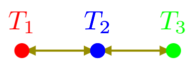
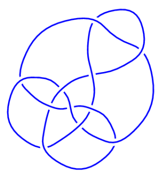
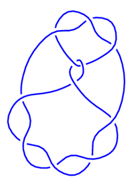
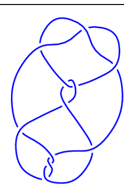
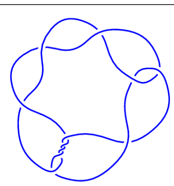
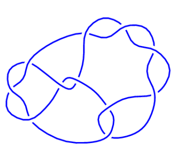
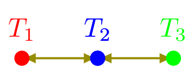
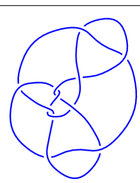
Knot	Diagram	Complex
11_{221}		
11_{222}		
11_{223}		
11_{224}		
11_{225}		
11_{226}		
11_{227}		
11_{228}		
11_{229}		
11_{230}		
11_{231}		
11_{232}		
11_{233}		
11_{234}		
11_{235}		
11_{236}		

Table 7.16. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{237}		
11_{238}		
11_{239}		
11_{240}		
11_{241}		
11_{242}		
11_{243}		
11_{244}		

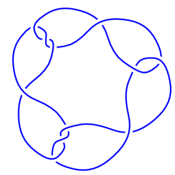
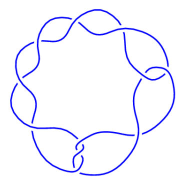
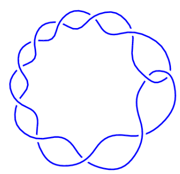
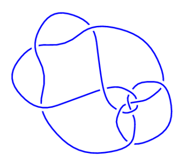
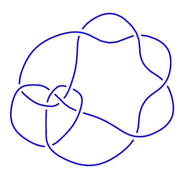
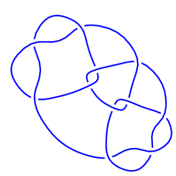
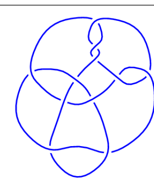
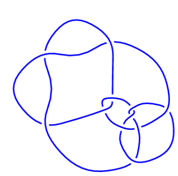
Knot	Diagram	Complex
11_{245}		
11_{246}		
11_{247}		
11_{248}		
11_{249}		
11_{250}		
11_{251}		
11_{252}		

TABLE 7.17. The Kakimizu complexes of 11 crossing alternating knots

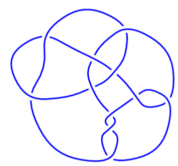
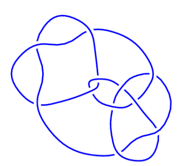
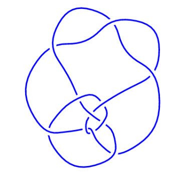
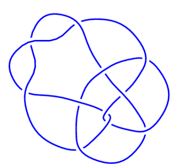
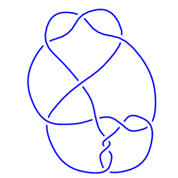
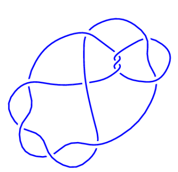
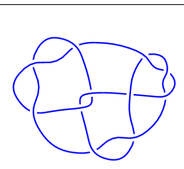
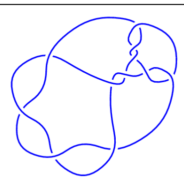
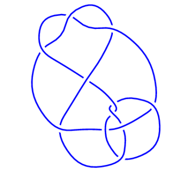
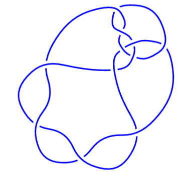
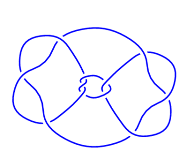
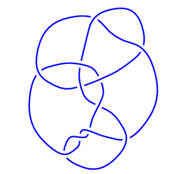
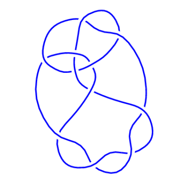
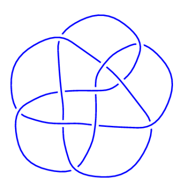
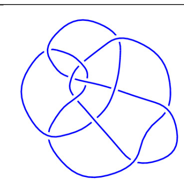
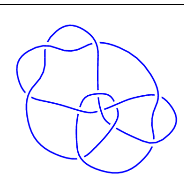
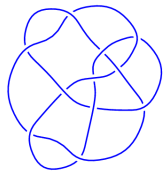
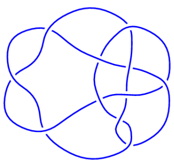
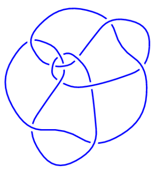
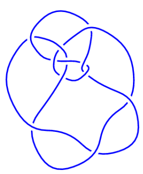
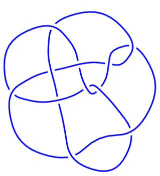
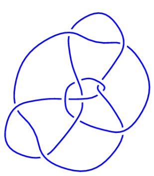
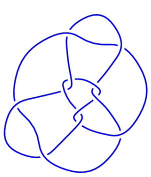
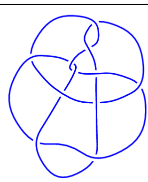
Knot	Diagram	Complex
11_{253}		
11_{254}		
11_{255}		
11_{256}		
11_{257}		
11_{258}		
11_{259}		
11_{260}		
11_{261}		
11_{262}		
11_{263}		
11_{264}		
11_{265}		
11_{266}		
11_{267}		
11_{268}		

Table 7.18. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{269}		
11_{270}		
11_{271}		
11_{272}		
11_{273}		
11_{274}		
11_{275}		
11_{276}		

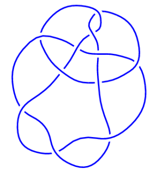
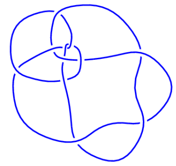
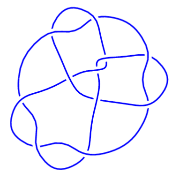
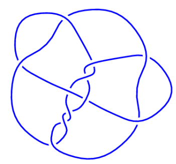
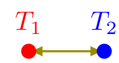
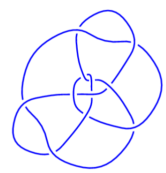
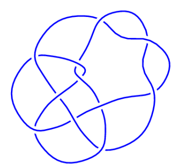
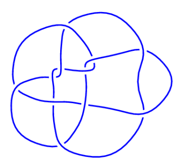
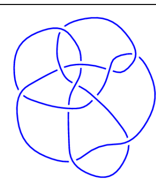
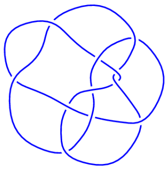
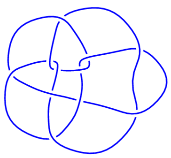
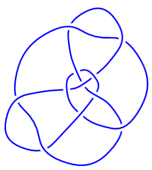
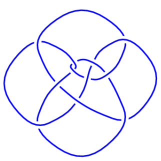
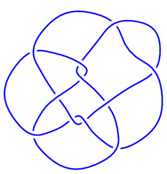
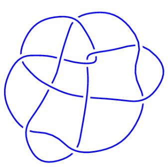
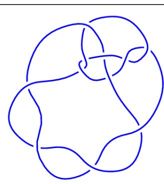
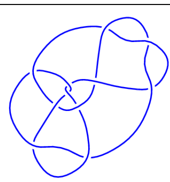
Knot	Diagram	Complex
11_{277}		
11_{278}		
11_{279}		
11_{280}		
11_{281}		
11_{282}		
11_{283}		
11_{284}		

TABLE 7.19. The Kakimizu complexes of 11 crossing alternating knots

Knot	Diagram	Complex
11_{285}		
11_{286}		
11_{287}		
11_{288}		
11_{289}		
11_{290}		
11_{291}		
11_{292}		

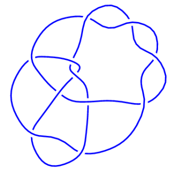
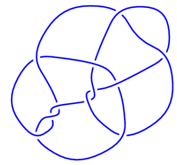
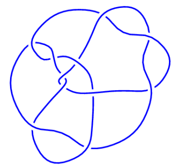
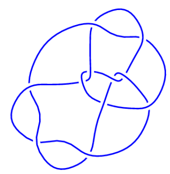
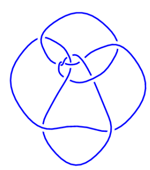
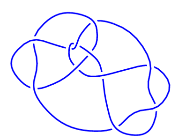
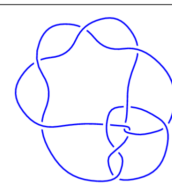
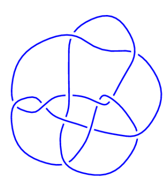
Knot	Diagram	Complex
11_{293}		
11_{294}		
11_{295}		
11_{296}		
11_{297}		
11_{298}		
11_{299}		
11_{300}		

Table 7.20. The Kakimizu complexes of 11 crossing alternating knots

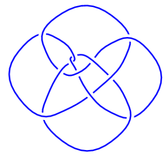
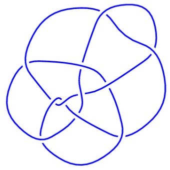
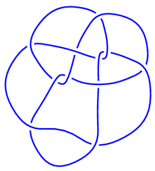
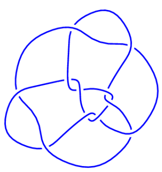
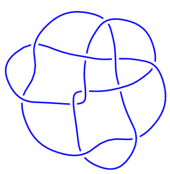
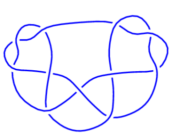
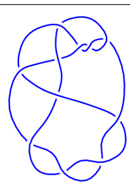
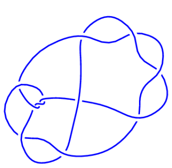
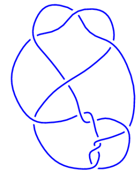
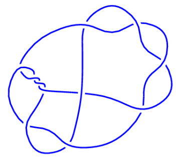
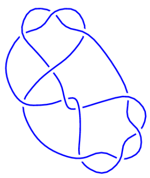
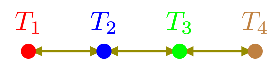
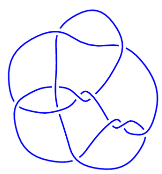
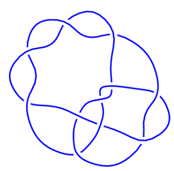
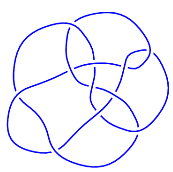
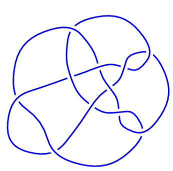
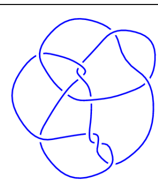
Knot	Diagram	Complex
11_{301}		
11_{302}		
11_{303}		
11_{304}		
11_{305}		
11_{306}		
11_{307}		
11_{308}		
11_{309}		
11_{310}		
11_{311}		
11_{312}		
11_{313}		
11_{314}		
11_{315}		
11_{316}		

TABLE 7.21. The Kakimizu complexes of 11 crossing alternating knots

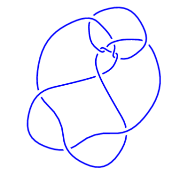
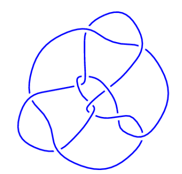
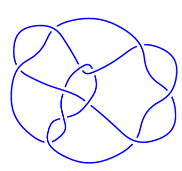
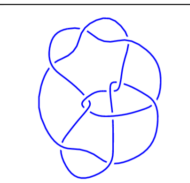
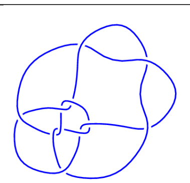
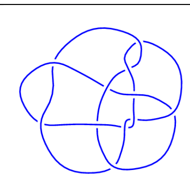
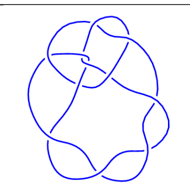
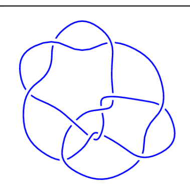
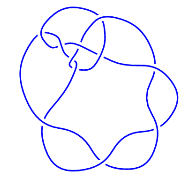
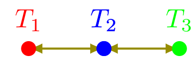
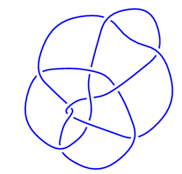
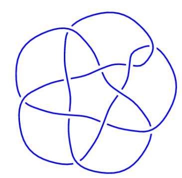
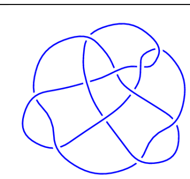
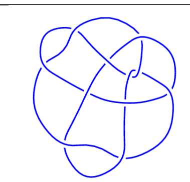
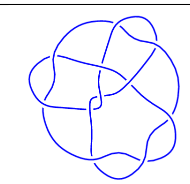
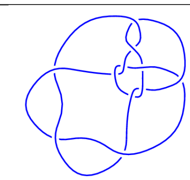
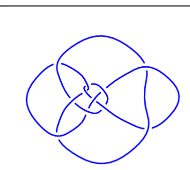
Knot	Diagram	Complex
11_{317}		
11_{318}		
11_{319}		
11_{320}		
11_{321}		
11_{322}		
11_{323}		
11_{324}		
11_{325}		
11_{326}		
11_{327}		
11_{328}		
11_{329}		
11_{330}		
11_{331}		
11_{332}		

Table 7.22. The Kakimizu complexes of 11 crossing alternating knots

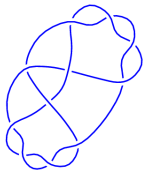
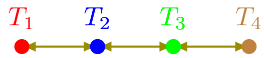
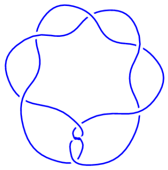
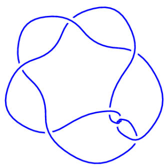
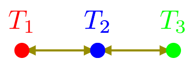
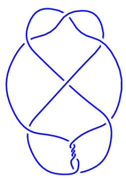
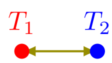
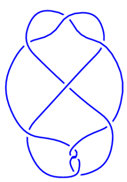
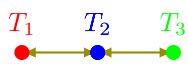
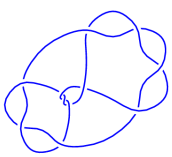
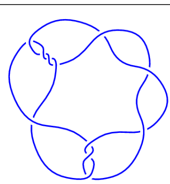
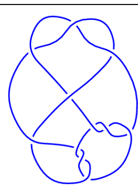
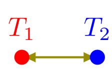
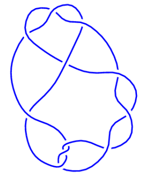
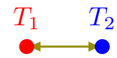
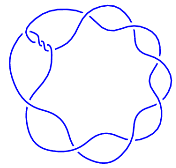
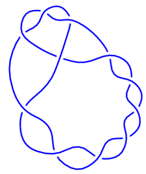
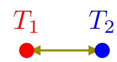
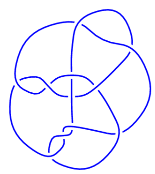
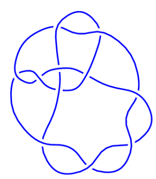
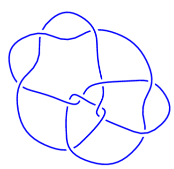
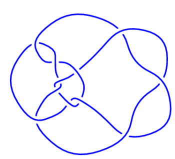
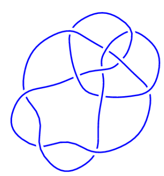
Knot	Diagram	Complex
11_{333}		
11_{334}		
11_{335}		
11_{336}		
11_{337}		
11_{338}		
11_{339}		
11_{340}		
11_{341}		
11_{342}		
11_{343}		
11_{344}		
11_{345}		
11_{346}		
11_{347}		
11_{348}		

Table 7.23. The Kakimizu complexes of 11 crossing alternating knots

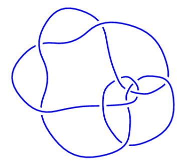
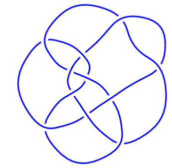
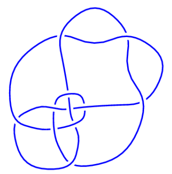
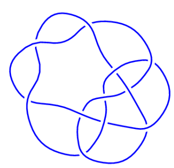
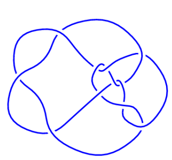
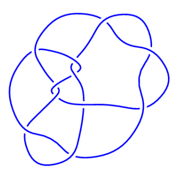
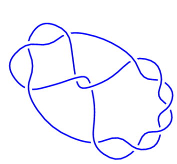
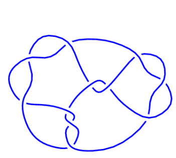
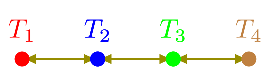
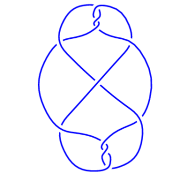
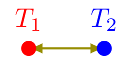
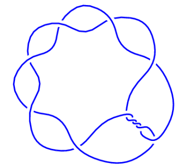
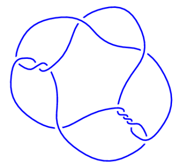
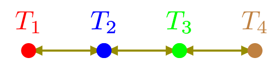
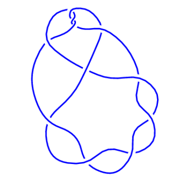
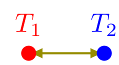
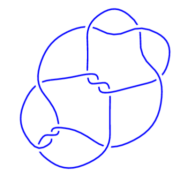
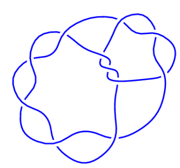
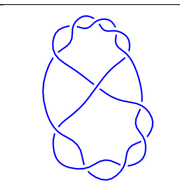
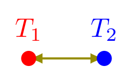
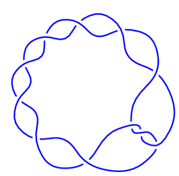
Knot	Diagram	Complex
11_{349}		
11_{350}		
11_{351}		
11_{352}		
11_{353}		
11_{354}		
11_{355}		
11_{356}		
11_{357}		
11_{358}		
11_{359}		
11_{360}		
11_{361}		
11_{362}		
11_{363}		
11_{364}		

Table 7.24. The Kakimizu complexes of 11 crossing alternating knots

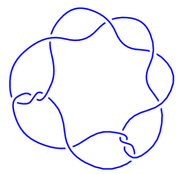
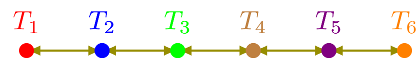
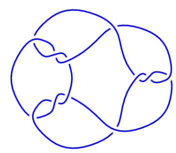
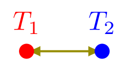
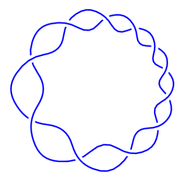
Knot	Diagram	Complex
11_{365}		
11_{366}		
11_{367}		

TABLE 7.25. The Kakimizu complexes of 12 crossing alternating knots

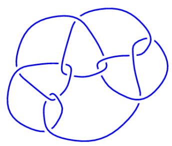
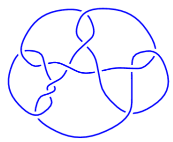
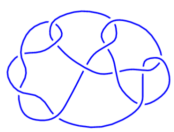
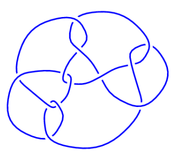
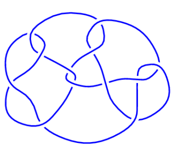
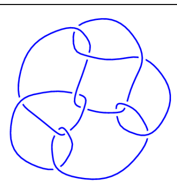
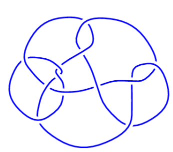
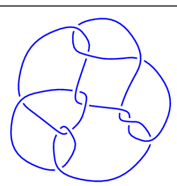
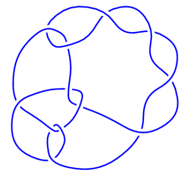
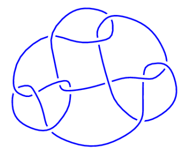
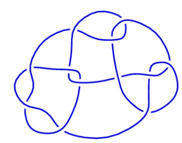
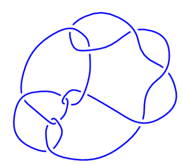
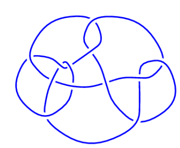
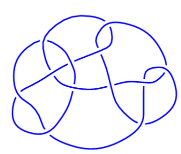
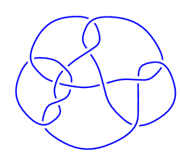
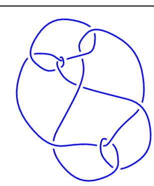
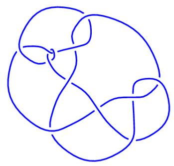
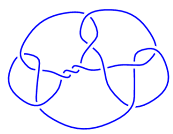
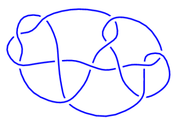
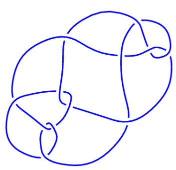
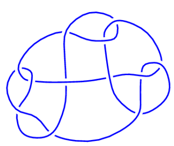
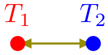
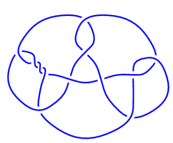
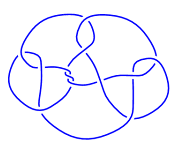
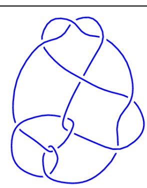
Knot	Diagram	Complex
12_1		
12_2		
12_3		
12_4		
12_5		
12_6		
12_7		
12_8		
12_9		
12_{10}		
12_{11}		
12_{12}		
12_{13}		
12_{14}		
12_{15}		
12_{16}		

Table 7.26. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{17}		T 
12_{18}		T 
12_{19}		T 
12_{20}		T 
12_{21}		T_1 T_2 
12_{22}		T 
12_{23}		T 
12_{24}		T 

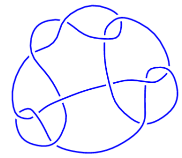
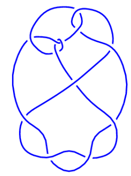
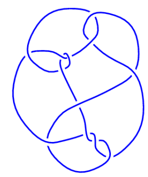
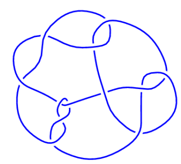
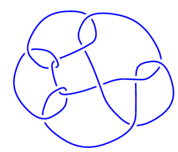
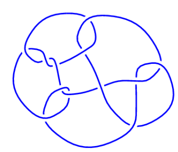
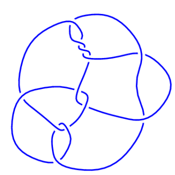
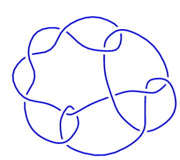
Knot	Diagram	Complex
12_{25}		T 
12_{26}		T 
12_{27}		T 
12_{28}		T 
12_{29}		T 
12_{30}		T 
12_{31}		T 
12_{32}		T 

Table 7.27. The Kakimizu complexes of 12 crossing alternating knots

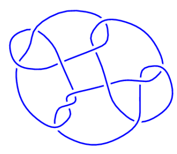
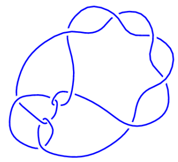
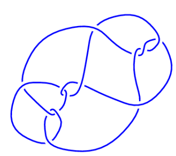
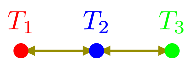
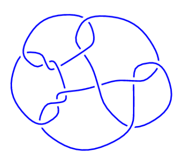
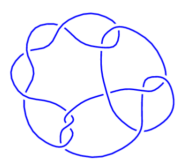
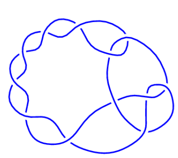
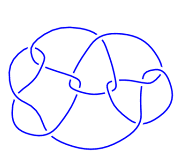
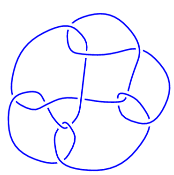
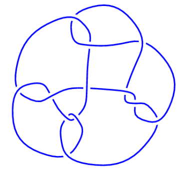
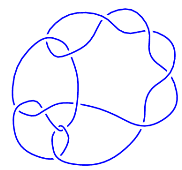
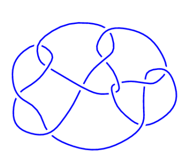
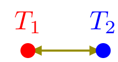
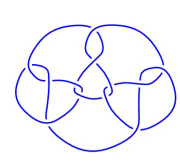
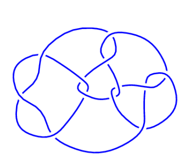
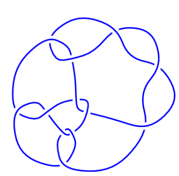
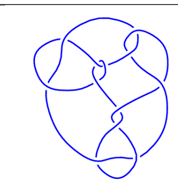
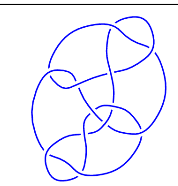
Knot	Diagram	Complex
12_{33}		
12_{34}		
12_{35}		
12_{36}		
12_{37}		
12_{38}		
12_{39}		
12_{40}		
12_{41}		
12_{42}		
12_{43}		
12_{44}		
12_{45}		
12_{46}		
12_{47}		
12_{48}		

Table 7.28. The Kakimizu complexes of 12 crossing alternating knots

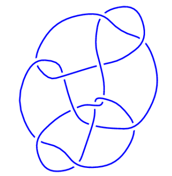
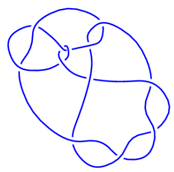
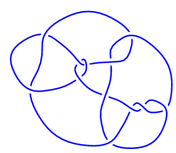
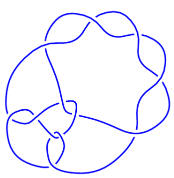
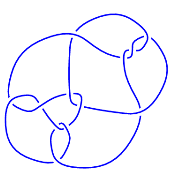
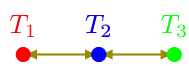
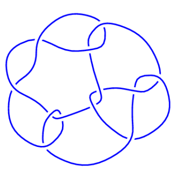
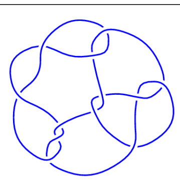
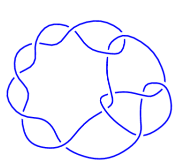
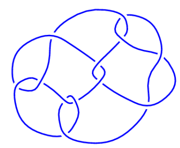
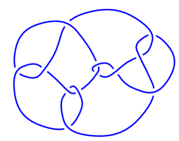
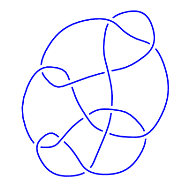
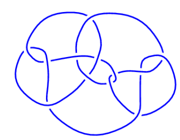
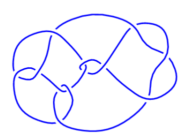
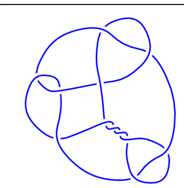
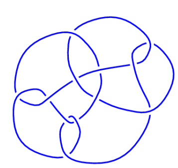
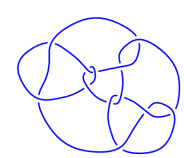
Knot	Diagram	Complex
12_{49}		
12_{50}		
12_{51}		
12_{52}		
12_{53}		
12_{54}		
12_{55}		
12_{56}		
12_{57}		
12_{58}		
12_{59}		
12_{60}		
12_{61}		
12_{62}		
12_{63}		
12_{64}		

Table 7.29. The Kakimizu complexes of 12 crossing alternating knots

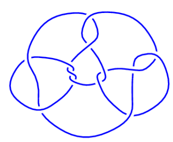
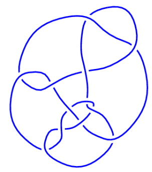
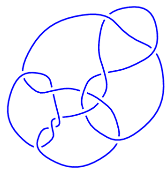
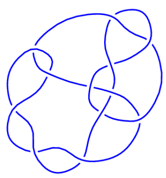
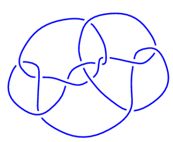
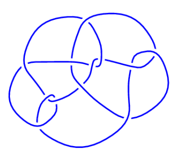
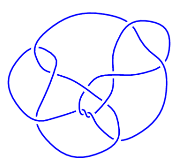
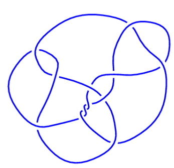
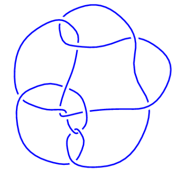
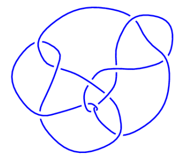
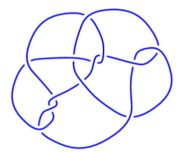
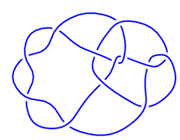
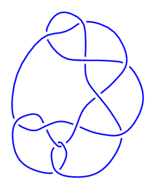
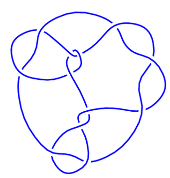
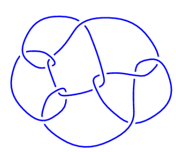
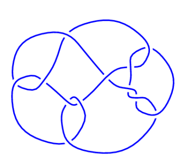
Knot	Diagram	Complex
12_{65}		
12_{66}		
12_{67}		
12_{68}		
12_{69}		
12_{70}		
12_{71}		
12_{72}		
12_{73}		
12_{74}		
12_{75}		
12_{76}		
12_{77}		
12_{78}		
12_{79}		
12_{80}		

Table 7.30. The Kakimizu complexes of 12 crossing alternating knots

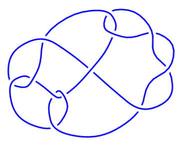
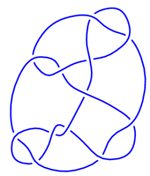
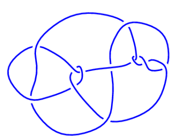
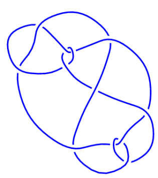
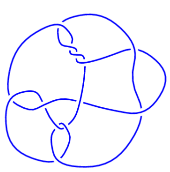
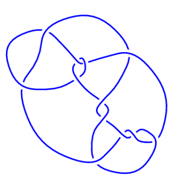
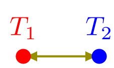
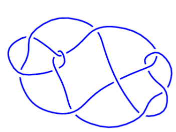
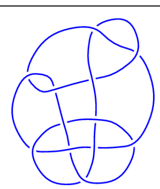
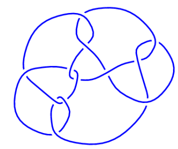
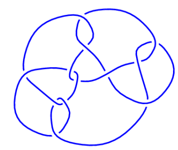
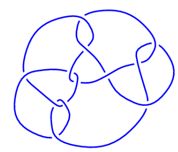
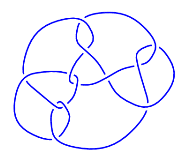
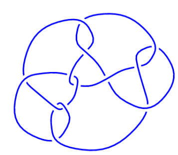
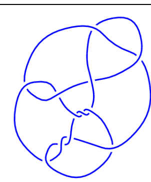
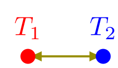
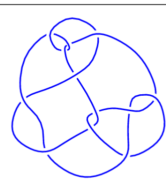
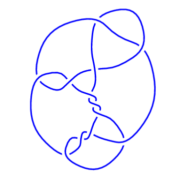
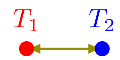
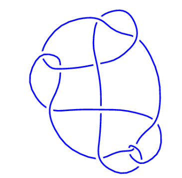
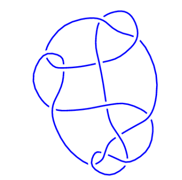
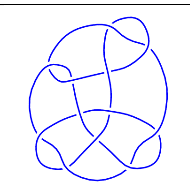
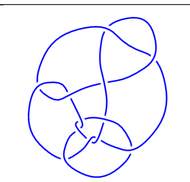
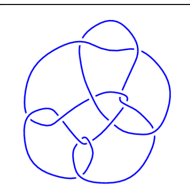
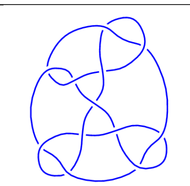
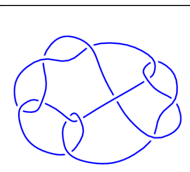
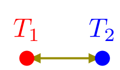
Knot	Diagram	Complex
12_{81}		
12_{82}		
12_{83}		
12_{84}		
12_{85}		
12_{86}		
12_{87}		
12_{88}		
12_{89}		NOT FOUND
12_{90}		
12_{91}		
12_{92}		
12_{93}		
12_{94}		
12_{95}		
12_{96}		

Table 7.31. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{97}		
12_{98}		
12_{99}		
12_{100}		
12_{101}		
12_{102}		
12_{103}		
12_{104}		

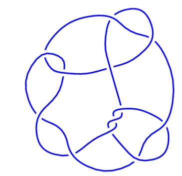
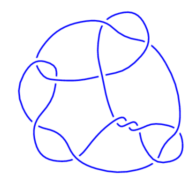
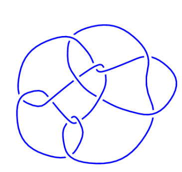
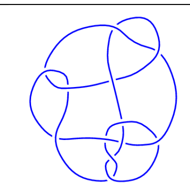
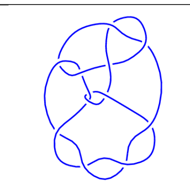
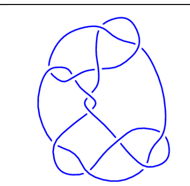
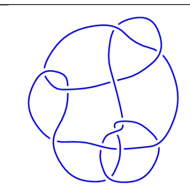
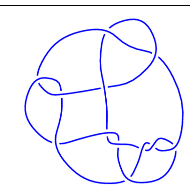
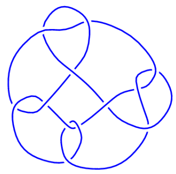
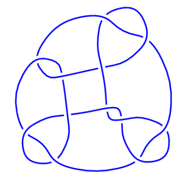
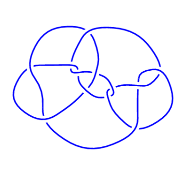
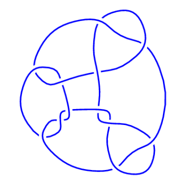
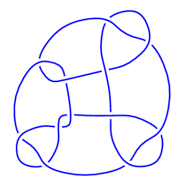
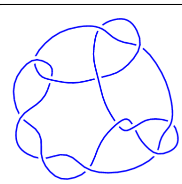
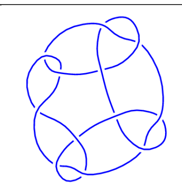
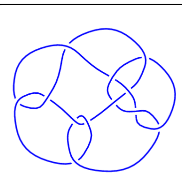
Knot	Diagram	Complex
12_{105}		
12_{106}		
12_{107}		
12_{108}		
12_{109}		
12_{110}		
12_{111}		
12_{112}		

Table 7.32. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{113}		
12_{114}		
12_{115}		
12_{116}		
12_{117}		
12_{118}		
12_{119}		
12_{120}		

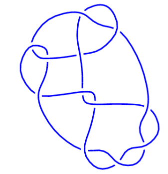
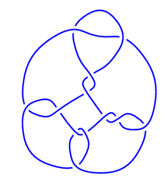
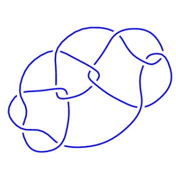
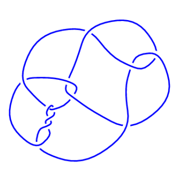
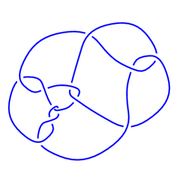
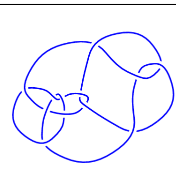
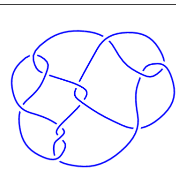
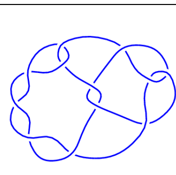
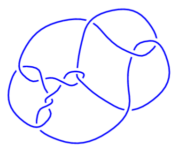
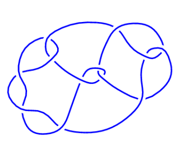
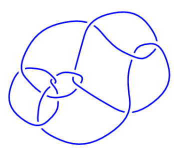
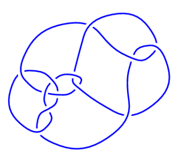
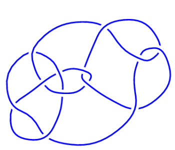
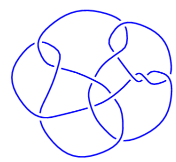
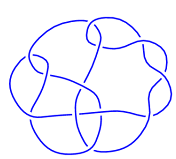
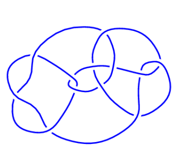
Knot	Diagram	Complex
12_{121}		
12_{122}		
12_{123}		
12_{124}		
12_{125}		
12_{126}		
12_{127}		
12_{128}		

Table 7.33. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{129}		
12_{130}		
12_{131}		
12_{132}		
12_{133}		
12_{134}		
12_{135}		
12_{136}		

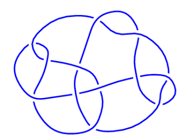
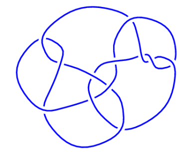
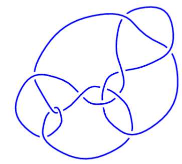
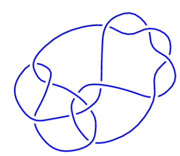
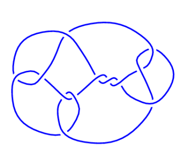
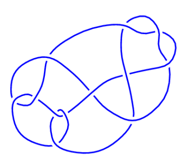
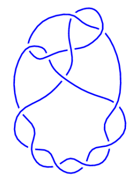
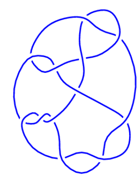
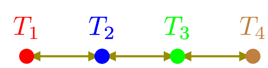
Knot	Diagram	Complex
12_{137}		
12_{138}		
12_{139}		
12_{140}		
12_{141}		
12_{142}		
12_{143}		
12_{144}		

Table 7.34. The Kakimizu complexes of 12 crossing alternating knots

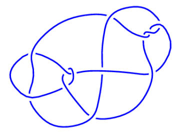
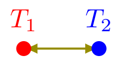
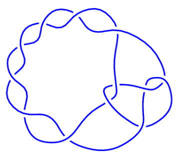
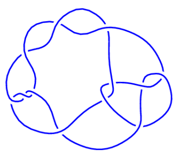
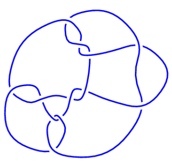
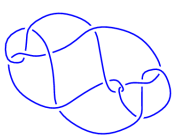
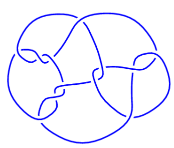
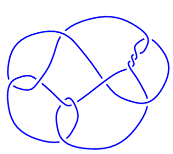
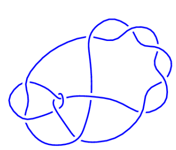
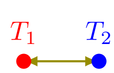
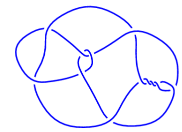
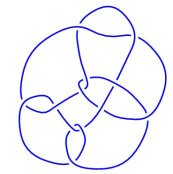
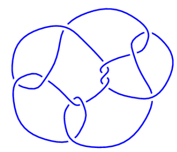
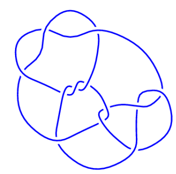
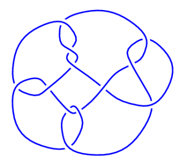
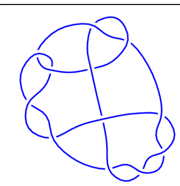
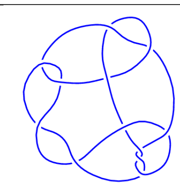
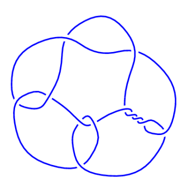
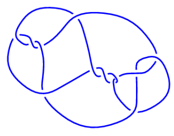
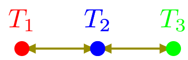
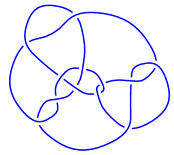
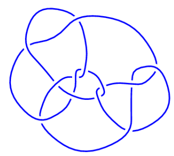
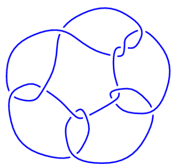
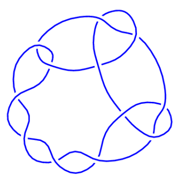
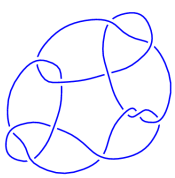
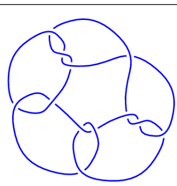
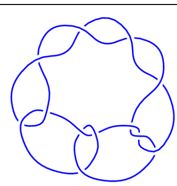
Knot	Diagram	Complex
12_{145}		
12_{146}		
12_{147}		
12_{148}		
12_{149}		
12_{150}		NOT FOUND
12_{151}		
12_{152}		
12_{153}		
12_{154}		
12_{155}		
12_{156}		
12_{157}		
12_{158}		
12_{159}		
12_{160}		

Table 7.35. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{161}		
12_{162}		
12_{163}		
12_{164}		
12_{165}		
12_{166}		
12_{167}		
12_{168}		

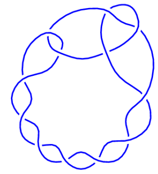
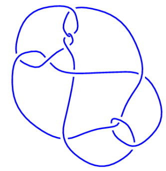
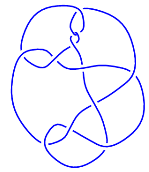
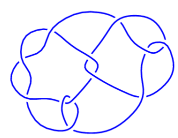
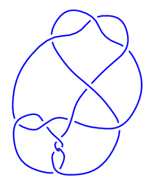
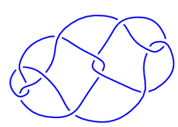
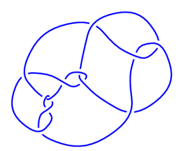
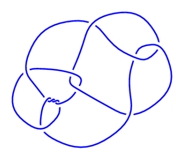
Knot	Diagram	Complex
12_{169}		
12_{170}		
12_{171}		
12_{172}		
12_{173}		
12_{174}		
12_{175}		
12_{176}		

Table 7.36. The Kakimizu complexes of 12 crossing alternating knots

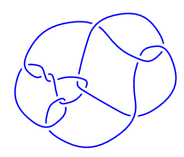
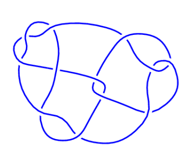
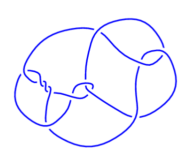
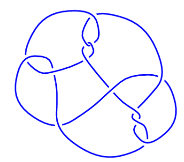
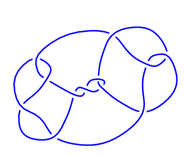
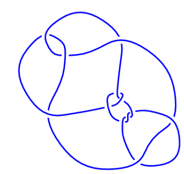
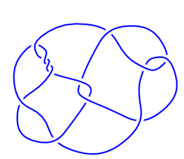
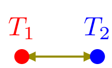
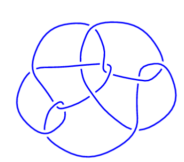
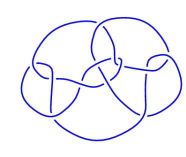
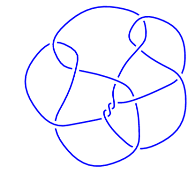
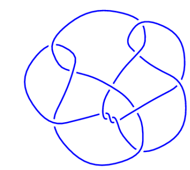
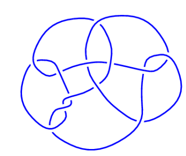
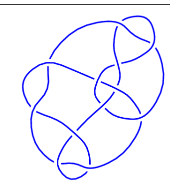
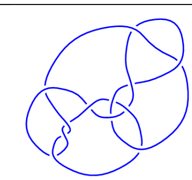
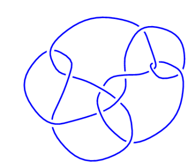
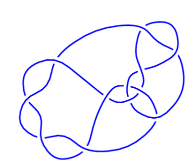
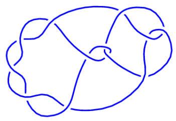
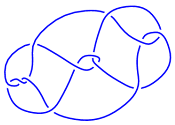
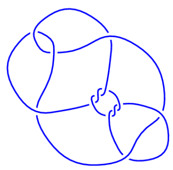
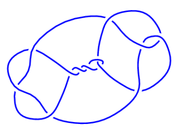
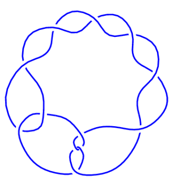
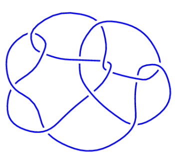
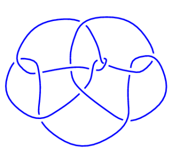
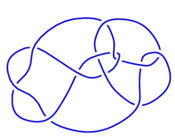
Knot	Diagram	Complex
12_{177}		 T
12_{178}		 T
12_{179}		 T
12_{180}		 T
12_{181}		 T
12_{182}		 T
12_{183}		 T_1 T_2
12_{184}		 T
12_{185}		 T
12_{186}		 T
12_{187}		 T
12_{188}		 T
12_{189}		 T
12_{190}		 T
12_{191}		 T
12_{192}		 T

Table 7.37. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{193}		T
12_{194}		T
12_{195}		T
12_{196}		T
12_{197}		T
12_{198}		T
12_{199}		T
12_{200}		T

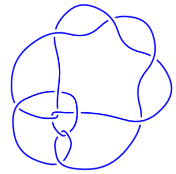
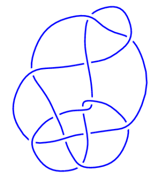
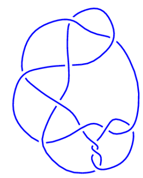
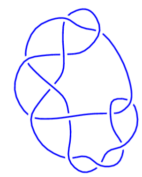
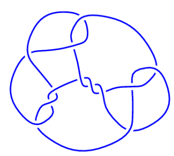
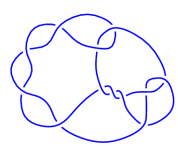
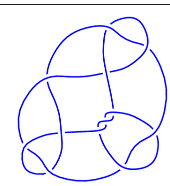
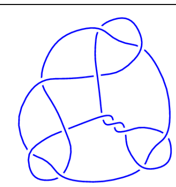
Knot	Diagram	Complex
12_{201}		T
12_{202}		T
12_{203}		T
12_{204}		T
12_{205}		T
12_{206}		T
12_{207}		T
12_{208}		T

Table 7.38. The Kakimizu complexes of 12 crossing alternating knots

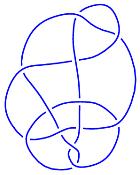
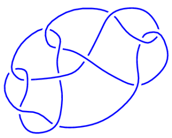
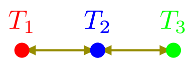
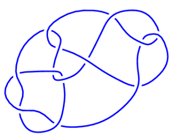
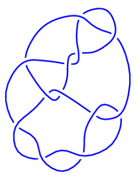
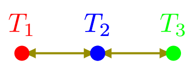
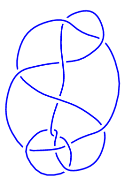
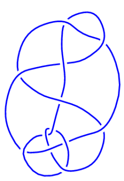
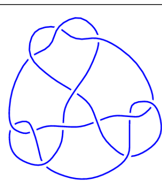
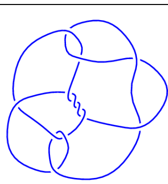
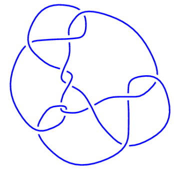
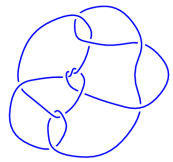
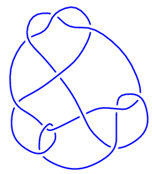
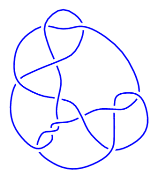
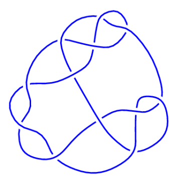
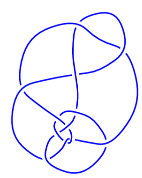
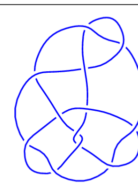
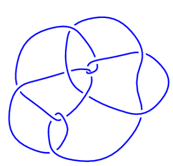
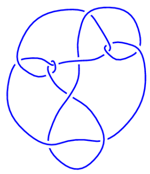
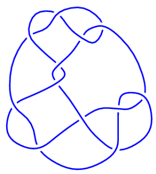
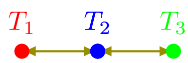
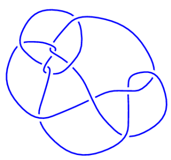
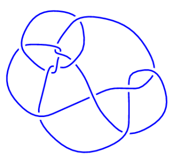
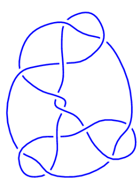
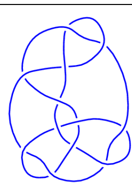
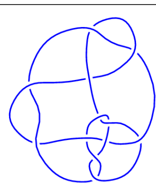
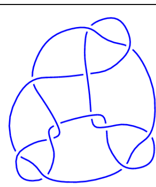
Knot	Diagram	Complex
12_{209}		
12_{210}		
12_{211}		
12_{212}		
12_{213}		
12_{214}		
12_{215}		
12_{216}		
12_{217}		
12_{218}		
12_{219}		
12_{220}		
12_{221}		
12_{222}		
12_{223}		
12_{224}		

Table 7.39. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{225}		
12_{226}		
12_{227}		
12_{228}		
12_{229}		
12_{230}		
12_{231}		
12_{232}		NOT FOUND

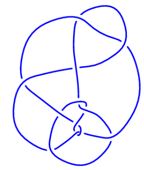
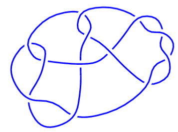
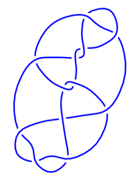
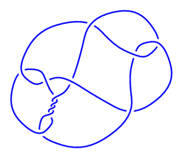
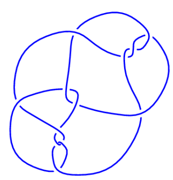
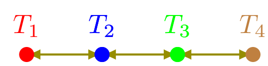
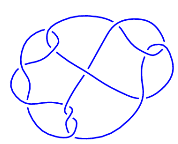
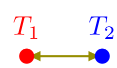
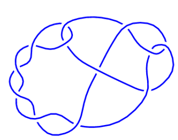
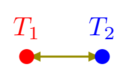
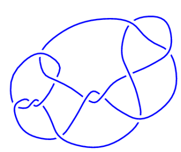
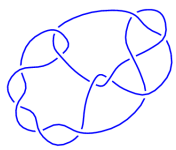
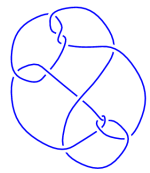
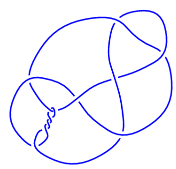
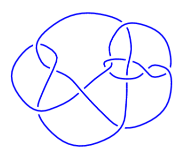
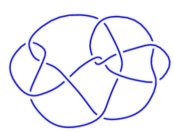
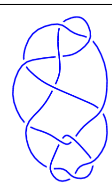
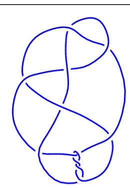
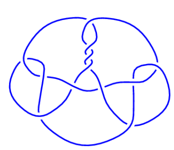
Knot	Diagram	Complex
12_{233}		
12_{234}		
12_{235}		NOT FOUND
12_{236}		
12_{237}		
12_{238}		
12_{239}		
12_{240}		

Table 7.40. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{241}		
12_{242}		
12_{243}		
12_{244}		
12_{245}		
12_{246}		
12_{247}		
12_{248}		

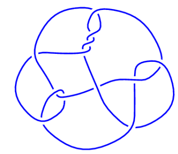
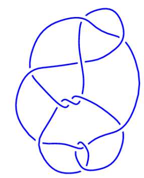
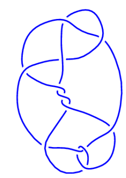
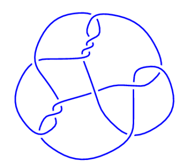
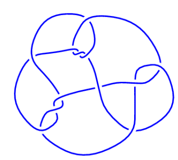
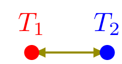
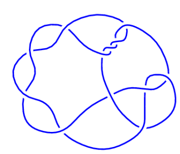
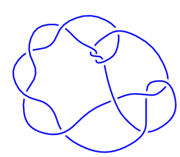
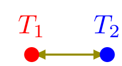
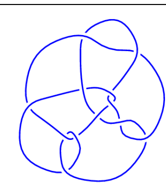
Knot	Diagram	Complex
12_{249}		
12_{250}		
12_{251}		
12_{252}		
12_{253}		
12_{254}		
12_{255}		
12_{256}		

Table 7.41. The Kakimizu complexes of 12 crossing alternating knots

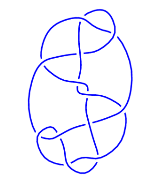
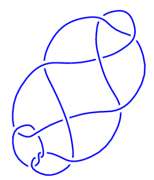
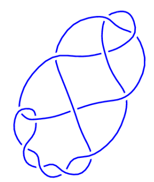
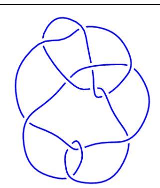
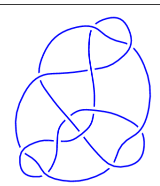
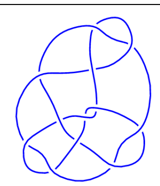
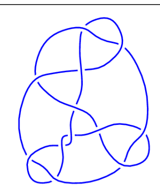
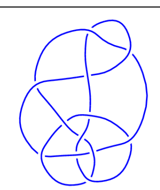
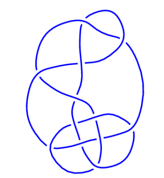
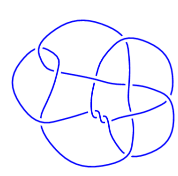
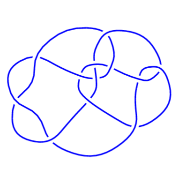
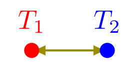
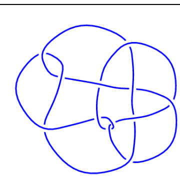
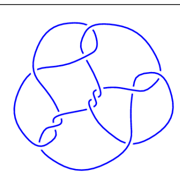
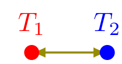
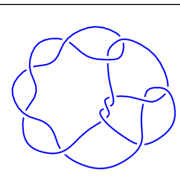
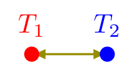
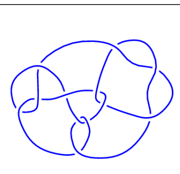
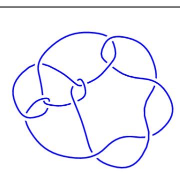
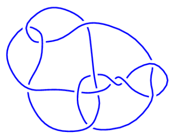
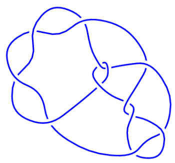
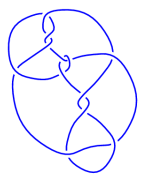
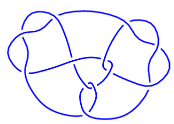
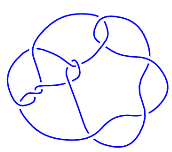
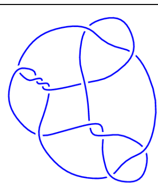
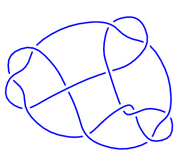
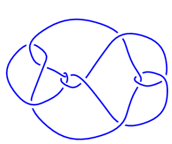
Knot	Diagram	Complex
12_{257}		
12_{258}		
12_{259}		
12_{260}		
12_{261}		
12_{262}		
12_{263}		
12_{264}		
12_{265}		
12_{266}		
12_{267}		
12_{268}		
12_{269}		
12_{270}		
12_{271}		
12_{272}		

Table 7.42. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{273}		NOT FOUND
12_{274}		T
12_{275}		NOT FOUND
12_{276}		T
12_{277}		T
12_{278}		T
12_{279}		T
12_{280}		T

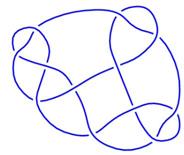
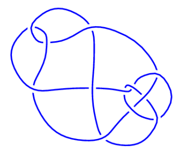
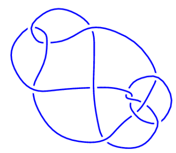
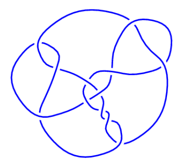
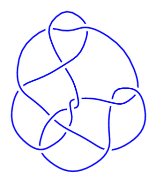
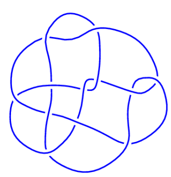
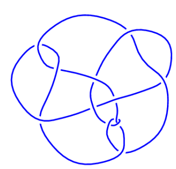
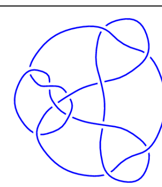
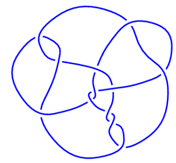
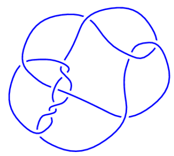
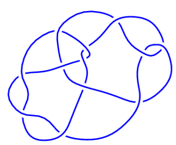
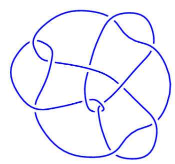
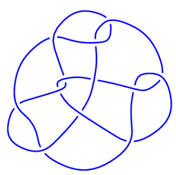
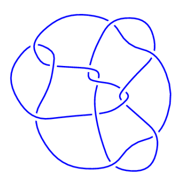
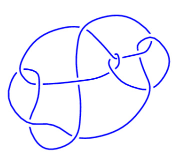
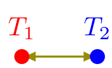
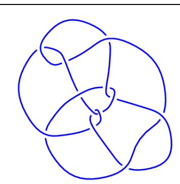
Knot	Diagram	Complex
12_{281}		T
12_{282}		T
12_{283}		T
12_{284}		T
12_{285}		T
12_{286}		T
12_{287}		T
12_{288}		T

Table 7.43. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{289}		
12_{290}		
12_{291}		NOT FOUND
12_{292}		
12_{293}		
12_{294}		
12_{295}		
12_{296}		

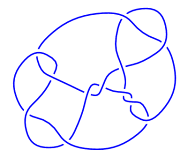
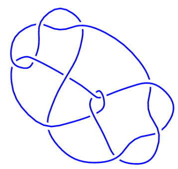
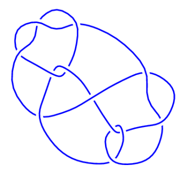
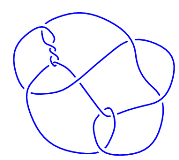
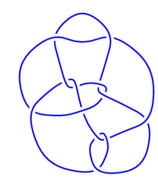
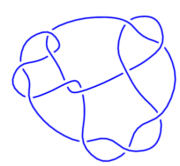
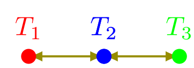
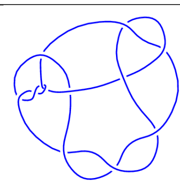
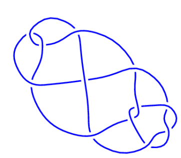
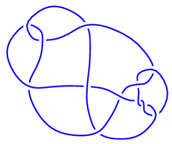
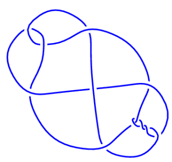
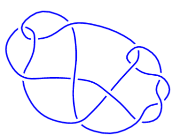
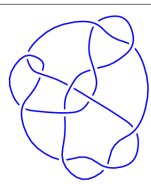
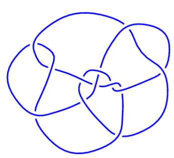
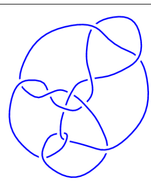
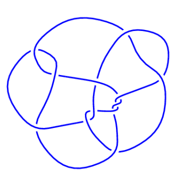
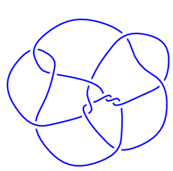
Knot	Diagram	Complex
12_{297}		
12_{298}		
12_{299}		
12_{300}		
12_{301}		
12_{302}		
12_{303}		
12_{304}		

Table 7.44. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{305}		
12_{306}		
12_{307}		
12_{308}		
12_{309}		
12_{310}		
12_{311}		
12_{312}		

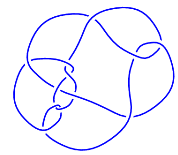
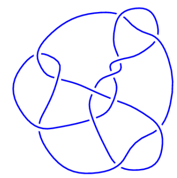
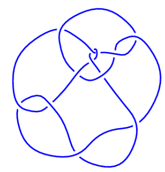
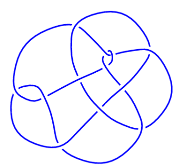
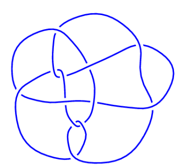
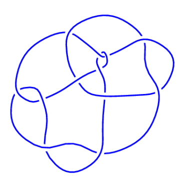
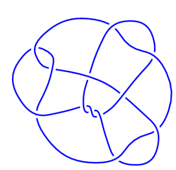
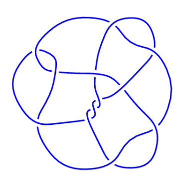
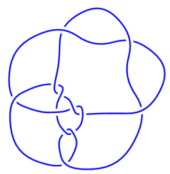
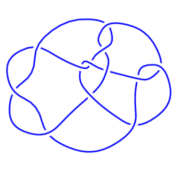
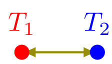
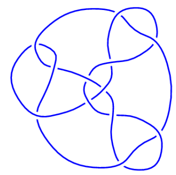
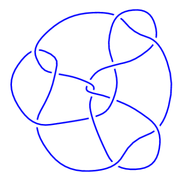
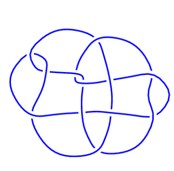
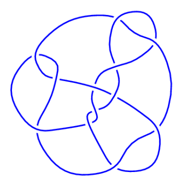
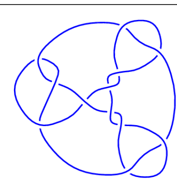
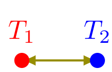
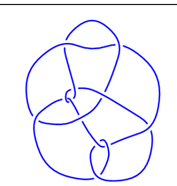
Knot	Diagram	Complex
12_{313}		NOT FOUND
12_{314}		
12_{315}		
12_{316}		
12_{317}		
12_{318}		
12_{319}		
12_{320}		

Table 7.45. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{321}		
12_{322}		
12_{323}		
12_{324}		
12_{325}		
12_{326}		
12_{327}		
12_{328}		

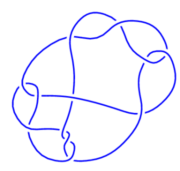
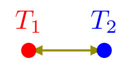
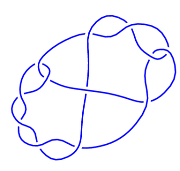
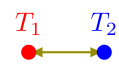
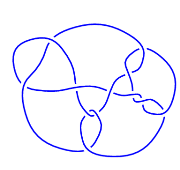
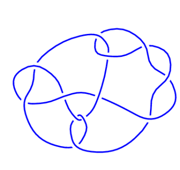
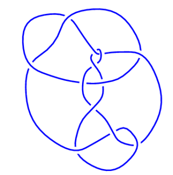
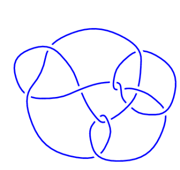
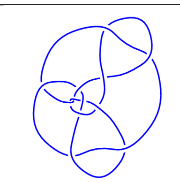
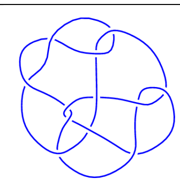
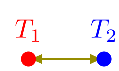
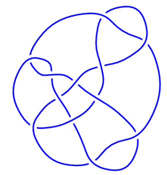
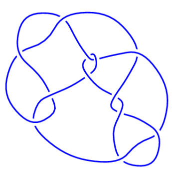
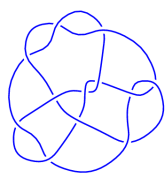
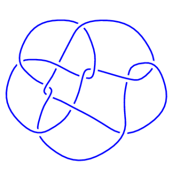
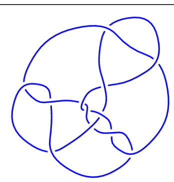
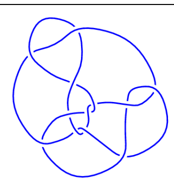
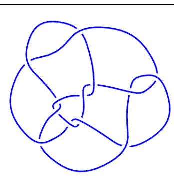
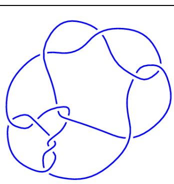
Knot	Diagram	Complex
12_{329}		
12_{330}		
12_{331}		
12_{332}		
12_{333}		
12_{334}		
12_{335}		
12_{336}		

TABLE 7.46. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{337}		
12_{338}		
12_{339}		
12_{340}		
12_{341}		
12_{342}		
12_{343}		
12_{344}		

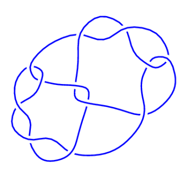
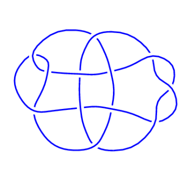
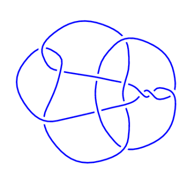
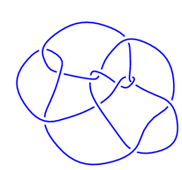
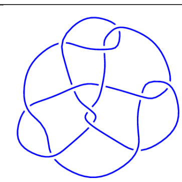
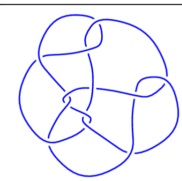
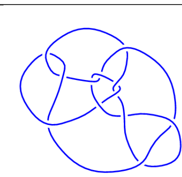
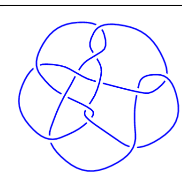
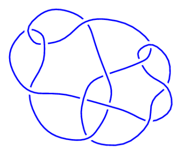
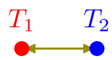
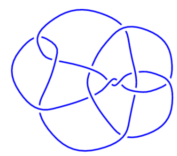
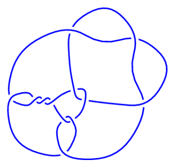
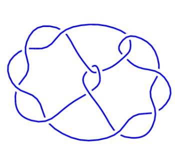
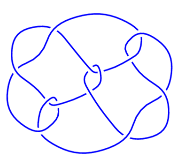
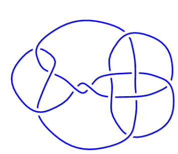
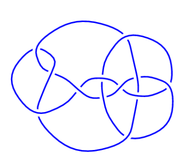
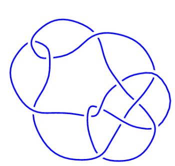
Knot	Diagram	Complex
12_{345}		
12_{346}		
12_{347}		NOT FOUND
12_{348}		
12_{349}		
12_{350}		
12_{351}		
12_{352}		

TABLE 7.47. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{353}		
12_{354}		
12_{355}		
12_{356}		
12_{357}		
12_{358}		
12_{359}		
12_{360}		

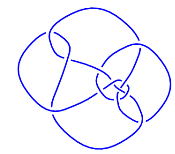
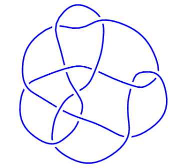
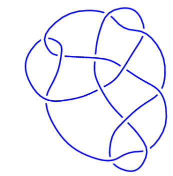
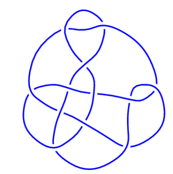
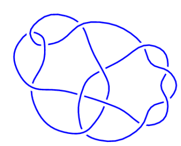
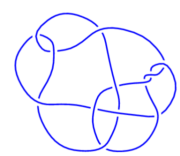
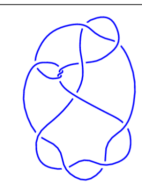
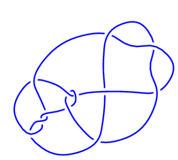
Knot	Diagram	Complex
12_{361}		
12_{362}		
12_{363}		
12_{364}		
12_{365}		
12_{366}		NOT FOUND
12_{367}		
12_{368}		

Table 7.48. The Kakimizu complexes of 12 crossing alternating knots

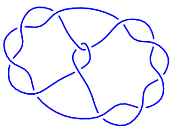
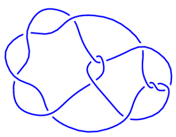
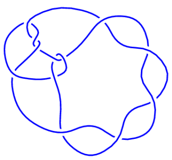
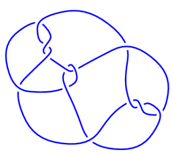
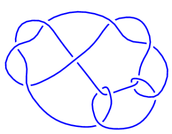
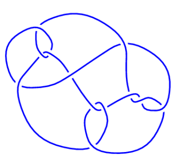
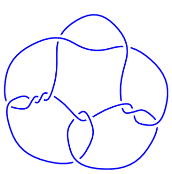
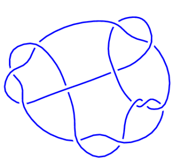
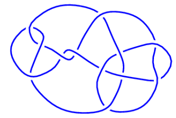
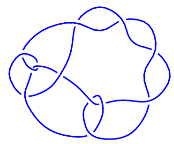
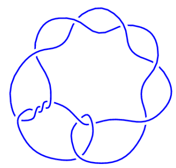
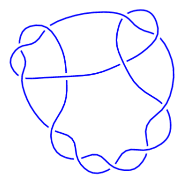
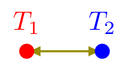
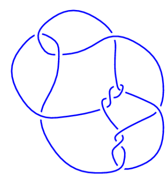
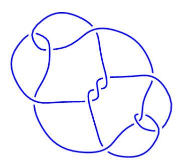
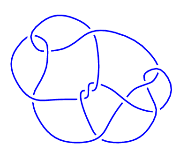
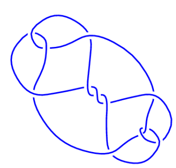
Knot	Diagram	Complex
12_{369}		
12_{370}		
12_{371}		
12_{372}		NOT FOUND
12_{373}		
12_{374}		
12_{375}		
12_{376}		NOT FOUND
12_{377}		
12_{378}		
12_{379}		
12_{380}		
12_{381}		
12_{382}		
12_{383}		
12_{384}		

Table 7.49. The Kakimizu complexes of 12 crossing alternating knots

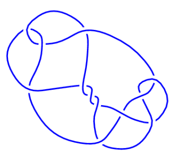
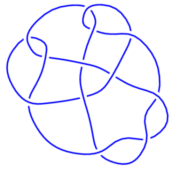
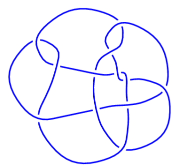
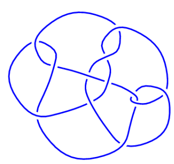
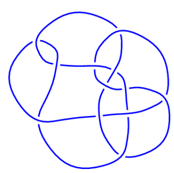
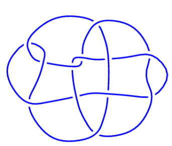
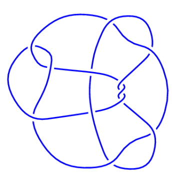
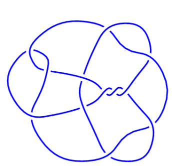
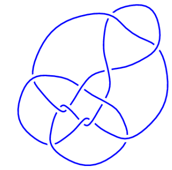
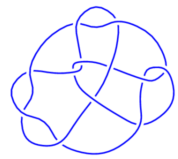
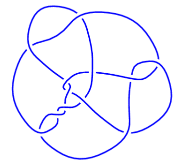
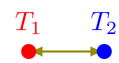
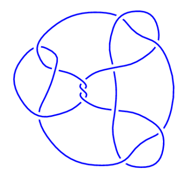
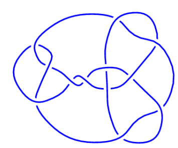
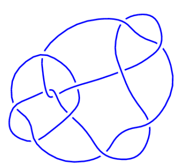
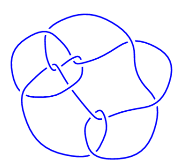
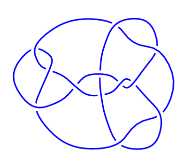
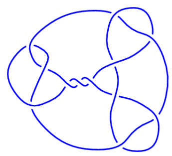
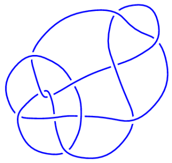
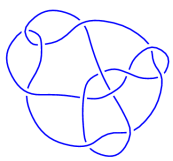
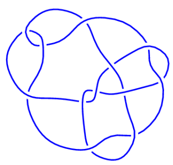
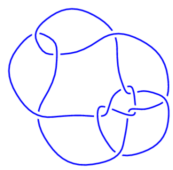
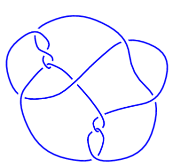
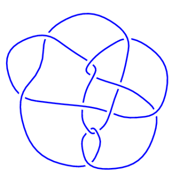
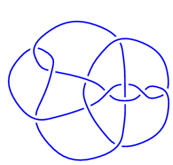
Knot	Diagram	Complex
12_{385}		
12_{386}		
12_{387}		
12_{388}		
12_{389}		
12_{390}		
12_{391}		
12_{392}		
12_{393}		
12_{394}		
12_{395}		
12_{396}		
12_{397}		
12_{398}		
12_{399}		
12_{400}		

Table 7.50. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{401}		
12_{402}		
12_{403}		
12_{404}		
12_{405}		
12_{406}		
12_{407}		
12_{408}		

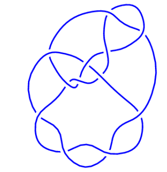
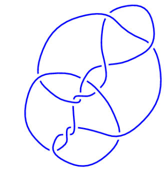
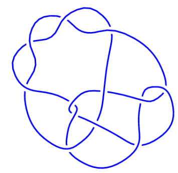
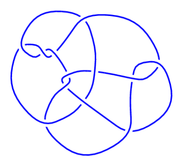
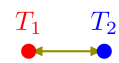
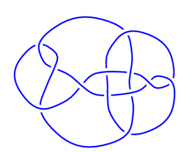
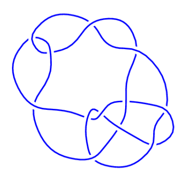
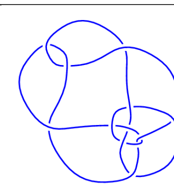
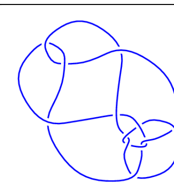
Knot	Diagram	Complex
12_{409}		
12_{410}		NOT FOUND
12_{411}		
12_{412}		
12_{413}		
12_{414}		
12_{415}		
12_{416}		

Table 7.51. The Kakimizu complexes of 12 crossing alternating knots

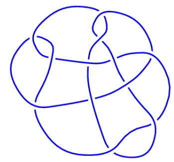
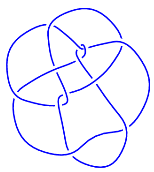
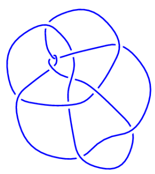
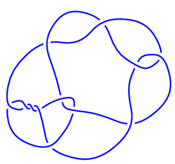
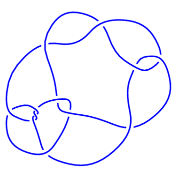
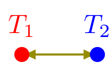
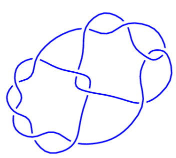
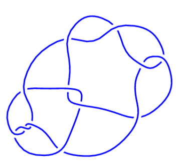
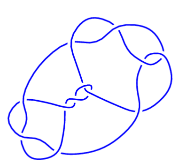
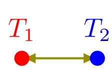
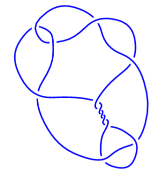
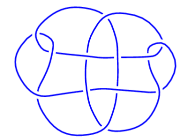
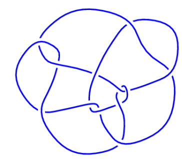
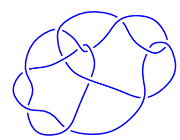
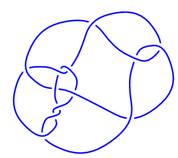
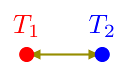
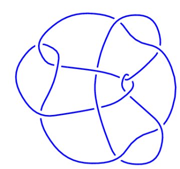
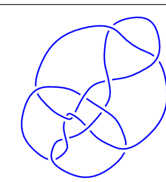
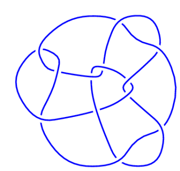
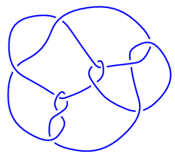
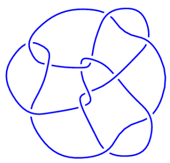
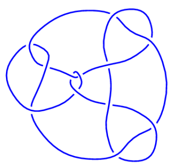
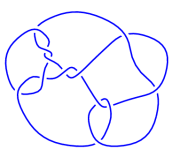
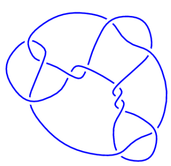
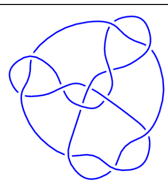
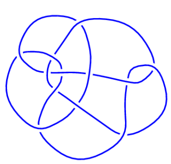
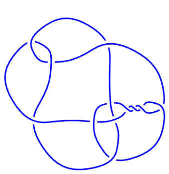
Knot	Diagram	Complex
12_{417}		
12_{418}		
12_{419}		
12_{420}		
12_{421}		
12_{422}		
12_{423}		NOT FOUND
12_{424}		
12_{425}		
12_{426}		
12_{427}		
12_{428}		
12_{429}		
12_{430}		
12_{431}		
12_{432}		

Table 7.52. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{433}		T
12_{434}		T
12_{435}		T
12_{436}		T
12_{437}		T
12_{438}		T
12_{439}		T
12_{440}		T

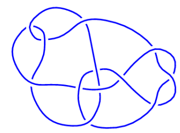
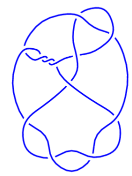
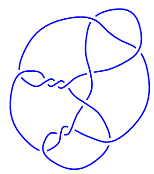
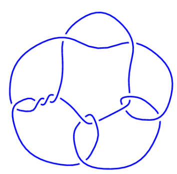
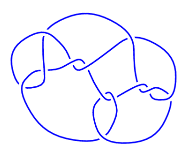
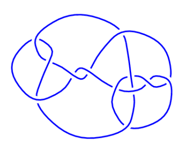
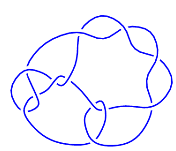
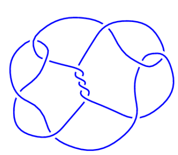
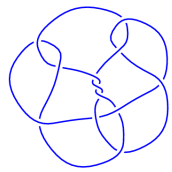
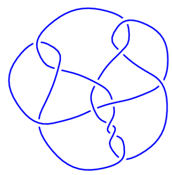
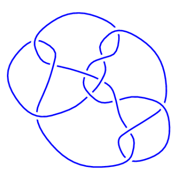
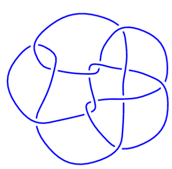
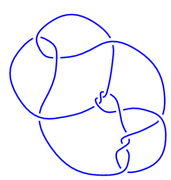
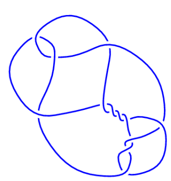
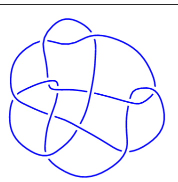
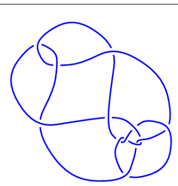
Knot	Diagram	Complex
12_{441}		NOT FOUND
12_{442}		T
12_{443}		T_1 T_2
12_{444}		T
12_{445}		T
12_{446}		T
12_{447}		T
12_{448}		T

Table 7.53. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{449}		T 
12_{450}		T 
12_{451}		T 
12_{452}		T 
12_{453}		T 
12_{454}		T 
12_{455}		T 
12_{456}		T 

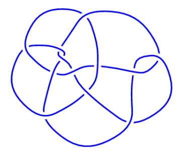
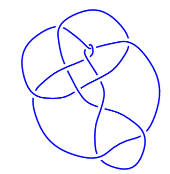
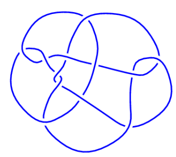
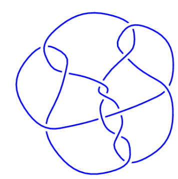
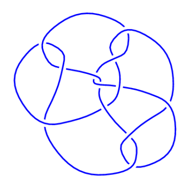
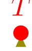
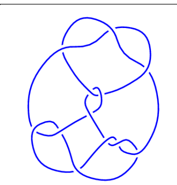
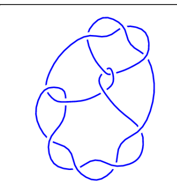
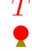
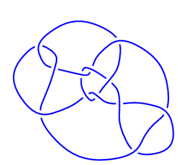
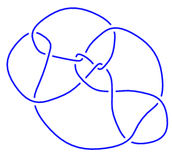
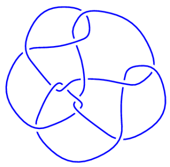
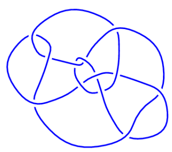
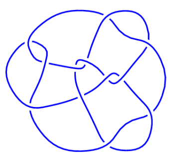
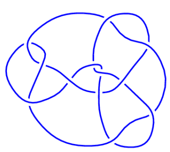
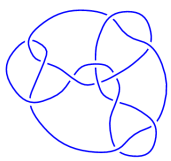
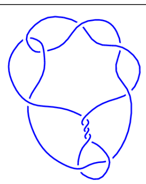
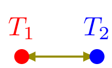
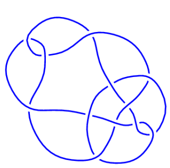
Knot	Diagram	Complex
12_{457}		T 
12_{458}		T 
12_{459}		T 
12_{460}		T 
12_{461}		T 
12_{462}		T 
12_{463}		T 
12_{464}		T 

Table 7.54. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{465}		
12_{466}		
12_{467}		
12_{468}		
12_{469}		
12_{470}		
12_{471}		
12_{472}		

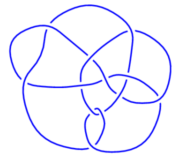
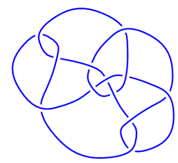
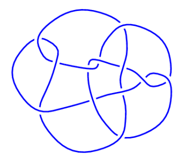
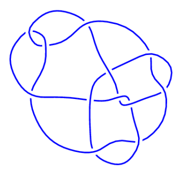
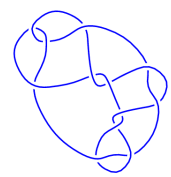
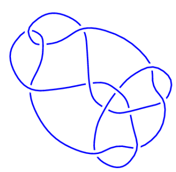
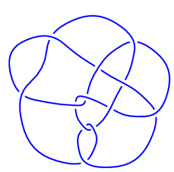
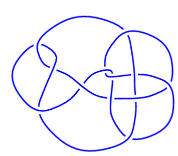
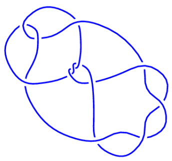
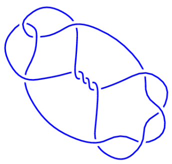
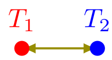
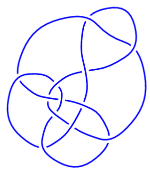
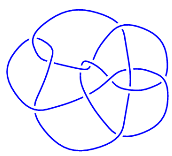
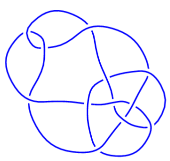
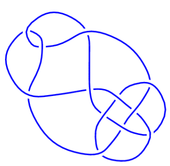
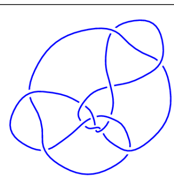
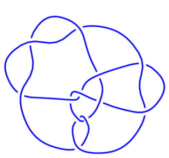
Knot	Diagram	Complex
12_{473}		
12_{474}		
12_{475}		
12_{476}		
12_{477}		
12_{478}		
12_{479}		
12_{480}		

Table 7.55. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{481}		
12_{482}		
12_{483}		
12_{484}		
12_{485}		
12_{486}		
12_{487}		
12_{488}		

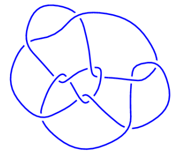
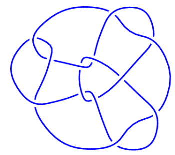
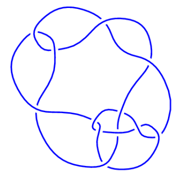
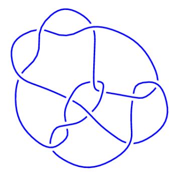
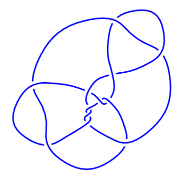
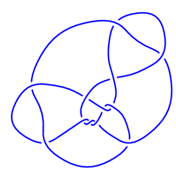
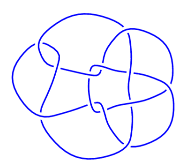
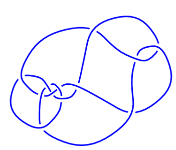
Knot	Diagram	Complex
12_{489}		
12_{490}		
12_{491}		
12_{492}		
12_{493}		
12_{494}		
12_{495}		
12_{496}		

Table 7.56. The Kakimizu complexes of 12 crossing alternating knots

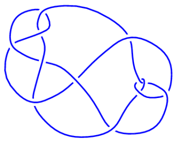
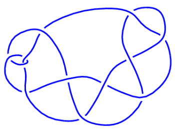
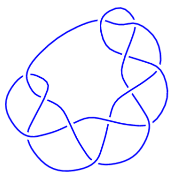
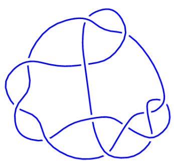
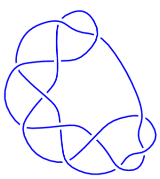
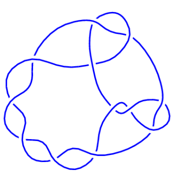
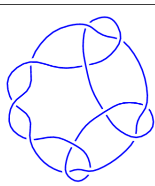
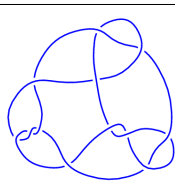
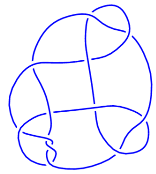
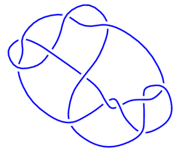
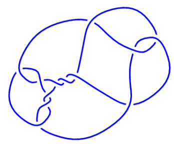
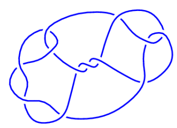
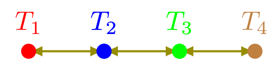
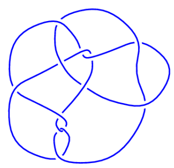
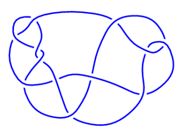
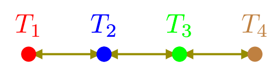
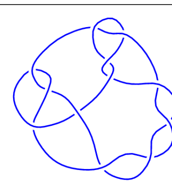
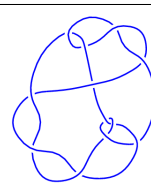
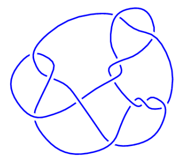
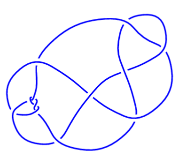
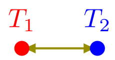
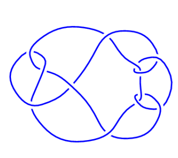
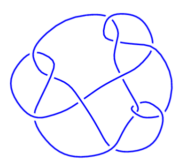
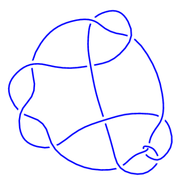
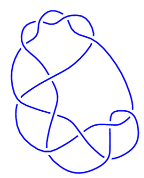
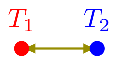
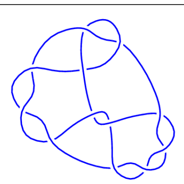
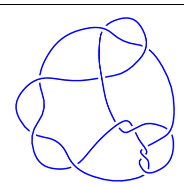
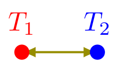
Knot	Diagram	Complex
12_{497}		T
12_{498}		T
12_{499}		T
12_{500}		T
12_{501}		T
12_{502}		T
12_{503}		T
12_{504}		NOT FOUND
12_{505}		T
12_{506}		T
12_{507}		T
12_{508}		
12_{509}		T
12_{510}		
12_{511}		T
12_{512}		T

Table 7.57. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{513}		NOT FOUND
12_{514}		
12_{515}		
12_{516}		
12_{517}		
12_{518}		
12_{519}		
12_{520}		

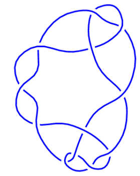
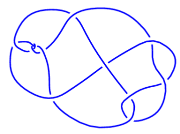
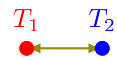
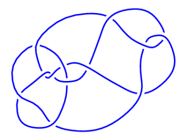
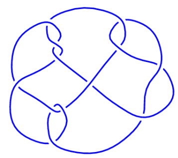
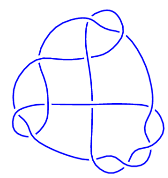
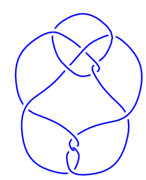
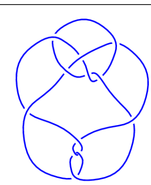
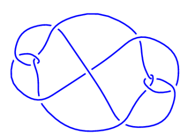
Knot	Diagram	Complex
12_{521}		
12_{522}		
12_{523}		
12_{524}		NOT FOUND
12_{525}		
12_{526}		
12_{527}		
12_{528}		

Table 7.58. The Kakimizu complexes of 12 crossing alternating knots

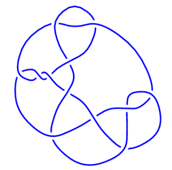
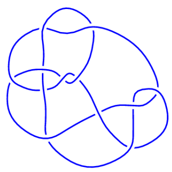
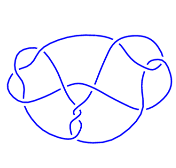
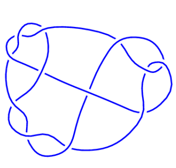
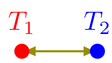
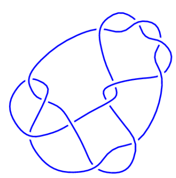
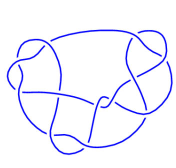
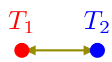
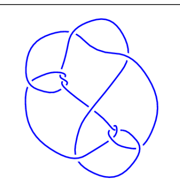
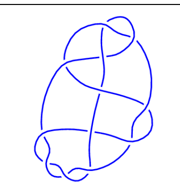
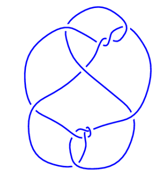
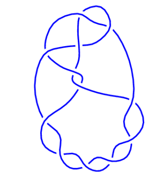
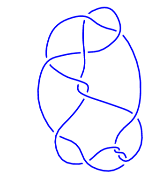
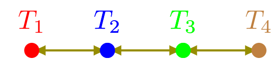
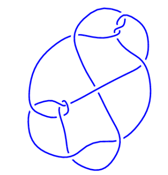
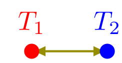
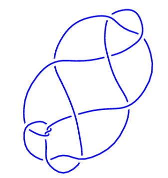
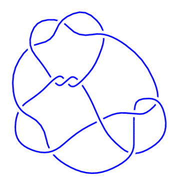
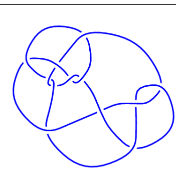
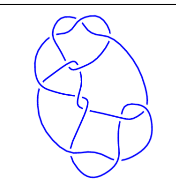
Knot	Diagram	Complex
12_{529}		
12_{530}		
12_{531}		
12_{532}		
12_{533}		
12_{534}		
12_{535}		
12_{536}		
12_{537}		
12_{538}		
12_{539}		
12_{540}		
12_{541}		
12_{542}		
12_{543}		
12_{544}		

Table 7.59. The Kakimizu complexes of 12 crossing alternating knots

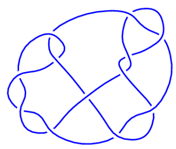
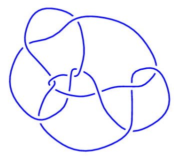
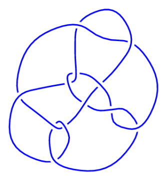
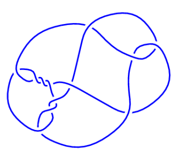
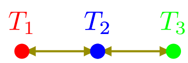
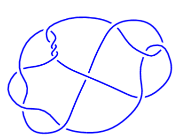
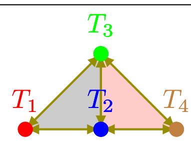
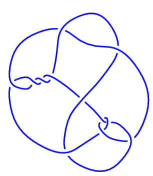
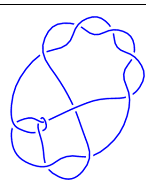
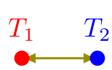
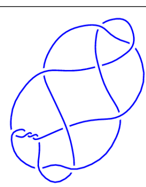
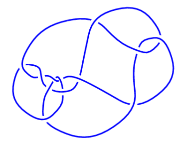
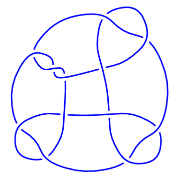
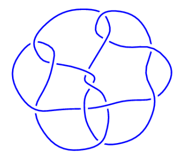
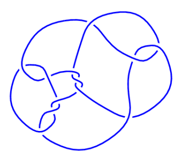
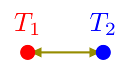
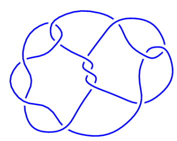
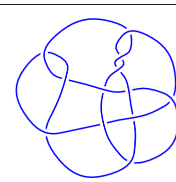
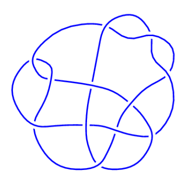
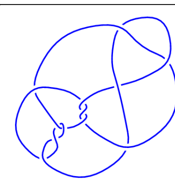
Knot	Diagram	Complex
12_{545}		
12_{546}		
12_{547}		
12_{548}		
12_{549}		
12_{550}		
12_{551}		
12_{552}		
12_{553}		
12_{554}		
12_{555}		
12_{556}		
12_{557}		
12_{558}		
12_{559}		NOT FOUND
12_{560}		

Table 7.60. The Kakimizu complexes of 12 crossing alternating knots

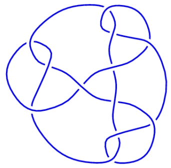
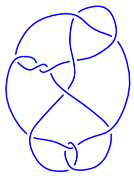
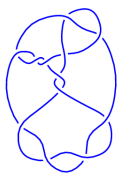
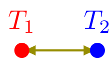
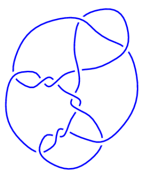
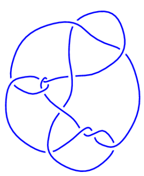
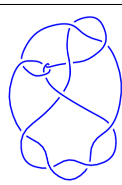
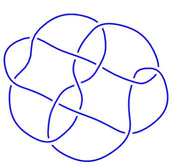
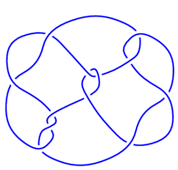
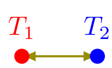
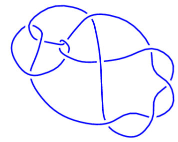
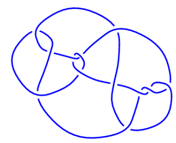
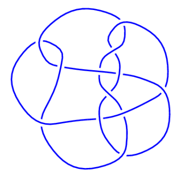
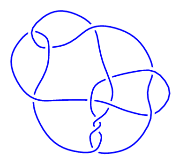
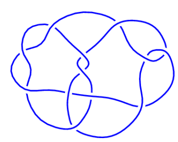
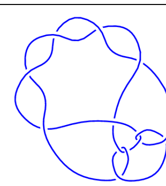
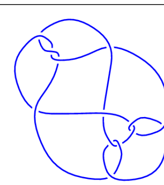
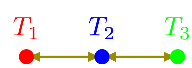
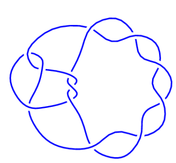
Knot	Diagram	Complex
12_{561}		
12_{562}		
12_{563}		
12_{564}		
12_{565}		
12_{566}		
12_{567}		
12_{568}		
12_{569}		
12_{570}		
12_{571}		
12_{572}		
12_{573}		
12_{574}		
12_{575}		
12_{576}		

Table 7.61. The Kakimizu complexes of 12 crossing alternating knots

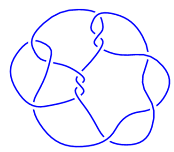
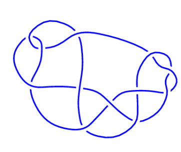
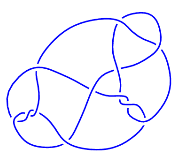
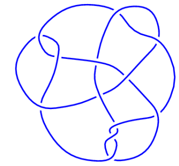
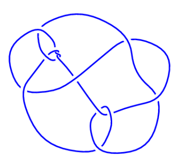
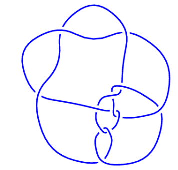
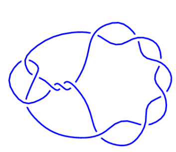
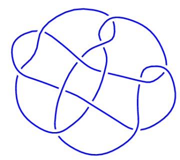
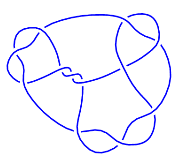
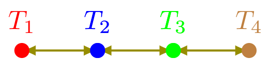
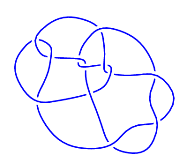
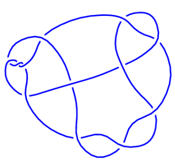
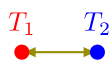
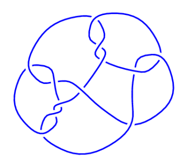
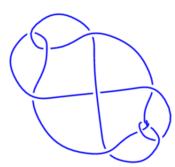
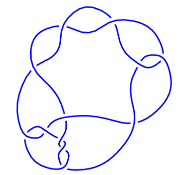
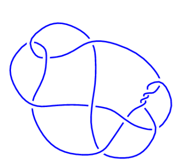
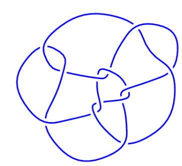
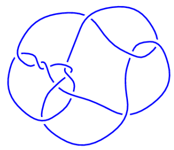
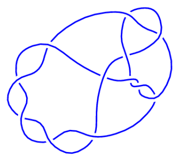
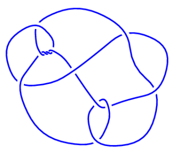
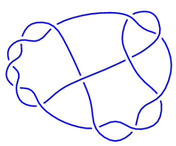
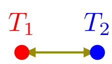
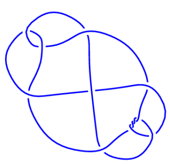
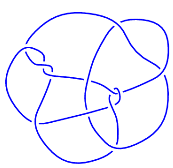
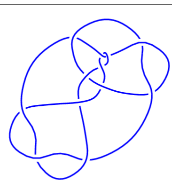
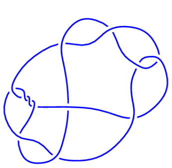
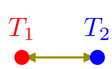
Knot	Diagram	Complex	Knot	Diagram	Complex
12_{577}			12_{585}		
12_{578}			12_{586}		
12_{579}			12_{587}		
12_{580}			12_{588}		
12_{581}			12_{589}		
12_{582}			12_{590}		
12_{583}			12_{591}		
12_{584}			12_{592}		

TABLE 7.62. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{593}		NOT FOUND
12_{594}		
12_{595}		
12_{596}		
12_{597}		
12_{598}		
12_{599}		
12_{600}		

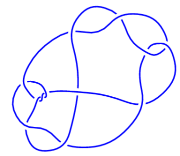
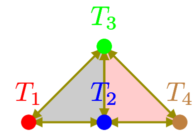
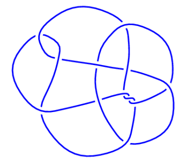
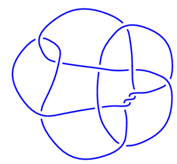
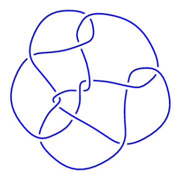
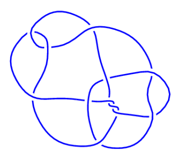
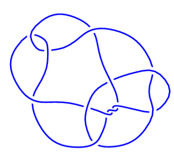
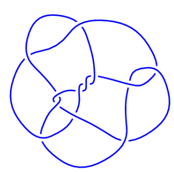
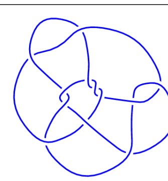
Knot	Diagram	Complex
12_{601}		
12_{602}		
12_{603}		NOT FOUND
12_{604}		
12_{605}		
12_{606}		NOT FOUND
12_{607}		
12_{608}		NOT FOUND

Table 7.63. The Kakimizu complexes of 12 crossing alternating knots

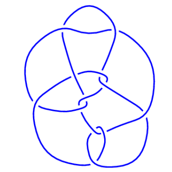
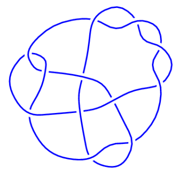
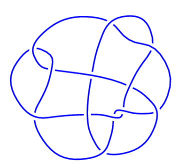
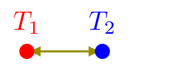
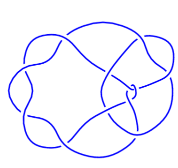
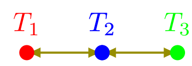
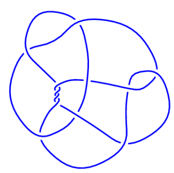
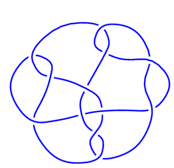
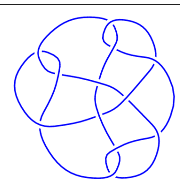
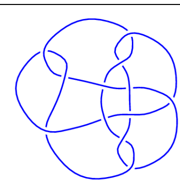
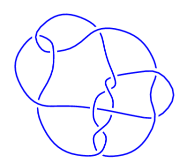
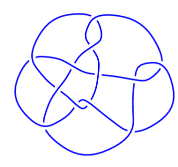
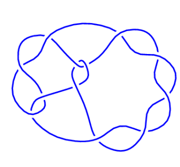
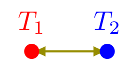
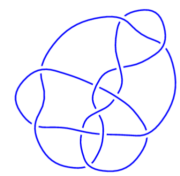
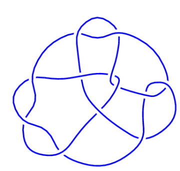
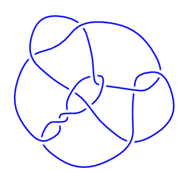
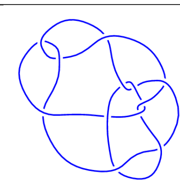
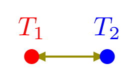
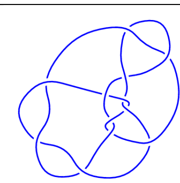
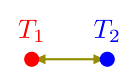
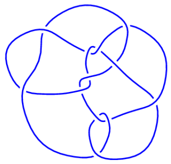
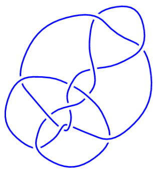
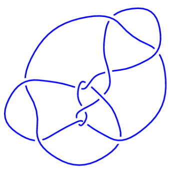
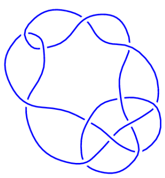
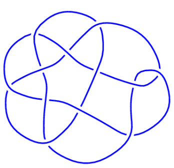
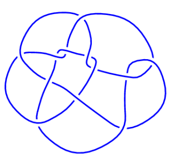
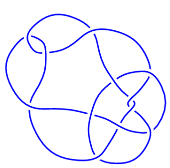
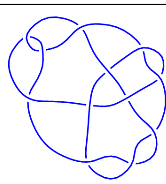
Knot	Diagram	Complex
12_{609}		
12_{610}		
12_{611}		
12_{612}		
12_{613}		
12_{614}		
12_{615}		
12_{616}		
12_{617}		
12_{618}		
12_{619}		
12_{620}		
12_{621}		
12_{622}		
12_{623}		
12_{624}		

Table 7.64. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{625}		
12_{626}		
12_{627}		
12_{628}		
12_{629}		
12_{630}		
12_{631}		NOT FOUND
12_{632}		NOT FOUND

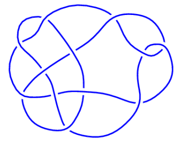
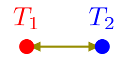
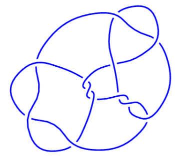
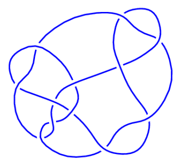
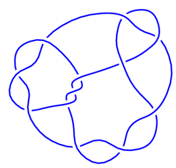
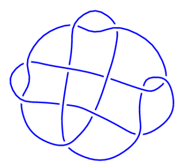
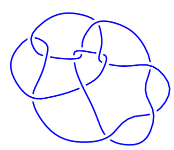
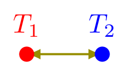
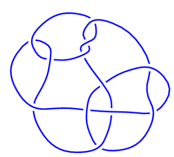
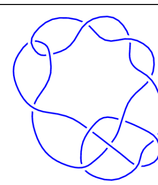
Knot	Diagram	Complex
12_{633}		
12_{634}		NOT FOUND
12_{635}		
12_{636}		
12_{637}		
12_{638}		
12_{639}		
12_{640}		

Table 7.65. The Kakimizu complexes of 12 crossing alternating knots

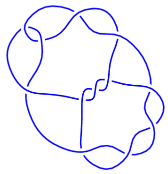
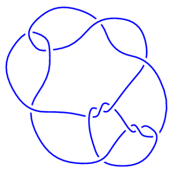
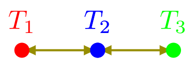
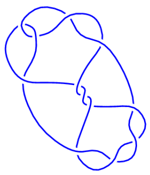
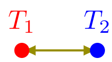
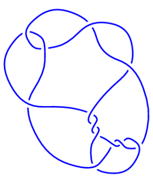
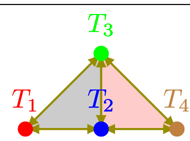
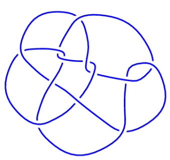
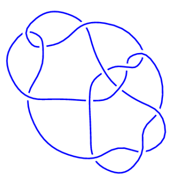
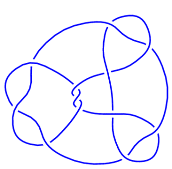
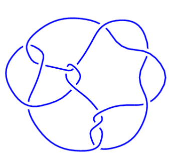
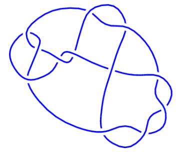
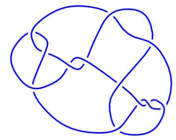
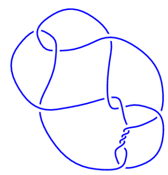
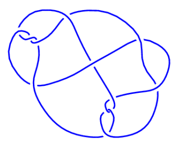
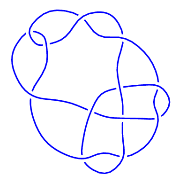
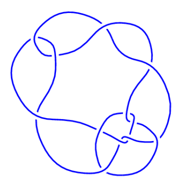
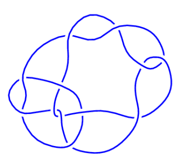
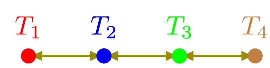
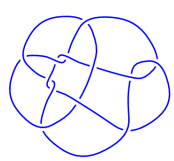
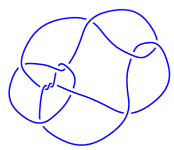
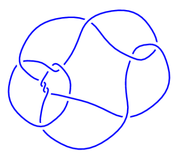
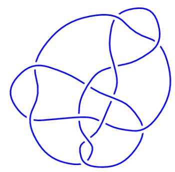
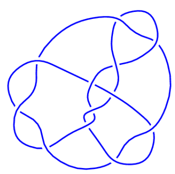
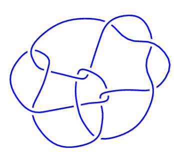
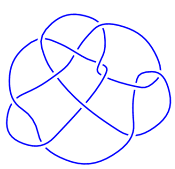
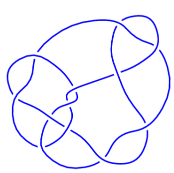
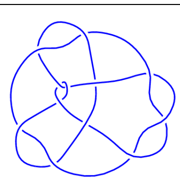
Knot	Diagram	Complex
12_{641}		
12_{642}		
12_{643}		
12_{644}		
12_{645}		
12_{646}		
12_{647}		
12_{648}		
12_{649}		
12_{650}		
12_{651}		
12_{652}		
12_{653}		
12_{654}		
12_{655}		
12_{656}		

Table 7.66. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{657}		T 
12_{658}		NOT FOUND
12_{659}		T 
12_{660}		T 
12_{661}		NOT FOUND
12_{662}		T 
12_{663}		T 
12_{664}		T 

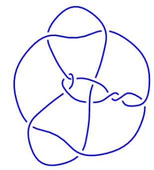
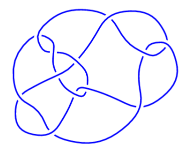
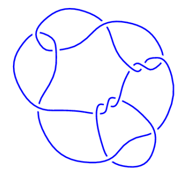
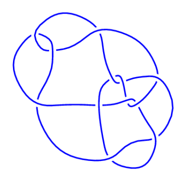
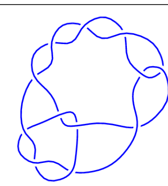
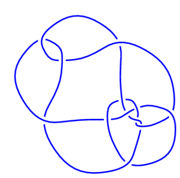
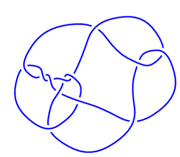
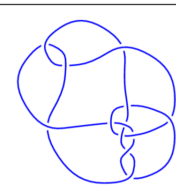
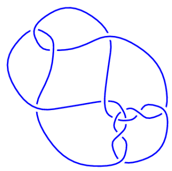
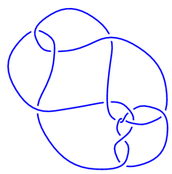
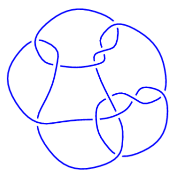
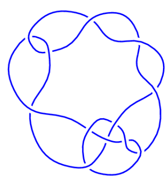
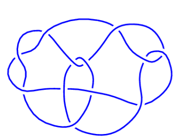
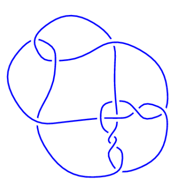
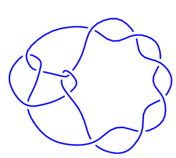
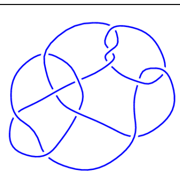
Knot	Diagram	Complex
12_{665}		T 
12_{666}		T 
12_{667}		T 
12_{668}		T 
12_{669}		T 
12_{670}		T 
12_{671}		T 
12_{672}		T 

Table 7.67. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{673}		T
12_{674}		T
12_{675}		T
12_{676}		T
12_{677}		NOT FOUND
12_{678}		NOT FOUND
12_{679}		T
12_{680}		T

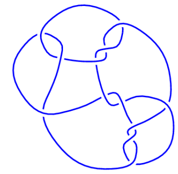
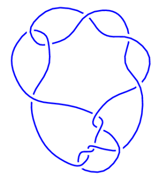
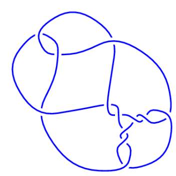
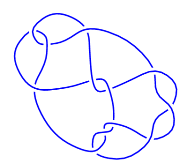
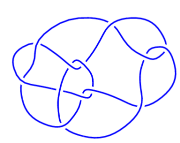
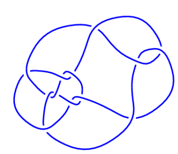
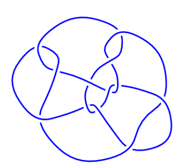
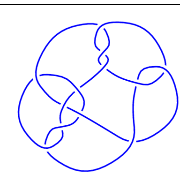
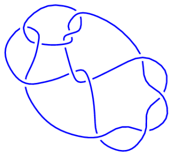
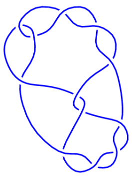
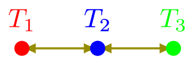
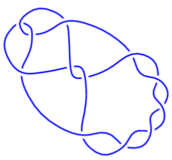
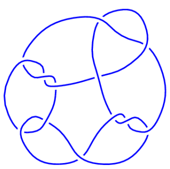
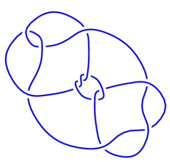
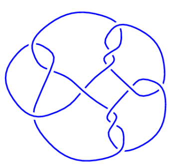
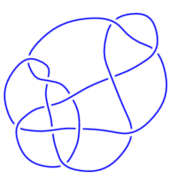
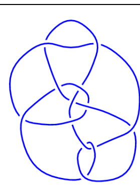
Knot	Diagram	Complex
12_{681}		T
12_{682}		T
12_{683}		T
12_{684}		T
12_{685}		NOT FOUND
12_{686}		T
12_{687}		T
12_{688}		T

Table 7.68. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{689}		
12_{690}		
12_{691}		
12_{692}		
12_{693}		
12_{694}		
12_{695}		
12_{696}		

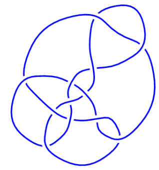
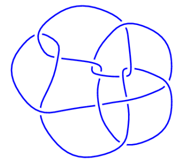
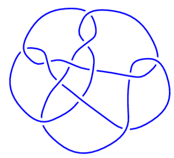
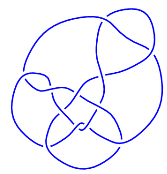
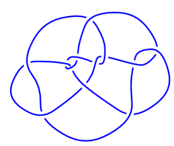
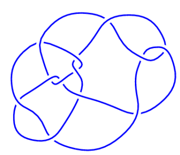
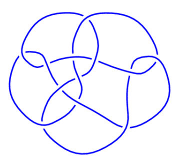
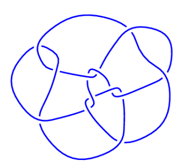
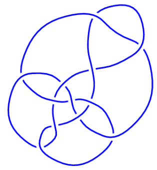
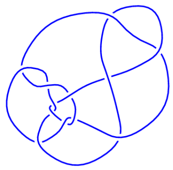
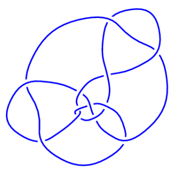
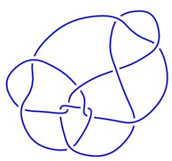
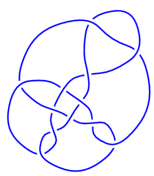
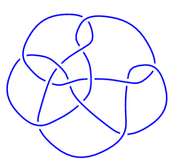
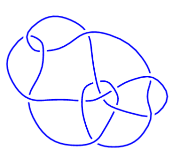
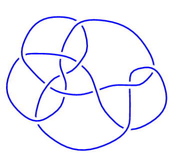
Knot	Diagram	Complex
12_{697}		
12_{698}		
12_{699}		
12_{700}		
12_{701}		
12_{702}		
12_{703}		
12_{704}		

Table 7.69. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{705}		
12_{706}		
12_{707}		
12_{708}		
12_{709}		
12_{710}		
12_{711}		
12_{712}		

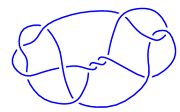
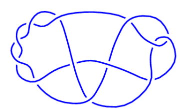
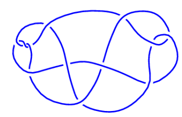
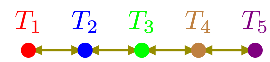
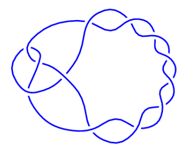
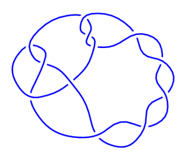
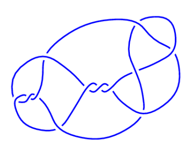
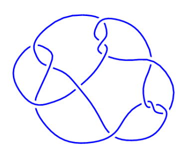
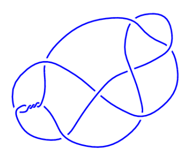
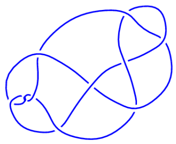
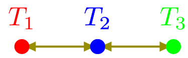
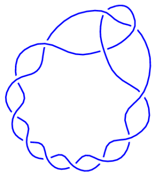
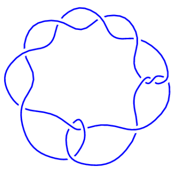
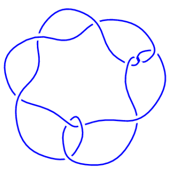
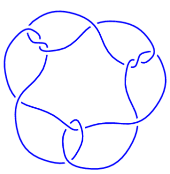
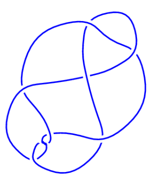
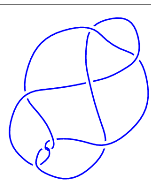
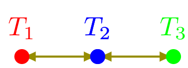
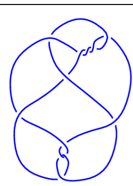
Knot	Diagram	Complex
12_{713}		
12_{714}		
12_{715}		
12_{716}		
12_{717}		
12_{718}		
12_{719}		NOT FOUND
12_{720}		

Table 7.70. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{721}		 T_1 T_2 T_3
12_{722}		 T
12_{723}		 T
12_{724}		 T
12_{725}		 T
12_{726}		 T
12_{727}		 T_1 T_2 T_3
12_{728}		 T

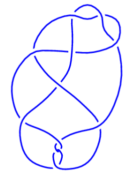
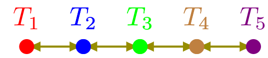
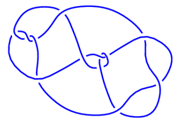
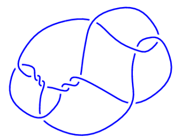
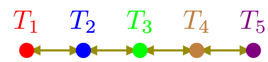
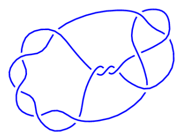
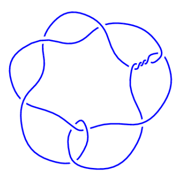
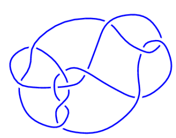
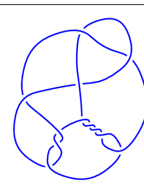
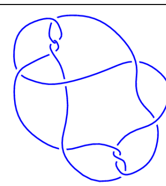
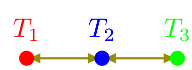
Knot	Diagram	Complex
12_{729}		 T_1 T_2 T_3 T_4 T_5
12_{730}		NOT FOUND
12_{731}		 T_1 T_2 T_3 T_4 T_5
12_{732}		 T
12_{733}		 T
12_{734}		 T
12_{735}		NOT FOUND
12_{736}		 T_1 T_2 T_3

Table 7.71. The Kakimizu complexes of 12 crossing alternating knots

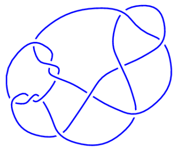
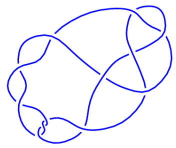
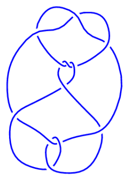
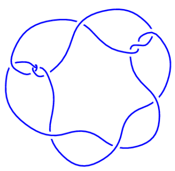
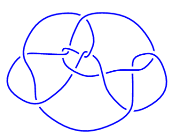
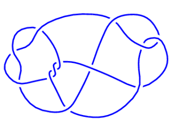
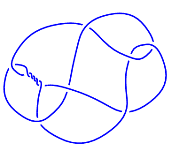
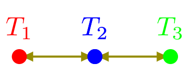
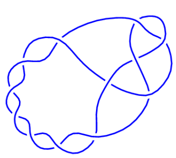
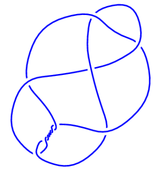
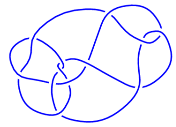
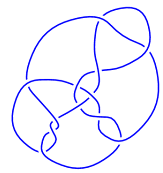
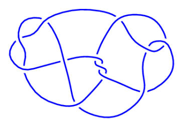
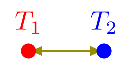
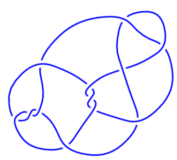
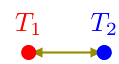
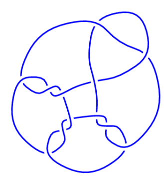
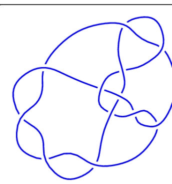
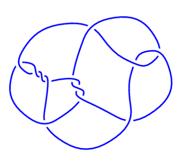
Knot	Diagram	Complex
12_{737}		
12_{738}		
12_{739}		
12_{740}		
12_{741}		
12_{742}		
12_{743}		
12_{744}		
12_{745}		
12_{746}		
12_{747}		
12_{748}		
12_{749}		
12_{750}		NOT FOUND
12_{751}		
12_{752}		NOT FOUND

TABLE 7.72. The Kakimizu complexes of 12 crossing alternating knots

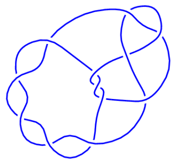
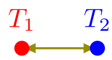
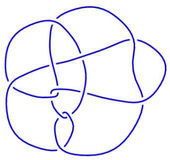
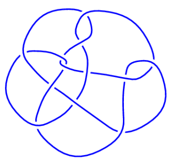
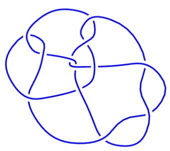
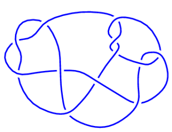
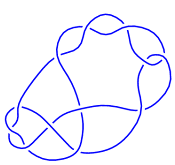
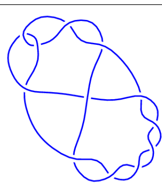
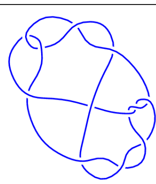
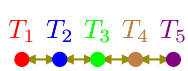
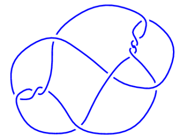
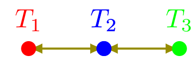
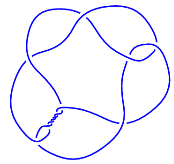
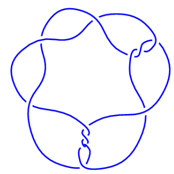
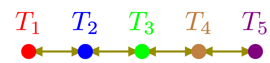
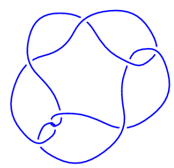
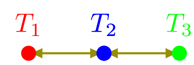
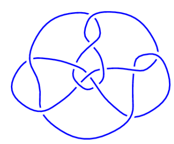
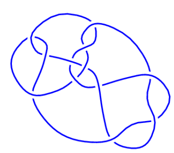
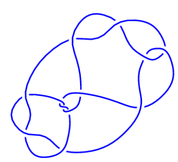
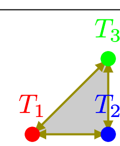
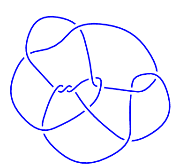
Knot	Diagram	Complex
12_{753}		
12_{754}		
12_{755}		
12_{756}		
12_{757}		
12_{758}		
12_{759}		
12_{760}		
12_{761}		
12_{762}		
12_{763}		
12_{764}		
12_{765}		
12_{766}		
12_{767}		
12_{768}		

Table 7.73. The Kakimizu complexes of 12 crossing alternating knots

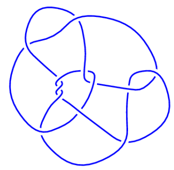
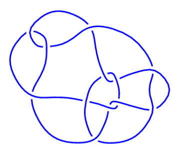
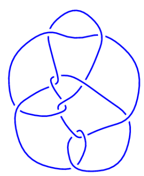
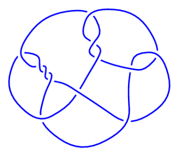
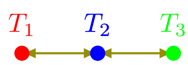
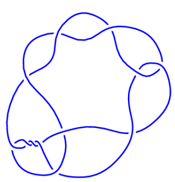
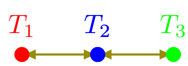
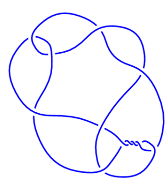
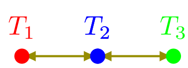
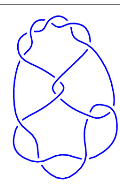
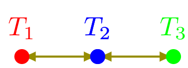
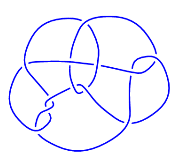
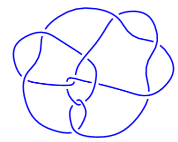
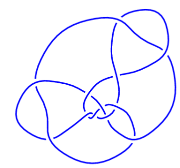
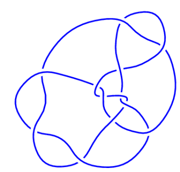
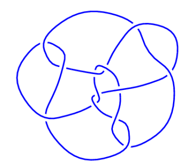
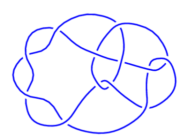
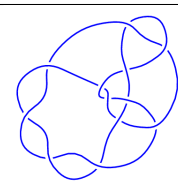
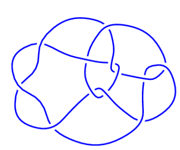
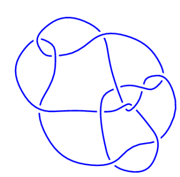
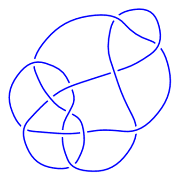
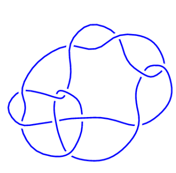
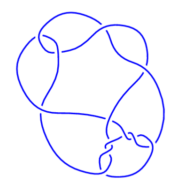
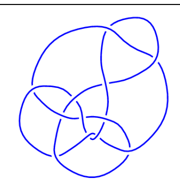
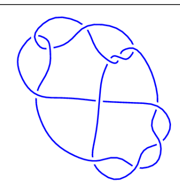
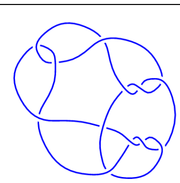
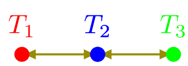
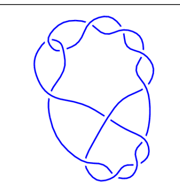
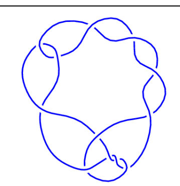
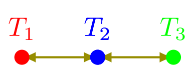
Knot	Diagram	Complex
12_{769}		
12_{770}		
12_{771}		
12_{772}		
12_{773}		
12_{774}		
12_{775}		
12_{776}		
12_{777}		
12_{778}		
12_{779}		
12_{780}		
12_{781}		
12_{782}		
12_{783}		
12_{784}		

TABLE 7.74. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{785}		
12_{786}		
12_{787}		NOT FOUND
12_{788}		
12_{789}		
12_{790}		
12_{791}		
12_{792}		

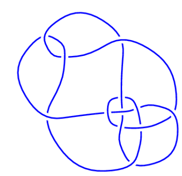
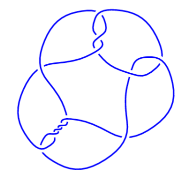
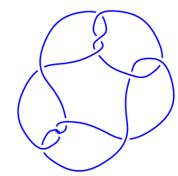
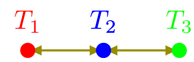
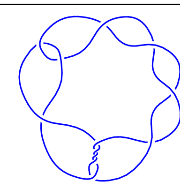
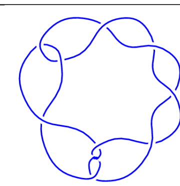
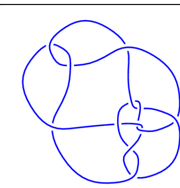
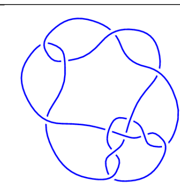
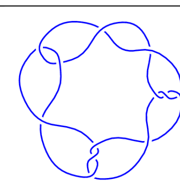
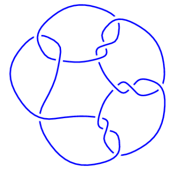
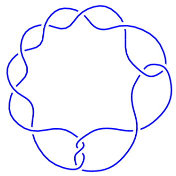
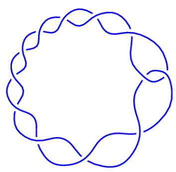
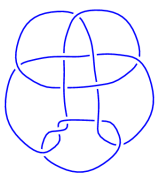
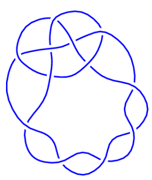
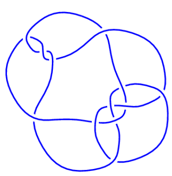
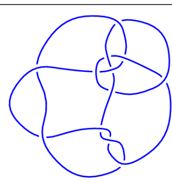
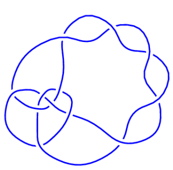
Knot	Diagram	Complex
12_{793}		
12_{794}		
12_{795}		
12_{796}		
12_{797}		
12_{798}		
12_{799}		
12_{800}		

Table 7.75. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{801}		T
12_{802}		T
12_{803}		T
12_{804}		T
12_{805}		T
12_{806}		T
12_{807}		T
12_{808}		T

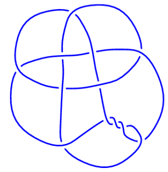
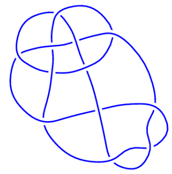
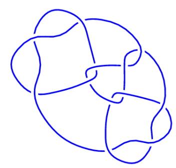
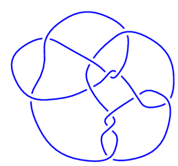
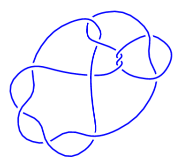
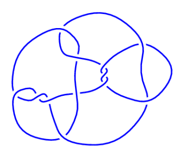
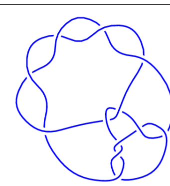
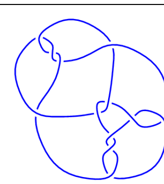
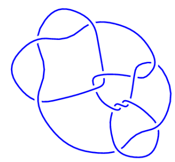
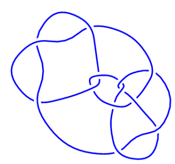
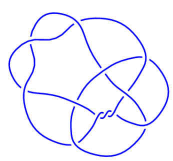
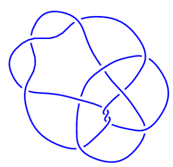
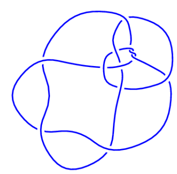
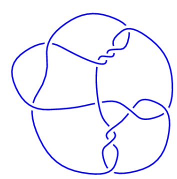
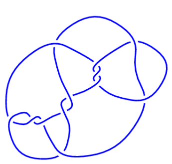
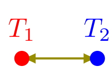
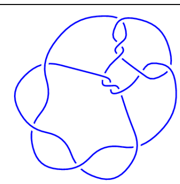
Knot	Diagram	Complex
12_{809}		T
12_{810}		T_1 T_2
12_{811}		T
12_{812}		T
12_{813}		T
12_{814}		T_1 T_2
12_{815}		T
12_{816}		T

Table 7.76. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{817}		
12_{818}		
12_{819}		
12_{820}		
12_{821}		NOT FOUND
12_{822}		
12_{823}		
12_{824}		

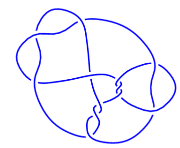
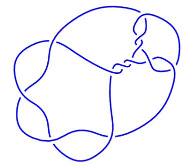
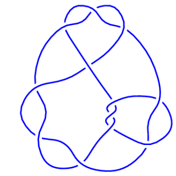
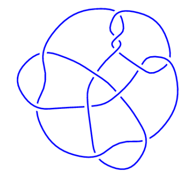
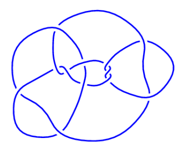
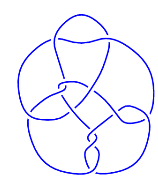
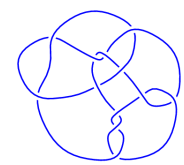
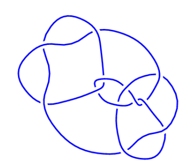
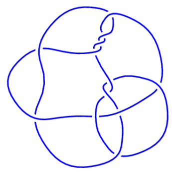
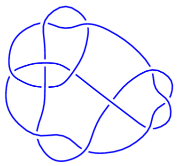
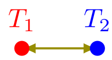
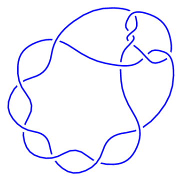
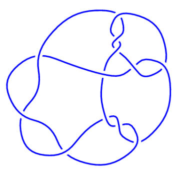
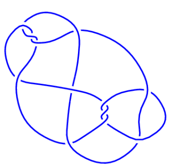
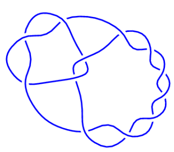
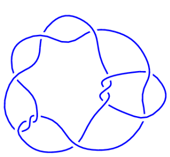
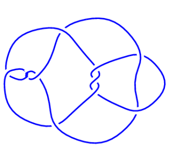
Knot	Diagram	Complex
12_{825}		
12_{826}		
12_{827}		
12_{828}		
12_{829}		
12_{830}		
12_{831}		
12_{832}		

Table 7.77. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{833}		
12_{834}		
12_{835}		
12_{836}		
12_{837}		
12_{838}		
12_{839}		
12_{840}		

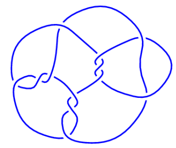
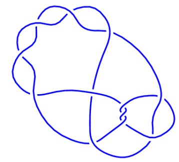
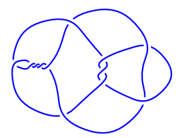
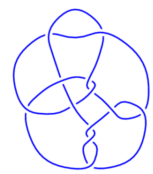
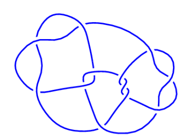
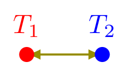
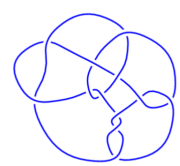
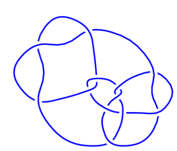
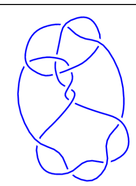
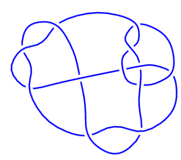
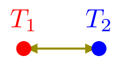
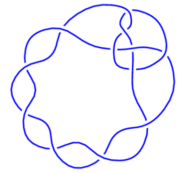
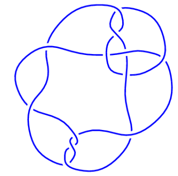
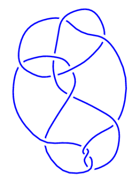
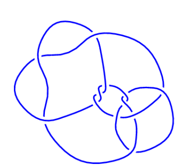
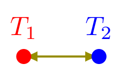
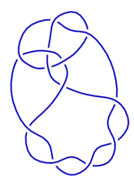
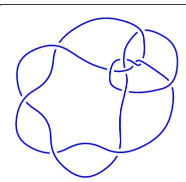
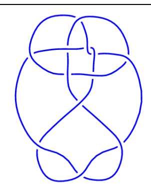
Knot	Diagram	Complex
12_{841}		NOT FOUND
12_{842}		
12_{843}		
12_{844}		
12_{845}		
12_{846}		
12_{847}		
12_{848}		

Table 7.78. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{849}		
12_{850}		
12_{851}		
12_{852}		
12_{853}		
12_{854}		
12_{855}		
12_{856}		NOT FOUND

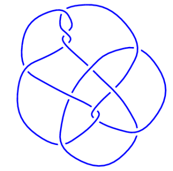
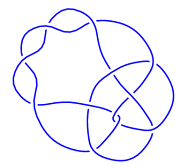
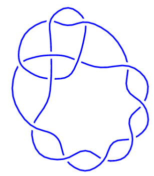
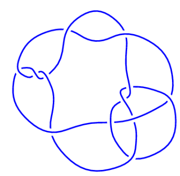
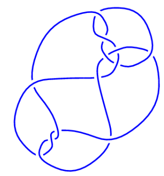
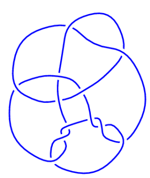
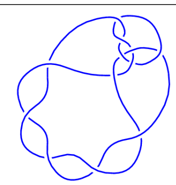
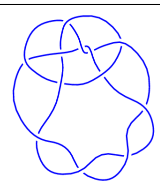
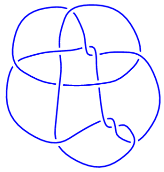
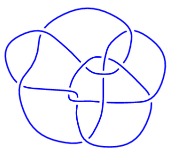
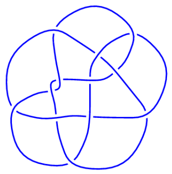
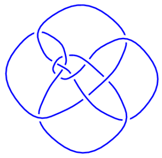
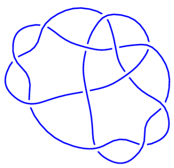
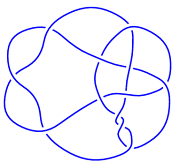
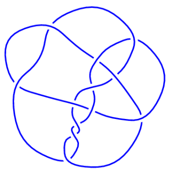
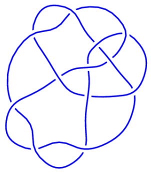
Knot	Diagram	Complex
12_{857}		
12_{858}		
12_{859}		
12_{860}		
12_{861}		
12_{862}		NOT FOUND
12_{863}		
12_{864}		

Table 7.79. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{865}		T
12_{866}		T
12_{867}		T
12_{868}		T
12_{869}		T
12_{870}		T
12_{871}		T
12_{872}		NOT FOUND

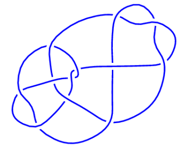
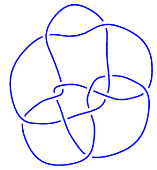
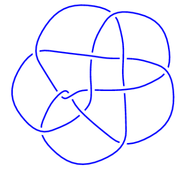
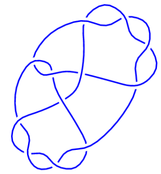
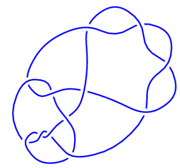
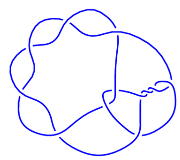
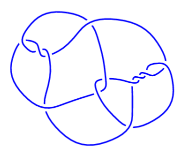
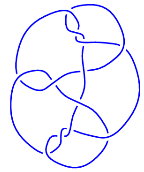
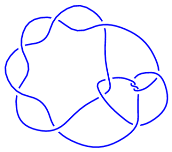
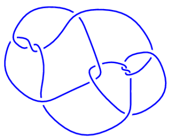
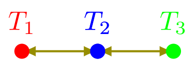
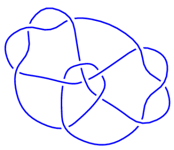
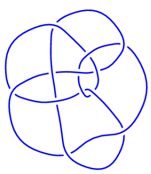
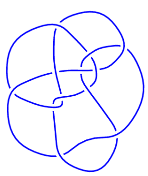
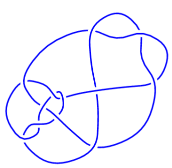
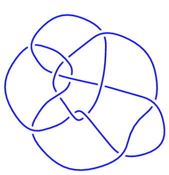
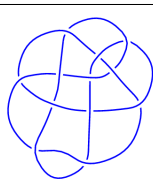
Knot	Diagram	Complex
12_{873}		T
12_{874}		T
12_{875}		T
12_{876}		T
12_{877}		$T_1 \leftrightarrow T_2$
12_{878}		T
12_{879}		T
12_{880}		$T_1 \leftrightarrow T_2$

TABLE 7.80. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{881}		
12_{882}		
12_{883}		NOT FOUND
12_{884}		
12_{885}		
12_{886}		
12_{887}		
12_{888}		

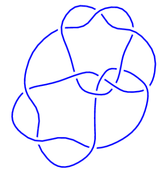
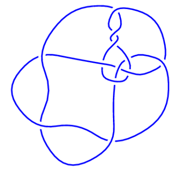
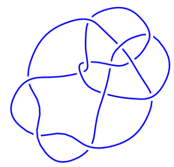
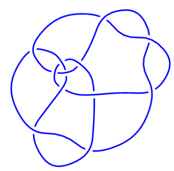
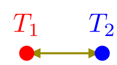
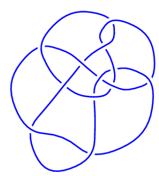
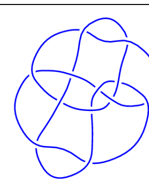
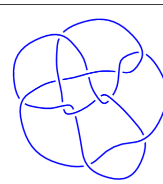
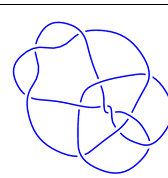
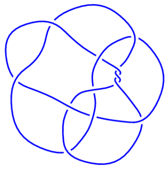
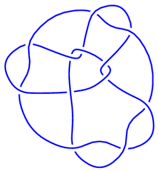
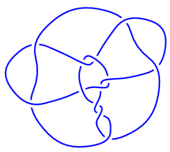
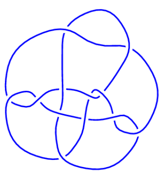
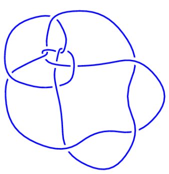
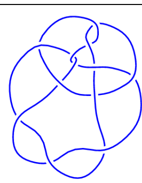
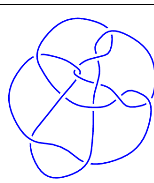
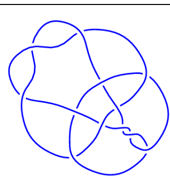
Knot	Diagram	Complex
12_{889}		
12_{890}		NOT FOUND
12_{891}		
12_{892}		
12_{893}		
12_{894}		
12_{895}		
12_{896}		

Table 7.81. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{897}		NOT FOUND
12_{898}		
12_{899}		
12_{900}		
12_{901}		
12_{902}		
12_{903}		
12_{904}		

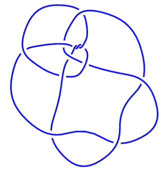
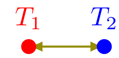
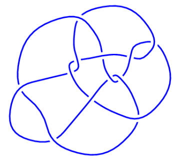
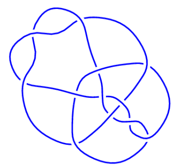
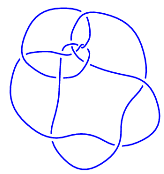
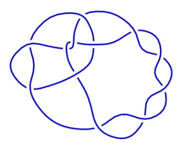
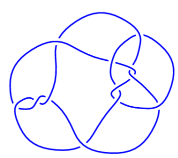
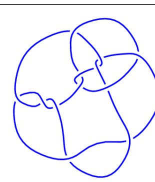
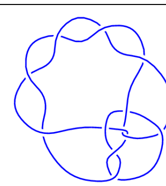
Knot	Diagram	Complex
12_{905}		
12_{906}		
12_{907}		
12_{908}		NOT FOUND
12_{909}		
12_{910}		
12_{911}		
12_{912}		

Table 7.82. The Kakimizu complexes of 12 crossing alternating knots

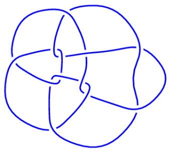
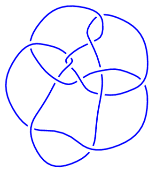
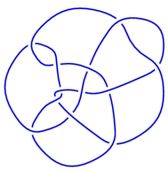
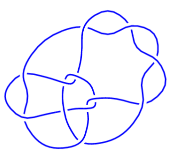
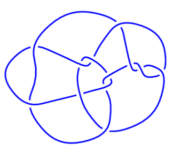
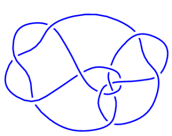
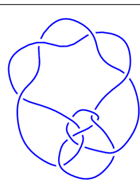
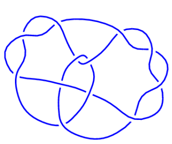
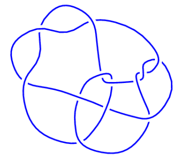
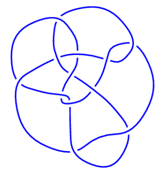
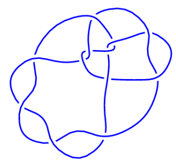
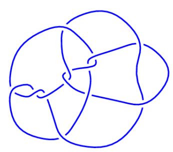
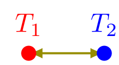
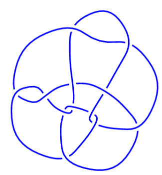
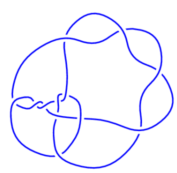
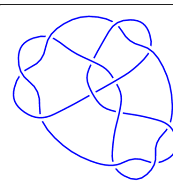
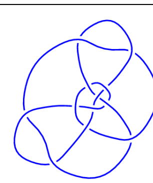
Knot	Diagram	Complex
12_{913}		
12_{914}		
12_{915}		
12_{916}		
12_{917}		
12_{918}		
12_{919}		
12_{920}		
12_{921}		
12_{922}		
12_{923}		
12_{924}		
12_{925}		
12_{926}		
12_{927}		NOT FOUND
12_{928}		

Table 7.83. The Kakimizu complexes of 12 crossing alternating knots

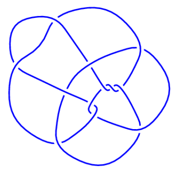
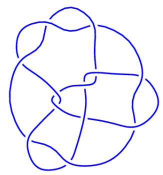
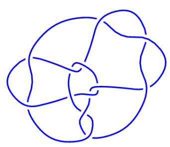
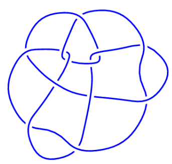
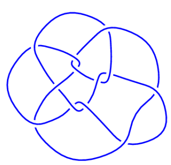
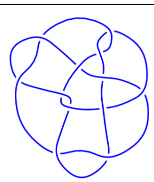
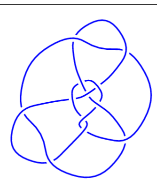
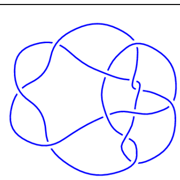
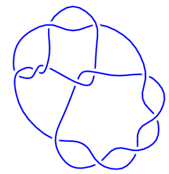
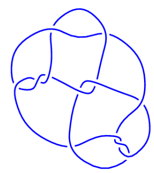
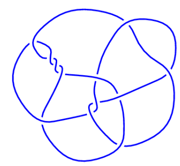
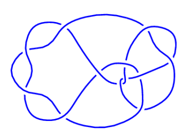
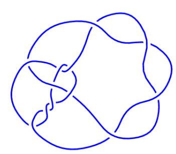
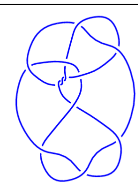
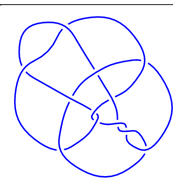
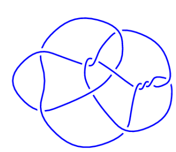
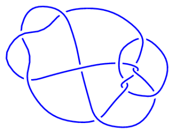
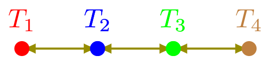
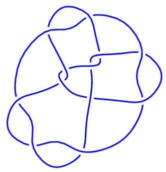
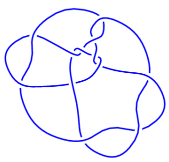
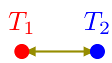
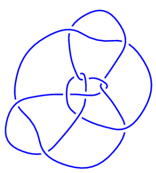
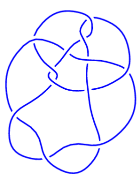
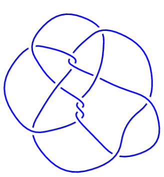
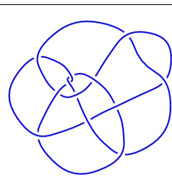
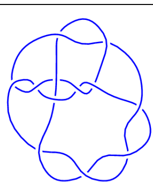
Knot	Diagram	Complex
12_{929}		T
12_{930}		T
12_{931}		NOT FOUND
12_{932}		T
12_{933}		T
12_{934}		T
12_{935}		T
12_{936}		NOT FOUND
12_{937}		T
12_{938}		NOT FOUND
12_{939}		T_1 T_2
12_{940}		T
12_{941}		T
12_{942}		NOT FOUND
12_{943}		NOT FOUND
12_{944}		T

Table 7.84. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{945}		
12_{946}		
12_{947}		
12_{948}		
12_{949}		
12_{950}		
12_{951}		
12_{952}		

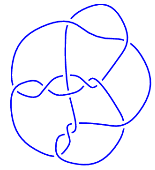
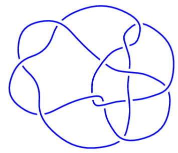
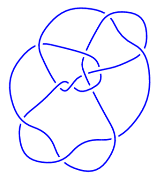
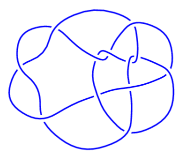
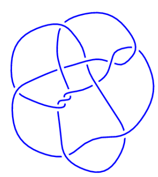
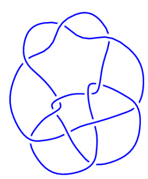
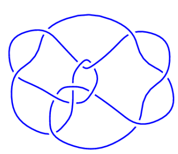
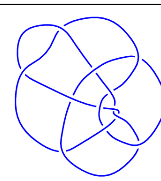
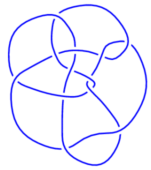
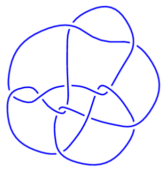
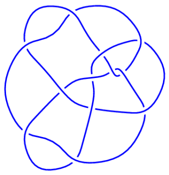
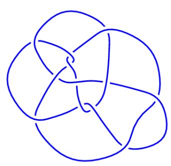
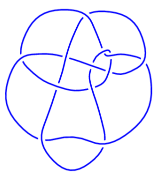
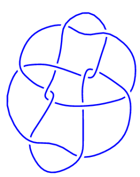
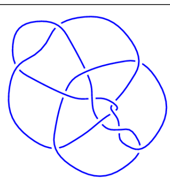
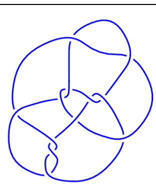
Knot	Diagram	Complex
12_{953}		NOT FOUND
12_{954}		
12_{955}		NOT FOUND
12_{956}		
12_{957}		NOT FOUND
12_{958}		
12_{959}		NOT FOUND
12_{960}		NOT FOUND

Table 7.85. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{961}		T 
12_{962}		T 
12_{963}		T 
12_{964}		T 
12_{965}		T 
12_{966}		T 
12_{967}		T 
12_{968}		T 

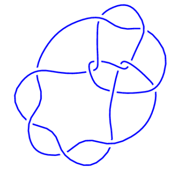
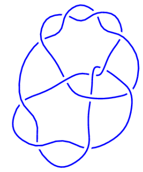
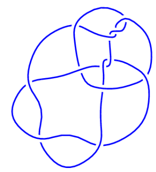
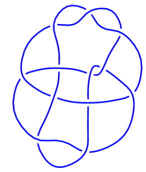
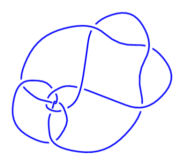
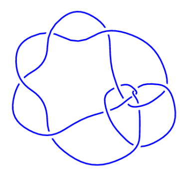
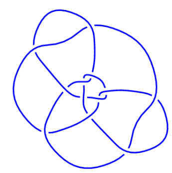
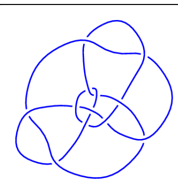
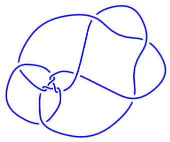
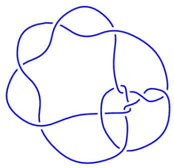
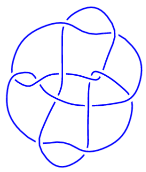
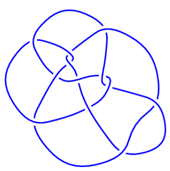
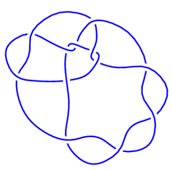
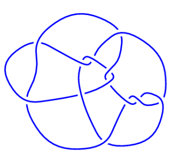
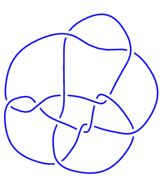
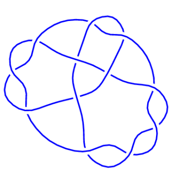
Knot	Diagram	Complex
12_{969}		T 
12_{970}		T 
12_{971}		NOT FOUND
12_{972}		T 
12_{973}		T 
12_{974}		T 
12_{975}		T 
12_{976}		T 

Table 7.86. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{977}		
12_{978}		
12_{979}		
12_{980}		
12_{981}		
12_{982}		
12_{983}		
12_{984}		

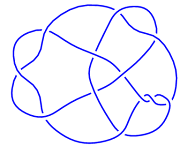
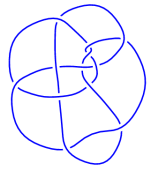
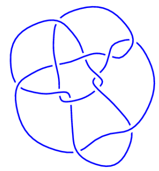
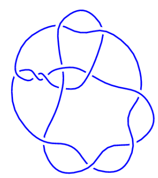
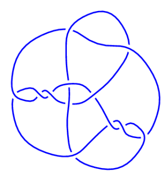
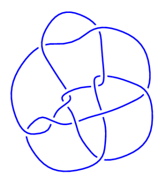
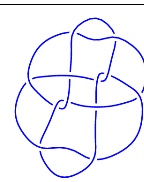
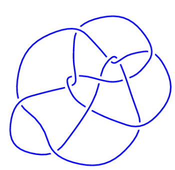
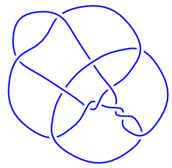
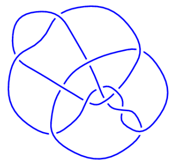
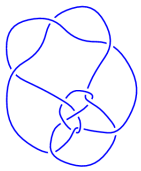
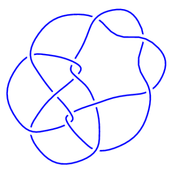
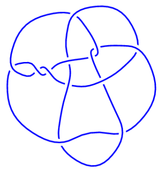
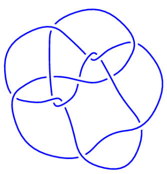
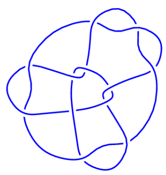
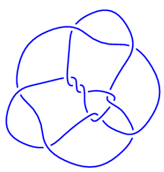
Knot	Diagram	Complex
12_{985}		
12_{986}		
12_{987}		
12_{988}		
12_{989}		NOT FOUND
12_{990}		
12_{991}		
12_{992}		

Table 7.87. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{993}		
12_{994}		
12_{995}		
12_{996}		
12_{997}		
12_{998}		
12_{999}		
12_{1000}		NOT FOUND

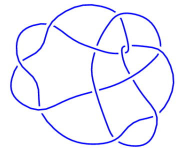
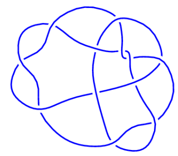
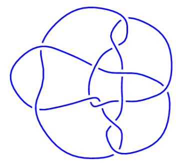
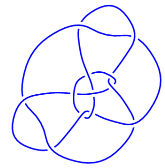
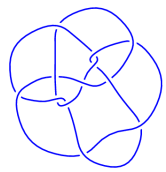
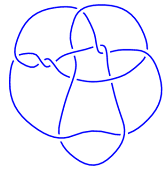
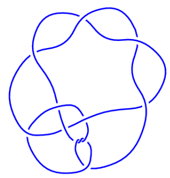
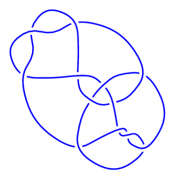
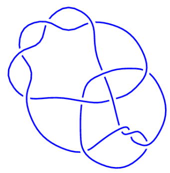
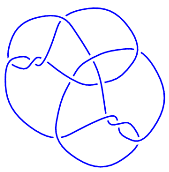
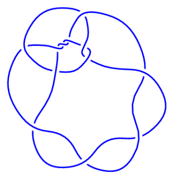
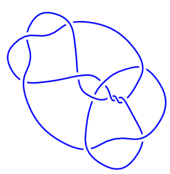
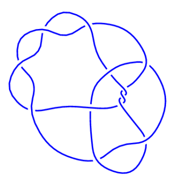
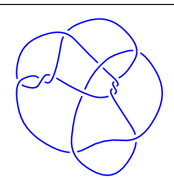
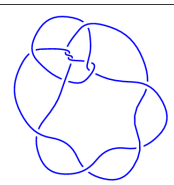
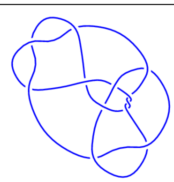
Knot	Diagram	Complex
12_{1001}		
12_{1002}		NOT FOUND
12_{1003}		NOT FOUND
12_{1004}		NOT FOUND
12_{1005}		NOT FOUND
12_{1006}		NOT FOUND
12_{1007}		NOT FOUND
12_{1008}		NOT FOUND

Table 7.88. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1009}		NOT FOUND
12_{1010}		NOT FOUND
12_{1011}		T 
12_{1012}		NOT FOUND
12_{1013}		T 
12_{1014}		NOT FOUND
12_{1015}		NOT FOUND
12_{1016}		NOT FOUND

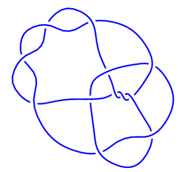
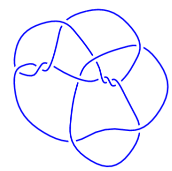
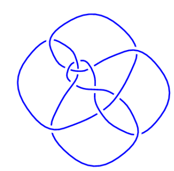
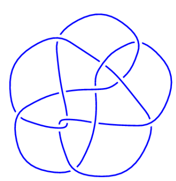
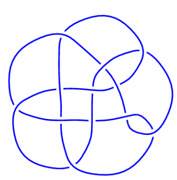
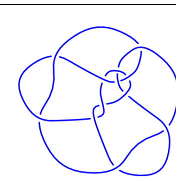
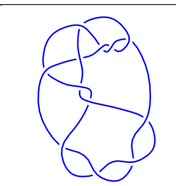
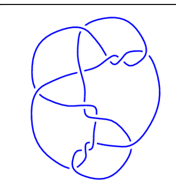
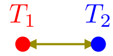
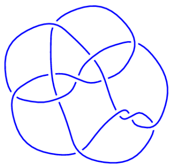
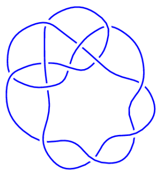
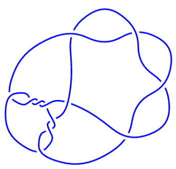
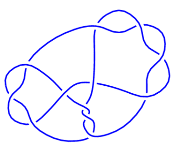
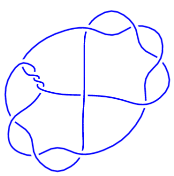
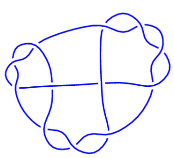
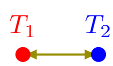
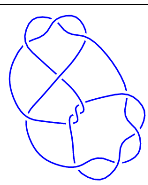
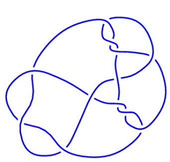
Knot	Diagram	Complex
12_{1017}		NOT FOUND
12_{1018}		NOT FOUND
12_{1019}		T 
12_{1020}		T 
12_{1021}		T 
12_{1022}		NOT FOUND
12_{1023}		T 
12_{1024}		T_1 T_2 

TABLE 7.89. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1025}		NOT FOUND
12_{1026}		NOT FOUND
12_{1027}		
12_{1028}		NOT FOUND
12_{1029}		
12_{1030}		
12_{1031}		NOT FOUND
12_{1032}		NOT FOUND

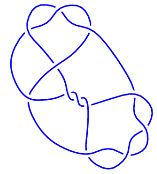
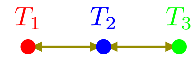
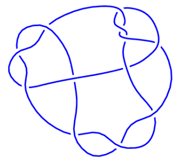
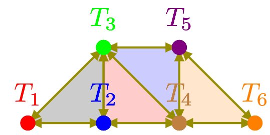
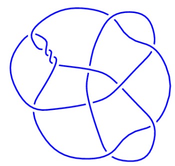
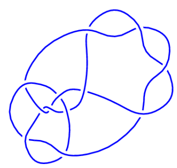
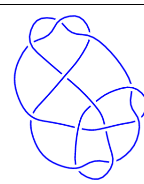
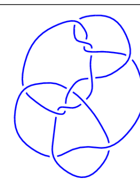
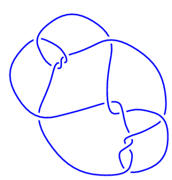
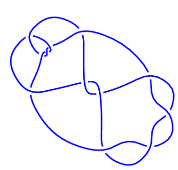
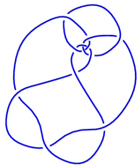
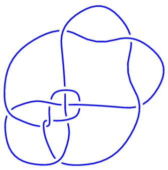
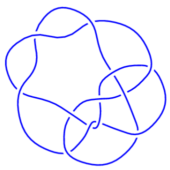
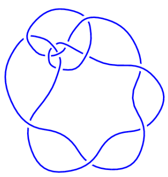
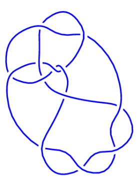
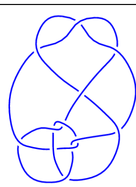
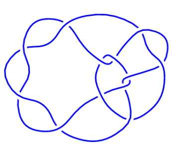
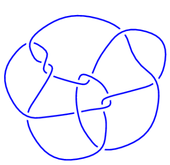
Knot	Diagram	Complex
12_{1033}		
12_{1034}		
12_{1035}		NOT FOUND
12_{1036}		NOT FOUND
12_{1037}		NOT FOUND
12_{1038}		NOT FOUND
12_{1039}		
12_{1040}		

Table 7.90. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1041}		NOT FOUND
12_{1042}		NOT FOUND
12_{1043}		NOT FOUND
12_{1044}		NOT FOUND
12_{1045}		
12_{1046}		NOT FOUND
12_{1047}		
12_{1048}		NOT FOUND

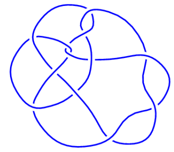
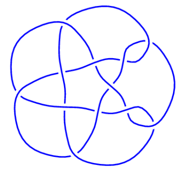
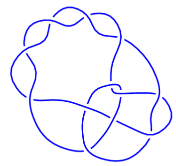
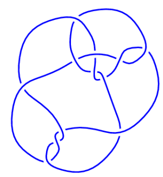
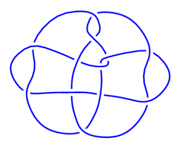
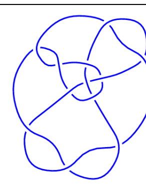
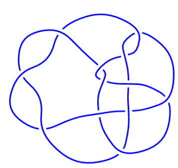
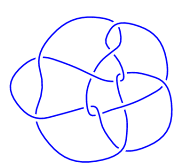
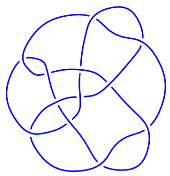
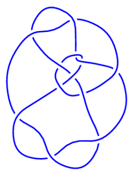
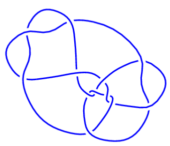
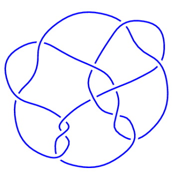
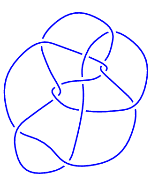
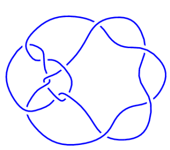
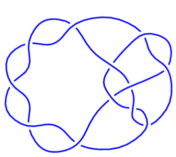
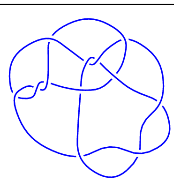
Knot	Diagram	Complex
12_{1049}		
12_{1050}		
12_{1051}		
12_{1052}		NOT FOUND
12_{1053}		NOT FOUND
12_{1054}		
12_{1055}		NOT FOUND
12_{1056}		NOT FOUND

Table 7.91. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1057}		NOT FOUND
12_{1058}		NOT FOUND
12_{1059}		NOT FOUND
12_{1060}		NOT FOUND
12_{1061}		NOT FOUND
12_{1062}		NOT FOUND
12_{1063}		NOT FOUND
12_{1064}		NOT FOUND

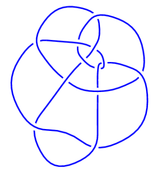
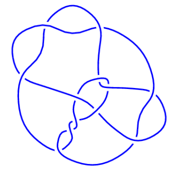
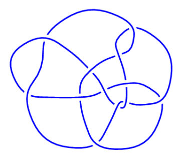
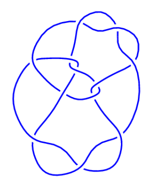
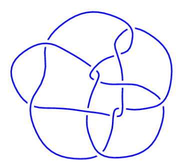
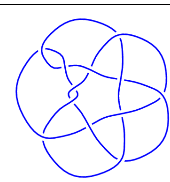
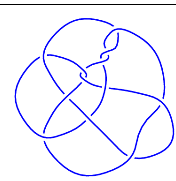
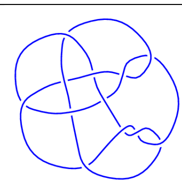
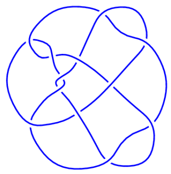
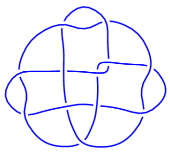
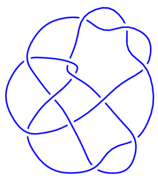
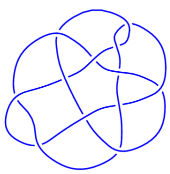
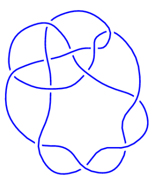
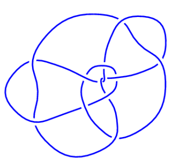
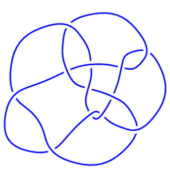
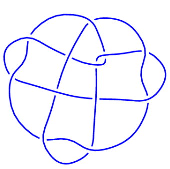
Knot	Diagram	Complex
12_{1065}		
12_{1066}		NOT FOUND
12_{1067}		
12_{1068}		NOT FOUND
12_{1069}		NOT FOUND
12_{1070}		
12_{1071}		NOT FOUND
12_{1072}		NOT FOUND

Table 7.92. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1073}		NOT FOUND
12_{1074}		
12_{1075}		NOT FOUND
12_{1076}		
12_{1077}		NOT FOUND
12_{1078}		NOT FOUND
12_{1079}		NOT FOUND
12_{1080}		

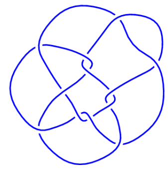
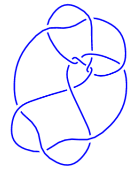
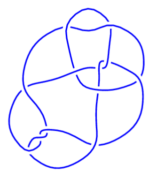
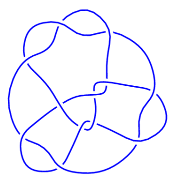
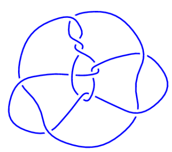
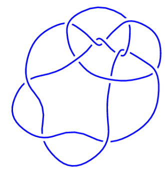
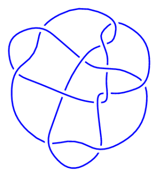
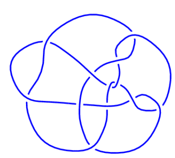
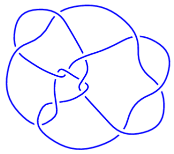
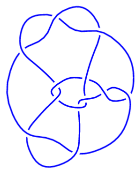
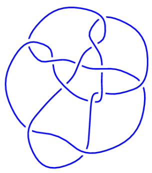
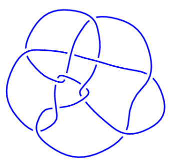
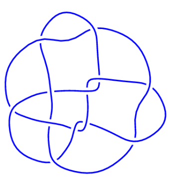
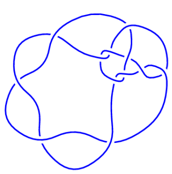
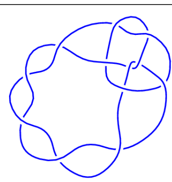
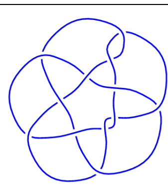
Knot	Diagram	Complex
12_{1081}		
12_{1082}		
12_{1083}		NOT FOUND
12_{1084}		
12_{1085}		NOT FOUND
12_{1086}		NOT FOUND
12_{1087}		
12_{1088}		

Table 7.93. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1089}		
12_{1090}		NOT FOUND
12_{1091}		NOT FOUND
12_{1092}		
12_{1093}		
12_{1094}		NOT FOUND
12_{1095}		NOT FOUND
12_{1096}		

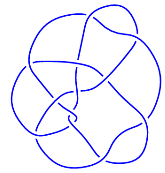
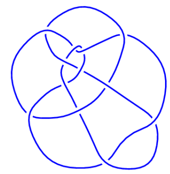
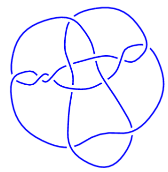
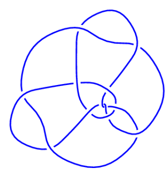
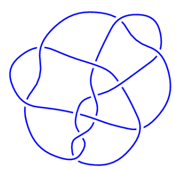
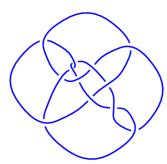
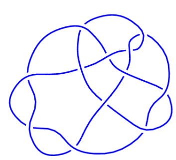
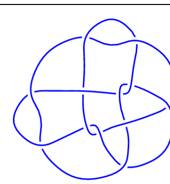
Knot	Diagram	Complex
12_{1097}		NOT FOUND
12_{1098}		NOT FOUND
12_{1099}		NOT FOUND
12_{1100}		NOT FOUND
12_{1101}		NOT FOUND
12_{1102}		
12_{1103}		NOT FOUND
12_{1104}		

Table 7.94. The Kakimizu complexes of 12 crossing alternating knots

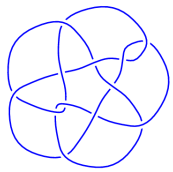
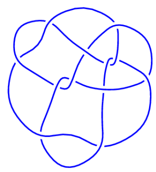
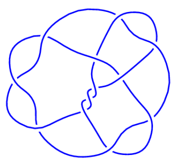
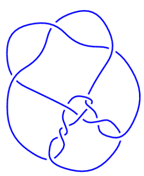
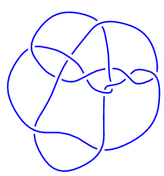
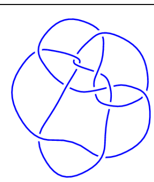
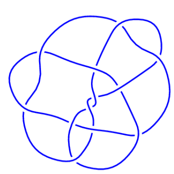
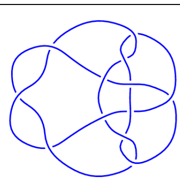
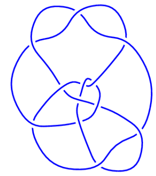
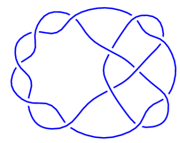
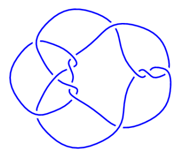
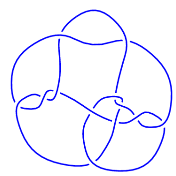
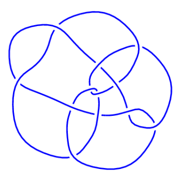
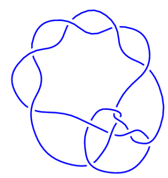
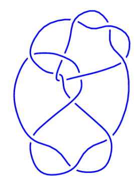
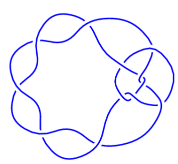
Knot	Diagram	Complex
12_{1105}		
12_{1106}		NOT FOUND
12_{1107}		NOT FOUND
12_{1108}		NOT FOUND
12_{1109}		NOT FOUND
12_{1110}		NOT FOUND
12_{1111}		NOT FOUND
12_{1112}		NOT FOUND
12_{1113}		NOT FOUND
12_{1114}		
12_{1115}		NOT FOUND
12_{1116}		NOT FOUND
12_{1117}		NOT FOUND
12_{1118}		NOT FOUND
12_{1119}		NOT FOUND
12_{1120}		

TABLE 7.95. The Kakimizu complexes of 12 crossing alternating knots

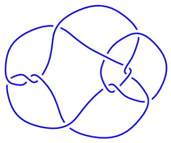
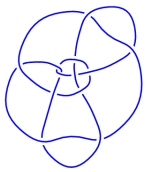
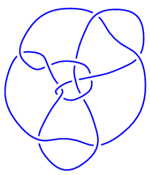
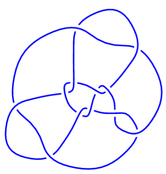
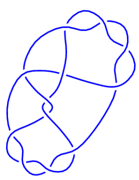
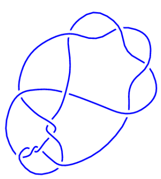
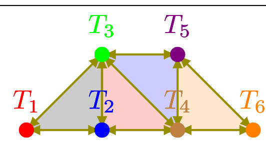
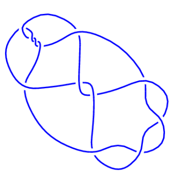
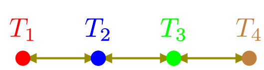
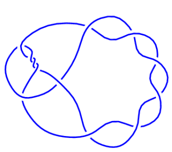
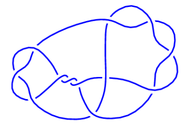
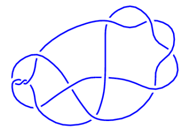
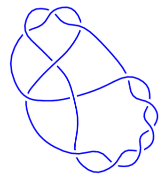
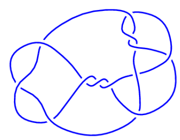
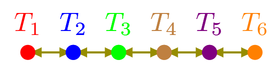
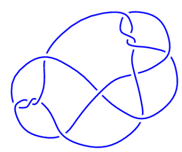
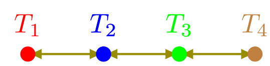
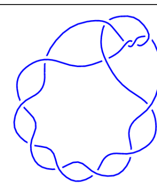
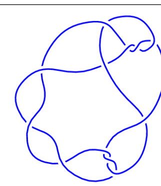
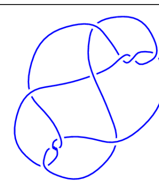
Knot	Diagram	Complex
12_{1121}		NOT FOUND
12_{1122}		
12_{1123}		
12_{1124}		
12_{1125}		
12_{1126}		
12_{1127}		
12_{1128}		
12_{1129}		
12_{1130}		
12_{1131}		
12_{1132}		
12_{1133}		
12_{1134}		
12_{1135}		
12_{1136}		

Table 7.96. The Kakimizu complexes of 12 crossing alternating knots

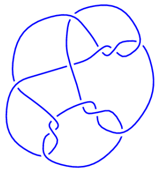
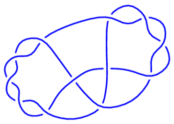
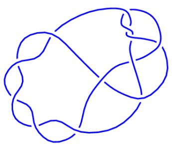
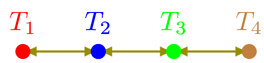
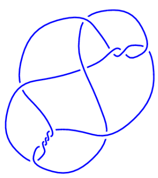
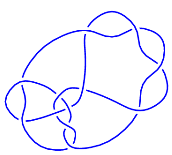
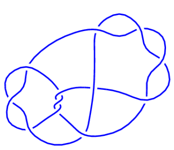
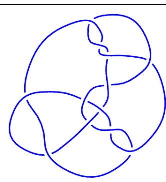
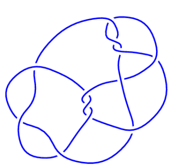
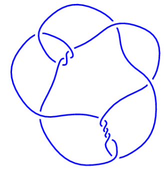
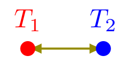
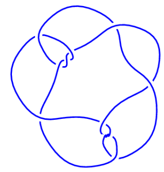
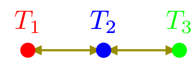
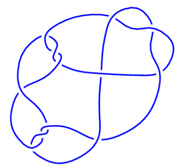
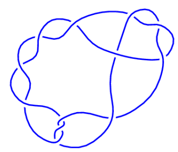
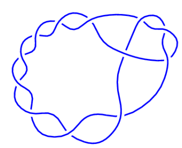
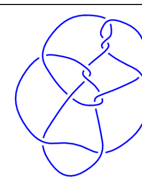
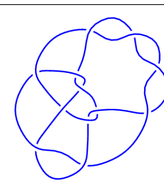
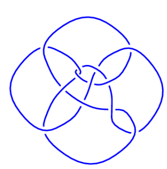
Knot	Diagram	Complex
12_{1137}		NOT FOUND
12_{1138}		
12_{1139}		
12_{1140}		
12_{1141}		
12_{1142}		NOT FOUND
12_{1143}		NOT FOUND
12_{1144}		NOT FOUND
12_{1145}		
12_{1146}		
12_{1147}		NOT FOUND
12_{1148}		
12_{1149}		
12_{1150}		
12_{1151}		NOT FOUND
12_{1152}		

TABLE 7.97. The Kakimizu complexes of 12 crossing alternating knots

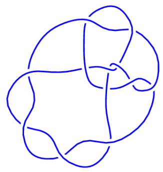
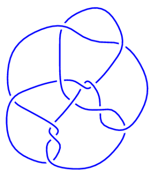
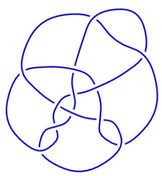
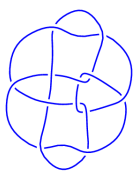
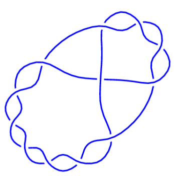
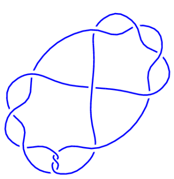
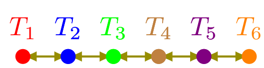
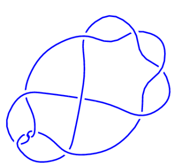
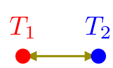
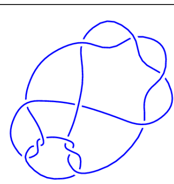
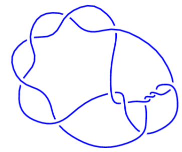
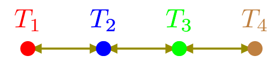
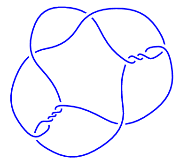
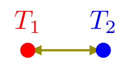
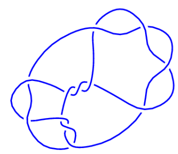
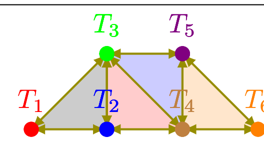
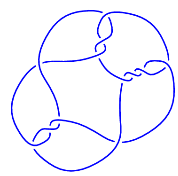
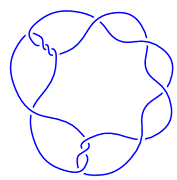
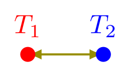
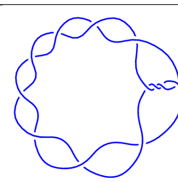
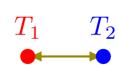
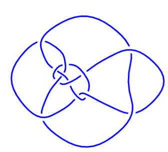
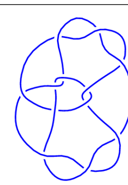
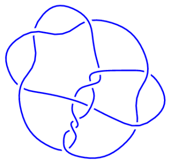
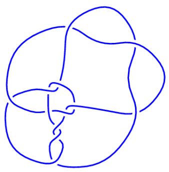
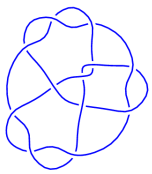
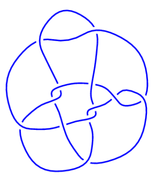
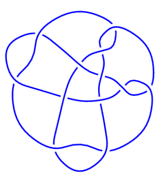
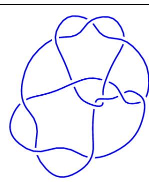
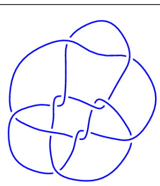
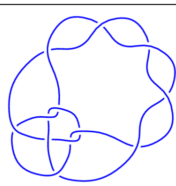
Knot	Diagram	Complex
12_{1153}		
12_{1154}		NOT FOUND
12_{1155}		NOT FOUND
12_{1156}		
12_{1157}		
12_{1158}		
12_{1159}		
12_{1160}		NOT FOUND
12_{1161}		
12_{1162}		
12_{1163}		
12_{1164}		NOT FOUND
12_{1165}		
12_{1166}		
12_{1167}		
12_{1168}		

Table 7.98. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1169}		NOT FOUND
12_{1170}		NOT FOUND
12_{1171}		NOT FOUND
12_{1172}		NOT FOUND
12_{1173}		NOT FOUND
12_{1174}		NOT FOUND
12_{1175}		NOT FOUND
12_{1176}		

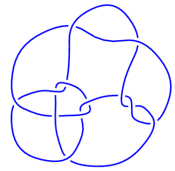
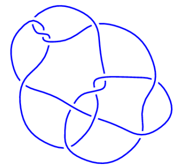
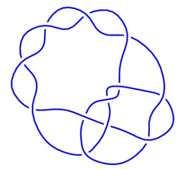
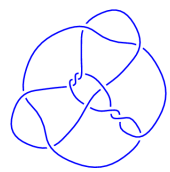
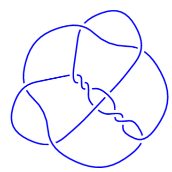
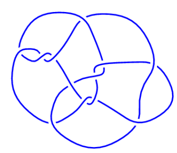
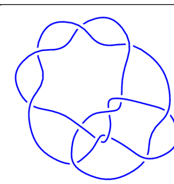
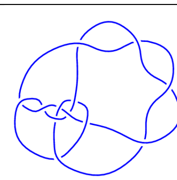
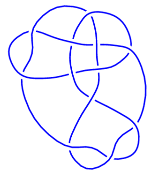
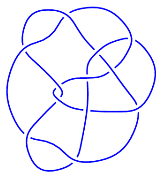
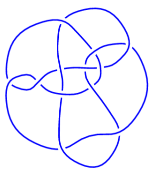
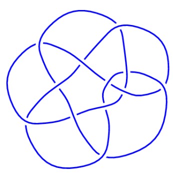
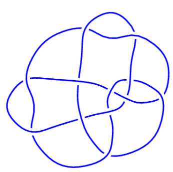
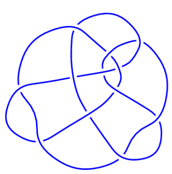
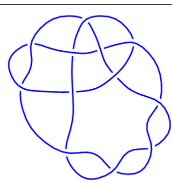
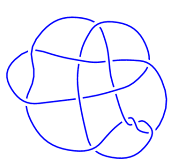
Knot	Diagram	Complex
12_{1177}		NOT FOUND
12_{1178}		NOT FOUND
12_{1179}		NOT FOUND
12_{1180}		NOT FOUND
12_{1181}		NOT FOUND
12_{1182}		NOT FOUND
12_{1183}		NOT FOUND
12_{1184}		NOT FOUND

Table 7.99. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1185}		NOT FOUND
12_{1186}		NOT FOUND
12_{1187}		NOT FOUND
12_{1188}		
12_{1189}		NOT FOUND
12_{1190}		
12_{1191}		
12_{1192}		NOT FOUND

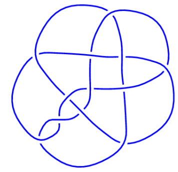
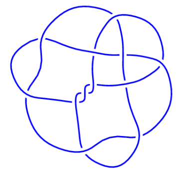
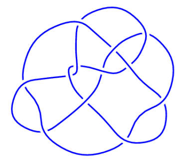
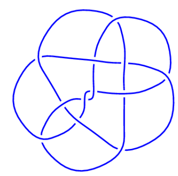
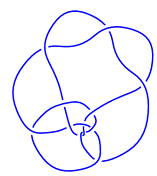
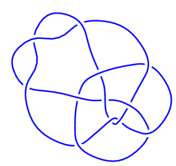
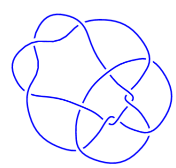
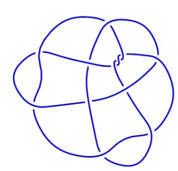
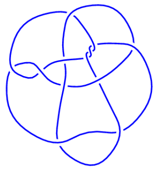
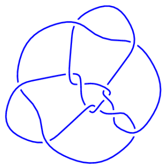
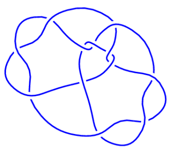
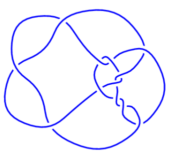
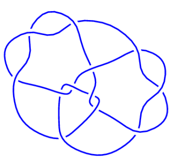
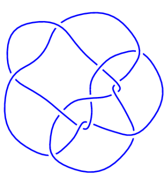
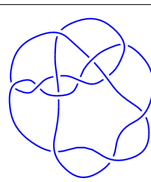
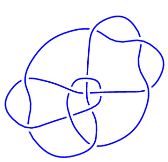
Knot	Diagram	Complex
12_{1193}		NOT FOUND
12_{1194}		NOT FOUND
12_{1195}		
12_{1196}		NOT FOUND
12_{1197}		NOT FOUND
12_{1198}		NOT FOUND
12_{1199}		
12_{1200}		NOT FOUND

Table 7.100. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1201}		NOT FOUND
12_{1202}		NOT FOUND
12_{1203}		
12_{1204}		NOT FOUND
12_{1205}		NOT FOUND
12_{1206}		NOT FOUND
12_{1207}		NOT FOUND
12_{1208}		NOT FOUND

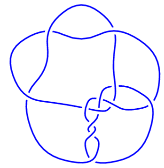
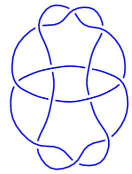
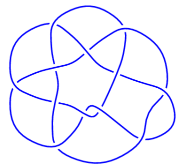
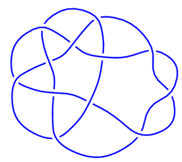
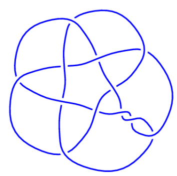
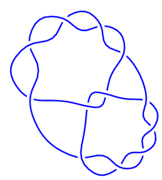
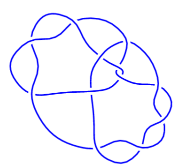
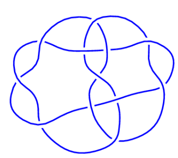
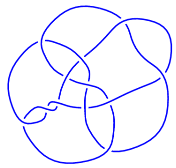
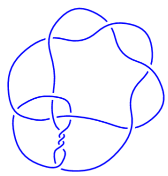
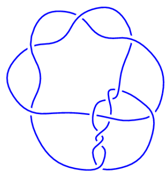
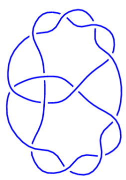
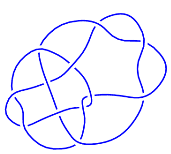
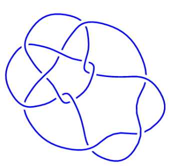
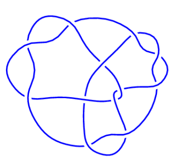
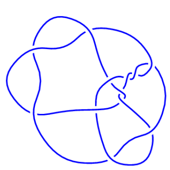
Knot	Diagram	Complex
12_{1209}		
12_{1210}		
12_{1211}		
12_{1212}		
12_{1213}		NOT FOUND
12_{1214}		
12_{1215}		
12_{1216}		NOT FOUND

Table 7.101. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1217}		NOT FOUND
12_{1218}		
12_{1219}		
12_{1220}		
12_{1221}		
12_{1222}		
12_{1223}		
12_{1224}		NOT FOUND

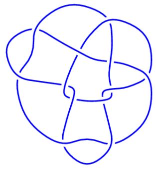
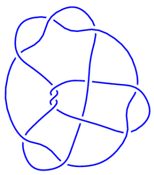
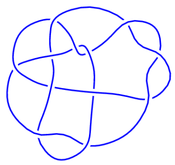
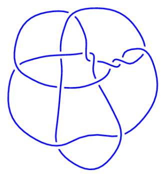
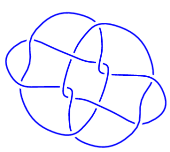
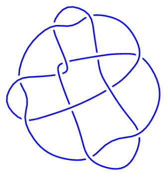
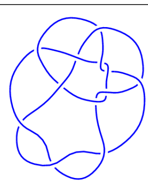
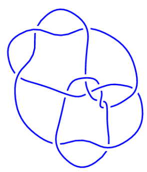
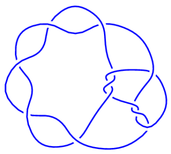
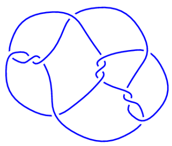
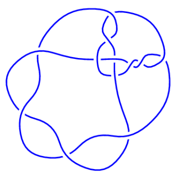
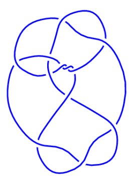
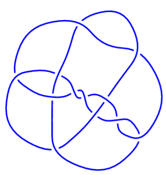
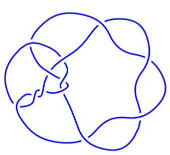
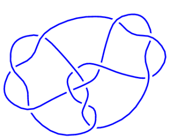
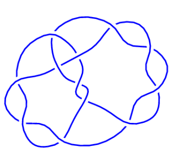
Knot	Diagram	Complex
12_{1225}		
12_{1226}		
12_{1227}		
12_{1228}		NOT FOUND
12_{1229}		
12_{1230}		
12_{1231}		
12_{1232}		NOT FOUND

Table 7.102. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1233}		
12_{1234}		NOT FOUND
12_{1235}		
12_{1236}		NOT FOUND
12_{1237}		NOT FOUND
12_{1238}		
12_{1239}		NOT FOUND
12_{1240}		NOT FOUND

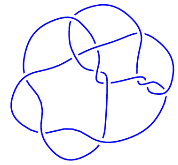
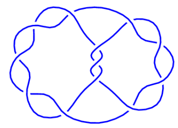
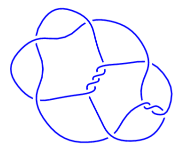
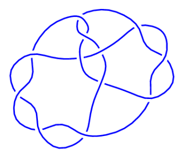
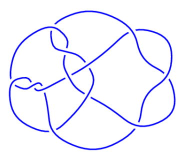
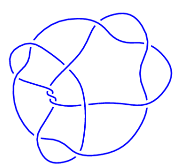
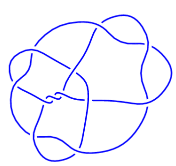
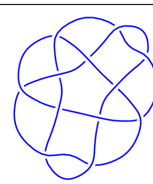
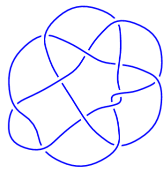
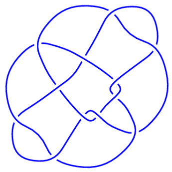
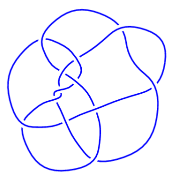
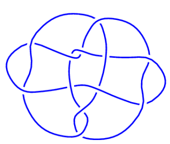
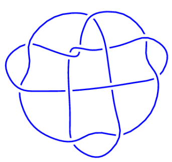
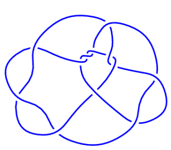
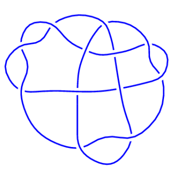
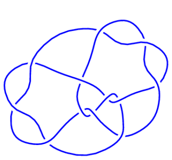
Knot	Diagram	Complex
12_{1241}		NOT FOUND
12_{1242}		NOT FOUND
12_{1243}		NOT FOUND
12_{1244}		NOT FOUND
12_{1245}		NOT FOUND
12_{1246}		
12_{1247}		NOT FOUND
12_{1248}		

Table 7.103. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1249}		
12_{1250}		
12_{1251}		NOT FOUND
12_{1252}		NOT FOUND
12_{1253}		
12_{1254}		
12_{1255}		
12_{1256}		NOT FOUND

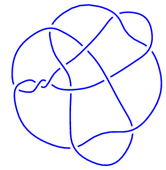
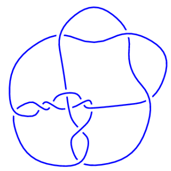
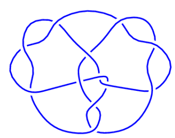
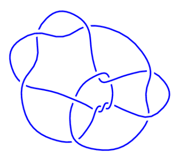
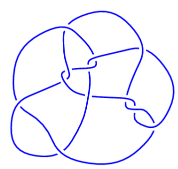
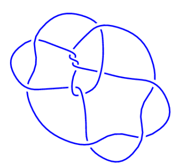
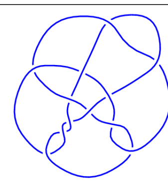
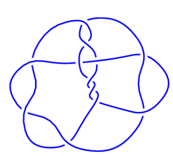
Knot	Diagram	Complex
12_{1257}		NOT FOUND
12_{1258}		
12_{1259}		NOT FOUND
12_{1260}		
12_{1261}		NOT FOUND
12_{1262}		NOT FOUND
12_{1263}		NOT FOUND
12_{1264}		NOT FOUND

Table 7.104. The Kakimizu complexes of 12 crossing alternating knots

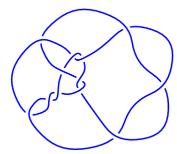
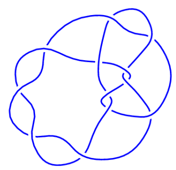
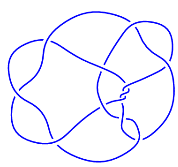
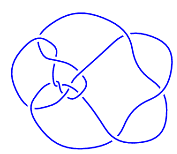
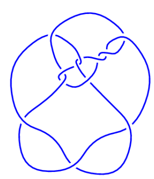
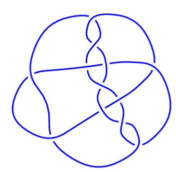
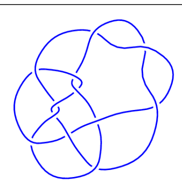
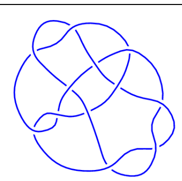
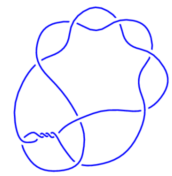
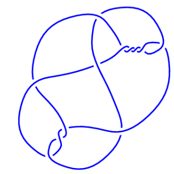
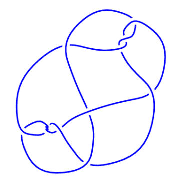
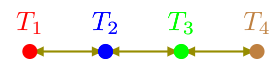
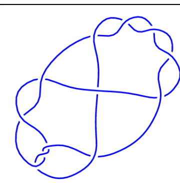
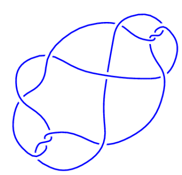
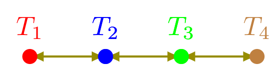
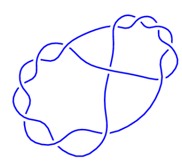
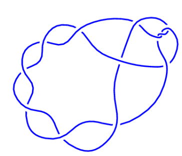
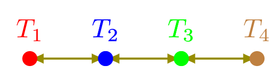
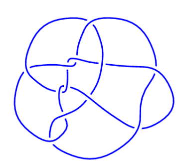
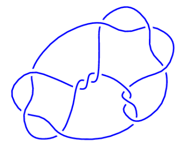
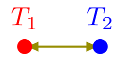
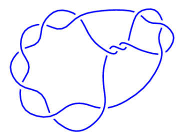
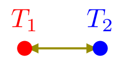
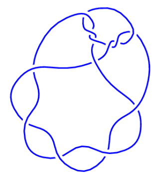
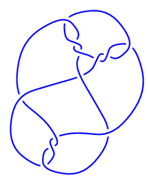
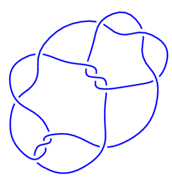
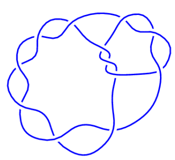
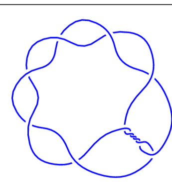
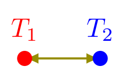
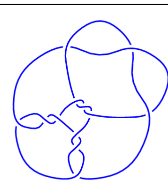
Knot	Diagram	Complex
12_{1265}		NOT FOUND
12_{1266}		NOT FOUND
12_{1267}		NOT FOUND
12_{1268}		NOT FOUND
12_{1269}		NOT FOUND
12_{1270}		NOT FOUND
12_{1271}		NOT FOUND
12_{1272}		NOT FOUND
12_{1273}		
12_{1274}		
12_{1275}		
12_{1276}		
12_{1277}		
12_{1278}		
12_{1279}		
12_{1280}		

Table 7.105. The Kakimizu complexes of 12 crossing alternating knots

Knot	Diagram	Complex
12_{1281}		
12_{1282}		
12_{1283}		
12_{1284}		NOT FOUND
12_{1285}		NOT FOUND
12_{1286}		NOT FOUND
12_{1287}		
12_{1288}		

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