Math 25 Problem Sheet 5

1) a) Show that if $\lim_{n\to\infty} s_n = \infty$, then $\lim_{n\to\infty} \frac{1}{s_n} = 0$.

b) Show that if $\lim_{n\to\infty} s_n = 0$ and $s_n > 0$ for all n, then $\lim_{n\to\infty} \frac{1}{s_n} = \infty$.

2) Define $\{s_n\}$ by $s_{n+1} = \sqrt{5 + s_n}$ for $n \geq 1$ and $s_1 = \sqrt{5}$.

a) Show that $\{s_n\}$ converges.

b) Find $\lim_{n\to\infty} s_n$.

3) Define $\{s_n\}$ by $s_{n+1} = \frac{1}{4}s_n + 15$ for $n \geq 1$ and $s_1 = 2$.

a) Show that $\{s_n\}$ converges.

b) Find $\lim_{n\to\infty} s_n$.

4) Use #2 on Problem sheet 4 and the identity $a^x = e^{x \ln a}$ to show the following:
If $\lim_{n\to\infty} s_n = s$ where $s_n > 0$ for all n and $s > 0$ and $\lim_{n\to\infty} t_n = t$, then $\lim_{n\to\infty} s_n^{t_n} = s^t$.

5) Show that every nonempty subset of $\mathbb{R}$ which is bounded below has a greatest lower bound as follows:

Let E be a nonempty subset of $\mathbb{R}$ which is bounded below, and let $L = \{l \in \mathbb{R} : \text{l is a lower bound for E}\}$.

a) Show that L is bounded above. Why can you conclude from this that $\sup L$ exists?

b) Show that $\inf E = \sup L$.