Math 21B-Wesley Fall 2022 - Practice problems for Midterm 1
Name:

No calculators, notes, or other devices allowed. You do not need to simplify your answers unless specified, but you may wish to. Show all work for partial credit.

(1) Find the following indefinite integrals:

a) $\displaystyle\int \left(\frac{3}{x} - \frac{2}{x^3}\right) dx$

b) $\displaystyle\int (2x+1)e^{x^2+x+6}\, dx$

c) $\displaystyle\int \tan(3x)\, dx$

d) $\displaystyle\int \frac{3e^x - 8e^{-x}}{3e^x + 8e^{-x}}\, dx$

e) $\int (x+3)(x-5)^4\,dx$

f) $\int \frac{1}{\sqrt{9-x^2}}\,dx$

g) $\int \sqrt{\frac{x^4-1}{x^{14}}}\,dx$

h) $\int \frac{\tan(\frac{1}{x})\sec^2(\frac{1}{x})}{x^2}\,dx$

(2) Let $f(x) = 3x + 5$.

(a) Approximate the definite integral $\int_0^2 f(x)\,dx$ using an upper sum U_4 with 4 rectangles.

(b) Find a formula for the upper sum U_n(again over [0,2]) using n rectangles.

(c) Compute $\lim_{n\to\infty} U_n$.

(3) Rhaenyra is flying a dragon at a speed of 30 meters per second, but suddenly she is forced to decelerate with constant acceleration a. Four seconds later, she is 80 meters away. Find the acceleration a and the time it takes her to stop. Do not use any prior knowledge of kinematics equations from physics.

(4) Find the area in the first quadrant enclosed by the curves $y = x^2$, $y = 0$, and $y = 2 - x$ by integrating
   (a) With respect to x.
   (b) With respect to y.

(5) Evaluate the following definite integrals:

(a) $\int_0^{\frac{\pi}{2}} (\cos x + x^2)\, dx$

(b) $\int_{-2}^{2} (6x^{21} - \sin^3 x + x^2) dx$

(c) $\int_0^5 x\sqrt{x^2+4}\, dx$

(6) Evaluate the following expressions.

(a) $\sum_{k=1}^{100} k$

(b) $\dfrac{d}{dx}\displaystyle\int_4^{x^2+5} \ln(t)\,dt$

(c) $\displaystyle\int_{-1}^{1} (\sqrt{1-x^2} + |x|)\,dx$

(7) Find the volume of the *ellipsoid*, the solid obtained by revolving the region bounded by the curves $y = 4\sqrt{1 - \frac{x^2}{9}}$ and $y = 0$ about the $x-$axis. You do not have to simplify your final answer.

(8) Set up, but do not evaluate, an integral to compute the volume of the solid whose base is the region enclosed by the curves $y = \sqrt{x}$ and $y = x^2$ and whose cross-sections perpendicular to the x-axis are equilateral triangles.

(9) Set up, but do not evaluate, integrals to compute the volume of the solid obtained by revolving the region enclosed by $y = x^2$ and $y = 3x - x^2$ about the following lines:

(a) $y = 0$

(b) $y = -2$

(c) $y = 4$